➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. The value of $\frac{(0.013)^3+(0.007)^3}{(0.013)^2-0.013\times0.007+(0.007)^2}$, is:

(A) 0.0091 (B) 0.006 (C) 0.00185 (D) 0.02

Ans. :

d. 0.02

Solution:

Assume a = 0.013 and v = 0.007.

Than the given expression can be rewritten as $\frac{a^3+b^3}{a^2-ab+b^2}$

Recall the formula for sum of two cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$Using the above formula, the expression becomes $\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}$

Note that both a and b are positive. So, neither $a^3 + b^3$ nor any factor of it can be zero.

Therefore we can cancel the term $(a^2 - ab + b^2)$ from both numerator and denominator. then the expression becomes

$\frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2} = a+b$

$= 0.013 + 0.007$

$= 0.02$

2. If p(x) = x + 4 then p(x) + p(-x) = ?

(A) 4 (B) 2x (C) 0 (D) 8

Ans. :

d. 8

Solution:

Let: p(x) = (x + 4)

∴ p(-x) = (x + 4)

= -x + 4

Thus, we have: p(x) + p(-x) ={(x + 4) + (-x + 4)}

= 4 + 4

= 8

3. If $x+\frac{1}{x}=3,$ then $x^6+\frac{1}{x^6}=$

(A) 927 (B) 322 (C) 414 (D) 364

Ans. :

b. 322

Solution:

On cubing we get.

$\left(x+\frac{1}{x}\right)^3 = x^3+\left(\frac{1}{x^3}\right)+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\Rightarrow 27 = x^3+\left(\frac{1}{x^3}\right)+3\times3$

$\Rightarrow x^3+\left(\frac{1}{x^3}\right)=27-9$

$\Rightarrow x^3+\left(\frac{1}{x^3}\right)=18$

Now, $\left(x^3+\frac{1}{x^3}\right)^2 = x^6+\left(\frac{1}{x^6}\right)+2\times x^3\times\frac{1}{x^3}$

$\Rightarrow 18^2 = x^6+\left(\frac{1}{x^6}\right)+2$

$x^6+\left(\frac{1}{x^6}\right)=324-2=322$

4. The value of K for which x - 1 is a factor of the polynomial $4x^3 + 3x^2 - 4x + k$.

(A) 3 (B) 0 (C) 1 (D) -3

Ans. :

d. -3

Solution:

$4x^3 + 3x^2 - 4x + k$

For given condition,

p(1) = 0

⇒ $4(1)^3 + 3(1)^2 - 4(1) + k = 0$

⇒ 4 + 3 - 4 + k = 0

⇒ k = -3

5. If x + y + z = 9 and xy + yz + zx = 23, the value of $(x^3 + y^3 + z^3 - 3xyz)$ = ?

(A) 108 (B) 207 (C) 669 (D) 729

Ans. :

a. 108

Solution:

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$

= 9 × (81 - 3 × 23)

= 9 × 12

= 108

6. If $x^3 - \frac{1}{x^3} = 14$ than $x - \frac{1}{x} =$

(A) 4 (B) 2 (C) 3 (D) 5

Ans. :

b. 2

Solution:

Given: $x^3 - \left(\frac{1}{x^3}\right) = 14$

Let x = a and $\frac{1}{x} = b$

Say, $x - \frac{1}{x} = A$

Then, $a^3 - b^3 = 14$

$\Rightarrow (a - b)(a^2 + ab + b^2) = 14$

$\Rightarrow (a - b)(\{(a - b)^2 + 2ab\} + 2ab) = 14$

$\Rightarrow (a - b)\{(a - b)^2 + 3ab\} = 14$

$\Rightarrow (a - b)\{(a - b)^2 + 3\} = 14$

$\Rightarrow A(A^2 + 3) = 14$

$\Rightarrow A(A^2 + 3) = 14$

$\Rightarrow A^3 + 3A - 14 = 0$

$\Rightarrow A^3 - 2A^2 + 2A^2 - 4A + 7A - 14 = 0$

$\Rightarrow A^2(A - 2) + 2Y(Y - 2) + 7(Y - 2) = 0$

$\Rightarrow (A - 2)(A^2 + 2A + 7) = 0$

$\Rightarrow A - 2 = 0,$

$\Rightarrow A = 2$

$\Rightarrow x - \frac{1}{x} = 2$

7. If $\frac{a}{b} + \frac{b}{a} = -1$ then $(a^3 - b^3)$ = ?

(A) -3 (B) -2 (C) -1 (D) 0

Ans. :

d. 0

Solution:

$\frac{a}{b} + \frac{b}{a} = -1$

$\Rightarrow \frac{a^2+b^2}{ab} = -1$

⇒ $a^2 + b^2 = -ab$

⇒ $a^2 + b^2 + ab = 0$

Thus, we have:

