STD 9 Maths
Polynomials (DPP-2)

Total Marks : 30

➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. The zeros of the polynomial p(x) = $2x^2$ + 5x - 3 are.

(A) $1, \frac{-1}{2}$ (B) $\frac{-1}{2}, 3$ (C) $\frac{1}{2}, -3$ (D) $\frac{1}{2}, 3$

Ans. :

c. $\frac{1}{2}, -3$

Solution:

The given polynomial is p(x) = $2x^2$ + 5x - 4

Putting $x = \frac{1}{2}$ in p(x), we get

$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} - 3$

$= \frac{1}{2} + \frac{5}{2} - 3 = 3 - 3 = 0$

Putting x = -3 in p(x), we get

$p(-3) = 2 \times (-3)^2 + 5 \times (-3) - 3$

$= 18 - 15 - 3$

$= 0$

Therefore, x = -3 is a zero of the polynomial p(x)

Thus, $\frac{1}{2}$ and -3 are the zeros of the given polynomial p(x).

2. The value of $\frac{(0.87)^3+(0.13)^3}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$ is:

(A) 0 (B) 0.13 (C) 0.87 (D) 1

Ans. :

d. 1

Solution:

$\frac{(0.87)^3+(0.13)^3}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$

$= \frac{(0.87+0.13)[(0.87)^2-(0.87\times0.13)+(0.13)^2]}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$

$= 0.87 + 0.13$

$= 1$

3. The value of $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$ is:

(A) (x^4+y^4) (B) (x^4-y^4) (C) $(x+y)^4$ (D) $(x-y)^4$

Ans. :

b. (x^4-y^4)

Solution:

$(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

$[(\sqrt{x})^2-(\sqrt{y})^2](x+y)(x^2+y^2)$

$= (x-y)(x+y)(x^2+y^2)$

$= [(x)^2-(y)^2](x^2+y^2)$

$= (x^2-y^2)(x^2+y^2)$

$= (x^2)^2-(y^2)^2$

$= x^2-y^2$

4. If p(x) = 5x - $4x^2$ + 3 then p(-1) = ?

(A) 6 (B) -2 (C) 2 (D) -6

Ans. :

d. -6

Solution:

p(x) = 5x - $4x^2$ + 3

Putting x = -1 in p(x), we get

p(-1) = 5 × (-1)-4 × $(-1)^2$ + 3 = -5 - 4 + 3 = -6

5. If x - 2 is a factor of x^2 + 3ax - 2a, then a =

(A) 1 (B) -2 (C) 2 (D) -1

Ans. :

a. -1

Solution:

As (x - 2) is a factor of f(x) = x^2 + 3ax - 2a

i.e., f(2) = 0

$(2)^2$ + 3a(2) - 2a = 0

4 + 6a - 2a = 0

= -1

6. If $49a^2 - b = \left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right)$, then the value of b is:

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Ans. :

b. $\frac{1}{4}$

Solution:

$\left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right) = (7a)^2 - \left(\frac{1}{2}\right)^2$

[by using identity (a + b)(a - b) = a^2 - b^2]

$\Rightarrow \left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right) = 49a^2 - \frac{1}{4}$

$\Rightarrow 49a^2 - b = 49a^2 - \frac{1}{4}$

$\Rightarrow b = \frac{1}{4}$

Hence, correct option is (b).

7. If both x - a and $x - \frac{1}{2}$ are the factors of $px^2 + 5x + r,$ than:

(A) p = r (B) 2p = r (C) p = 2r (D) None of these.