$(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$

$= (a - b) \times 0$
$= 0$

8. Write the correct answer in the following:
The value of $249^2 - 248^2$ is.
(A) 1^2 (B) 477 (C) 487 (D) 497

Ans. :
d. 497
Solution:
$(249)^2 - (248)^2 = (249 + 248)(249 - 248)$ $[(a)^2 - (b)^2 = (a + b)(a - b)]$
$= (497)(1) = 497$

9. If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $x + 2$ then $k =$
(A) -8 (B) -7 (C) -6 (D) -10

Ans. :
b. -8
Solution:
-8
$f(x) = x^3 + 6x^2 + 4x + k$
$f(-2) = 0$
$\therefore (-2)^3 + 6(-2)^2 + 4(-2) + k = 0$
$\therefore -8 + 6(4) + (-8) + k = 0$
$24 - 16 + k = 0$
$k + 8 = 0$
$k = -8$

10. When $p(x) = x^3 - ax^2 + x$ is divided by $(x - a)$, the remainder is:
(A) a (B) 0 (C) 3a (D) 2a

Ans. :
a. a
Solution:
By remainder theorem, when $p(x) = x^3 - ax^2 + x$ is divided by $(x - a)$, then the remainder = p(a)
Putting x = a in p(x), we get
$p(a) = a^3 - a \times a^2 + a = {}^3 - a^3 + a = a$
$\therefore$ Remainder = a

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

11. Factorize:
$x^2 + 5\sqrt{5}x + 30$

Ans. : $x^2 + 5\sqrt{5}x + 30$ Splitting the middle term, $= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$ $[\therefore 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} \text{ also } 2\sqrt{5} \times 3\sqrt{5} = 30]$ $= x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$ $= (x + 2\sqrt{5})(x + 3\sqrt{5})$ $\therefore x^2 + 5\sqrt{5}x + 30 = (x + 2\sqrt{5})(x + 3\sqrt{5})$

12. Give the possible expression for the length & breadth of the rectangle having $35y^2 - 13y - 12$ as its area.

Ans. : Area is given as $35y^2 - 13y - 12$ Splitting the middle term, Area = $35y^2 + 218y - 15y - 12 = 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$ We also know that area of rectangle = length × breadth $\therefore$ Possible length = (5y + 4) and breadth = (7y - 3) Or possible length = (7y - 3) and breadth = (5y + 4)

13. In the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not:
$f(x) = x^3 - 6x^2 - 19x + 84$, $g(x) = x - 7$

Ans. : Let $g(x) = 0 \Rightarrow x - 7 = 0 \Rightarrow x = 7$ $f(7) = 7^3 - 6(7)^2 - 19(7) + 84 = 343 - 6(49) - 19(7) + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0$ $\because f(7) = 0$, by factor theorem x - 7 is a factor of f(x).

14. If $x = \frac{1}{2}$ is a zero of the polynomial $f(x) = 8x^3 + ax^2 - 4x + 2$, find the value of a.

Ans. : Since $x = \frac{1}{2}$ is a zero of polynomial f(x). Therefore $f\left(\frac{1}{2}\right) = 0$ $\Rightarrow 8\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2 = 0$ $\Rightarrow 1 + \frac{a}{4} - 2 + 2 = 0$ $\Rightarrow a = -4$ The value of a is -4.

➤ **Answer the following questions. [3 Marks Each]** **[12]**

15. Multiply:

$(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$ by $(3x - 5y + 4)$

Ans. : $= (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 20y - 12x + 16) = (3x + (5y) + 4)\{(3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)4 - 4(3x)\}$ $[\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$ Here, a = 3x, b = -5y, c = 4 $= (3x)^3 + (-5y)^3 + 4^3 - 3(3x)(-5y)(4) = 27x^3 - 125y^3 + 64 + 180xy$ $\therefore (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 20y - 12x + 16) = 27x^3 - 125y^3 + 64 + 180xy$

16. Simplify:

$\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$

Ans. : $\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127} = \frac{173^3+127^3}{173^2-173\times127+127^2} = \frac{(173+127)(173^2-173\times127+127^2)}{173^2-173\times127+127^2}$ $[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$ = (173 + 127) = 300

17. Factorize:

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Ans. : $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$ Splitting the middle term, $= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}]$ $= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$ $= (\sqrt{5}x + 3)(5x + \sqrt{5})$ $\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$

18. Find the value of $x^3 + y^3 - 12xy + 64$, when x + y = -4

Ans. : $\because$ x + y = -4 $\therefore$ x + y + 4 = 0 ...(1) Now, $x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 64 - 12xy = (x)^3 + y^3 + 4^3 - 3 \times x \times y \times 4 = (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x) = 0(x^2 + y^2 + 16 - xy - 4y - 4x)$ [from(1)] = 0 $\therefore x^3 + y^3 - 12xy + 64 = 0$ when x + y = -4

----- -----