Ans. :

a. p = r

Solution:

If both x - a and $x - \frac{1}{2}$ are the factors of $px^2 + 5x + r,$ than f(2) = 0

$\Rightarrow p(2)^2 + 5(2) + r = 0$

$\Rightarrow 4p + 10 + r = 0$

$\Rightarrow 4p + r = -10 \ldots (i)$

Also, $f\left(\frac{1}{2}\right) = 0$

$\Rightarrow p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$\Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0$

$\Rightarrow p + 10 + 4r = 0$

$\Rightarrow p + 4r = -10 \ldots ii$

From eq. (i) and eq. (ii), we get

$4p + r = p + 4r$

$\Rightarrow 3p = 3r$

$\Rightarrow p = r$

8. The expression $(a - b)^3 + (b - c)^3 + (c - a)^3$ can be factorized as:

(A) (a - b)(b - c)(c - a) (B) 3(a - b)(b - c)(c - a) (C) -3(a - b)(b - c)(c - a) (D) (a + b + c)($a^2 + b^2 + c^2$ - ab - bc - ca)

Ans. :

b. 3(a - b)(b - c)(c - a)

Solution:

By we know that $a^3 + b^3 + c^3$ - 3abc = (a + b + c)($a^2 + b^2 + c^2$ - ab - bc - ca)

If a + b + c = 0, then

$a^3 + b^3 + c^3$ = 3abc

In given expression,

Let a - b = A, b - c = B, c - a = C

Now, a - b + b - c + c - a = 0

i.e. A + B + C = 0

$\Rightarrow A^3 + B^3 + C^3 = 3ABC$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Hence, correct option is (b).

9. The factors of $a^2 - 1 - 2x - x^2$, are.

(A) (a + x + 1) (a - x - 1) (B) (a - x + 1) (a - x - 1) (C) (a + x - 1) (a - x + 1) (D) None of these.

Ans. :

a. (a + x + 1) (a - x - 1)

Solution:

The given expression to be factorized is $a^2 - 1 - 2x - x^2$

Take common -1 from the last three terms and then we have

$a^2 - 1 - 2x - x^2 = a^2 - (1 + 2x + x^3)$

$= a^2 - \{(1)^2 + 2.1 \times x + (x)^2\}$

$= a^2 - (1 + x)^2$

$= (a)^2 - (1 + x)^2$

$= \{a + (1 + x)\} \{a - (1 + x)\}$

$= (a + 1 + x) (a - 1 - x)$

$= (a + x + 1) (a - x - 1)$

10. The zeroes of the polynomial p(x) = x(x - 2) (x + 3) are:

(A) 0, 2, -4 (B) 0, 2, 4 (C) 0, 2, -3 (D) 0

Ans. :

c. 0, 2, -3

Solution:

p(x) = x(x - 2) (x + 3)

$\Rightarrow$ x = 0 and x - 2 = 0 and x + 3 = 0

$\Rightarrow$ x = 0, x = 2 and x = -3

Therefore, the zeroes are 0, 2, -3

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

11. If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find a + b + c.

Ans. : Recall the formula $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ Given that $a^2 + b^2 + c^2 = 250$, $ab + bc + ca = 3$ Then we have $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ $(a + b + c)^2 = 250 + 2.(3)$ $(a + b + c)^2 = 256$ $(a + b + c) = \pm 16$

12. Factorize:

$x^2 - 2\sqrt{2}x - 30$

Ans. : $x^2 - 2\sqrt{2}x - 30$ Splitting the middle term, $= x^2 = 5\sqrt{2}x + 3\sqrt{2}x - 30$ $[\therefore -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2} \text{ also } -5\sqrt{2} \times 3\sqrt{2} = -30]$ $= x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$ $= (x - 5\sqrt{2})(x + 3\sqrt{2})$ $\therefore x^2 - 2\sqrt{2}x - 30 = (x - 5\sqrt{2})(x + 3\sqrt{2})$

13. Verify that:

2 and -3 are the zeros of the polynomial $q(x) = x^2 + x - 6$.

Ans. : $q(x) = x^2 + x - 6 \Rightarrow q(2) = 2^2 + 2 - 6 = 4 - 4 = 0$ Also, $q(-3) = (-3)^2 + (-3) - 6 = 9 - 9 = 0$ Hence, 2 and -3 are the zeroes of the given polynomial.

14. Find the value of a for which (x - 4) is a factor of $(2x^3 - 3x^2 - 18x + a)$.

Ans. : $f(x) = (2x^3 - 3x^2 - 18x + a)$ x - 4 = 0 $\Rightarrow$ x = 4 $\therefore f(4) = 2(4)^3 - 3(4)^2 - 18 \times 4 + a = 128 - 48 - 72 + a = 128 - 120 + a = 8 + a$ Given that (x - 4) is a factor of f(x). By the Factor Theorem, (x - a) will be a factor of f(x) if f(a) = 0 and therefore f(4) = 0. $\Rightarrow f(4) = 8 + a = 0 \Rightarrow a = -8$

➤ **Answer the following questions. [3 Marks Each]** **[12]**

15. Using factor theorem, show that g(x) is a factor of p(x), when

$p(x) = 2x^4 + x^3 - 8x^2 - x + 6$, $g(x) = 2x - 3$

Ans. : f(x) = ($2x^4 + x^3 - 8x^2 - x + 6$) By the Factor Theorem, (x - a) will be a factor of f(x) if f(a) = 0. Here, 2x - 3 = 0 $x = \frac{3}{2}$ $f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$ $= 2 \times \frac{81}{16} + \frac{27}{8} - 8 \times \frac{9}{4} - \frac{3}{2} + 6$ $= \frac{81}{8} + \frac{27}{8} - 18 - \frac{3}{2} + 6$ $= \frac{81+27-144-12+48}{8}$ $= \frac{156-156}{8} = 0$ $\therefore (2x - 3)$ is a factor of $\left(2x^4 + x^3 - 8x^2 - x + 6\right)$.

16. Using the remainder theorem, find the remainder, when p(x) is divided by g(x), where,
p(x) = $x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$.

Ans. : p(x) = $x^3 - 6x^2 + 2x - 4$ $g(x) = 1 - \frac{3}{2}x = -\frac{3}{2}\left(x - \frac{2}{3}\right)$ By remainder theorem, when p(x) is divided by $\left(1 - \frac{3}{2}x\right)$, then the remainder $= p\left(\frac{2}{3}\right)$. Putting $x = \frac{2}{3}$ in p(x), we get $p\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 6 \times \left(\frac{2}{3}\right)^2 + 2 \times \left(\frac{2}{3}\right) - 4$ $= \frac{8}{27} - \frac{8}{3} + \frac{4}{3} - 4$ $= \frac{8-72+36-108}{27} = -\frac{136}{27}$ $\therefore$ Remainder $= -\frac{136}{27}$ Thus, the remainder when p(x) is divided by g(x) is $= -\frac{136}{27}$.

17. The polynomials ($2x^3 + x^2 - ax + 2$) and ($2x^3 - 3x^2 - 3x + a$) when divided by (x - 2) leave the same remainder. Find the value of a.

Ans. : Let f(x) = $2x^3 + x^2 - ax + 2$ g(x) = $2x^3 - 3x^2 - 3x + a$. By remainder theorem, when f(x) is divided by (x - 2), then the remainder = f(2). Putting x = 2 in f(x), we get f(2) = $2 \times 2^3 + 2^2 - a \times 2 + 2$ = 16 + 4 - 2a + 2 = -2a + 22 By remainder theorem, when g(x) is divided by (x - 2), then the remainder = g(2). g(2) = $2 \times 2^3 - 3 \times 2^2 - 3 \times 2 + a$ = 16 - 12 - 6 + a = -2 + a It is given that, ⇒ -2a + 22 = -2 + a ⇒ -3a = -24 ⇒ a = 8 Thus, the value of a is 8.

18. Using the remainder theorem, find the remainder, when p(x) is divided by g(x), where,
p(x) = $2x^3 + x^2 - 15x - 12$, g(x) = x + 2.

Ans. : p(x) = $2x^3 + x^2 - 15x - 12$ g(x) = x + 2 by remainder theorem, when p(x) is divided by (x + 2), then the remainder = p(-2). Putting x = -2 in p(x), we get p(-2) = $(-2)^3 + (-2)^2 - 15 \times (-2) - 12$ = -16 + 4 + 30 - 12 = 6 $\therefore$ Remainder = 6 Thus, the remainder when p(x) is divided by g(x) is 6.

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