➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is:

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) -2 (D) 2

Ans. :

a. $\frac{1}{2}$

Solution:

$2x^2 + 7x - 4$

$= 2x^2 + 8x - x - 4$

$= 2x(x + 4) - 1(x + 4)$

$= (2x - 1)(x + 4)$

$2x - 1 = 0$ and $x + 4 = 0$

$x = \frac{1}{2}$ and $x = -4$

Therefore, one zero of the given polynomial is $\frac{1}{2}$

2. Write the correct answer in the following:
Which one of the following is a polynomial?

(A) $\frac{x^2}{2} - \frac{2}{x^2}$ (B) $\sqrt{2x - 1}$ (C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (D) $\frac{x-1}{x+1}$

Ans. :

c. $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

Solution:

a. Now, $\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$, it is not a polynomial, because exponent of x is -2 which is not a whole number.

b. Now, $\sqrt{2x - 1} = \sqrt{2x^{\frac{1}{2}}} - 1$, it is not a polynomial, because exponent of x is $-\frac{1}{2}$ which is not a whole number.

c. Now, $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}} = x^2 + 3x^{\frac{3}{2} - \frac{1}{2}} = x^2 + 3x^{\frac{2}{2}} = x^2 + 3x$, it is not a polynomial, because exponent of x is which is a whole number.

d. $\frac{x-1}{x+1}$, it is not a polynomial because it is a rational function.

3. If (x + 1) and (x - 1) are factors of $px^3 + x^2 - 2x + p$ then value of p and q are.

(A) p = -1, q = 2 (B) p = 2, q = -1 (C) p = 2, q = 1 (D) p = -2, q = -2

Ans. :

b. p = 2, q = -1

Solution:

Given: $f(x) = px^3 + x^2 - 2x + q$

If x + 1 is a factor of f(x).

Then f(-1) = 0

$p(-1)^3 + (-1)^2 - 2(1) + q = 0$

$-p + 1 + 2 + q = 0$

$-p + q = -3$

$p - q = 3$ ......(i)

Also, if x - 1 is a factor of f(x), then

$p(1)^3 + (1)^2 - 2(1) + q = 0$

$p + 1 - 2 + q = 0$

$p + q = 1$ ......(ii)

$2p = 4$

$p = 2$

subtracting eq.(ii) from eq. (i),

we get,

$-2q = 2$

$q = -1$
$q = -1$
Therefore, $p = 2$, $q = -1$

4. If $p(x) = 5x - 4x^2 + 3$ then $p(-1) = ?$

(A) 2 (B) -2 (C) 6 (D) -6

Ans. :

d. -6

Solution:

$P(x) = 5x - 4x^2 + 3$
$\Rightarrow p(-1) = 5(-1) - 4(-1)^2 + 3$
$= -5 - 4 + 3$
$= -6$

5. Write the correct answer in the following:

If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of k is.

(A) -3 (B) 4 (C) 2 (D) -2

Ans. :

c. 2

Solution:

Let $p(x) = 2x^2 + kx$
Since, $(x + 1)$ is a factor of $p(x)$, then
$p(-1) = 0$
$2(-1)^2 + k(-1) = 0$
$\Rightarrow 2 - k = 0$
$\Rightarrow k = 2$

6. The Zero of the polynomial $(x - 2)^2 - (x + 2)^2$ is:

(A) 1 (B) -2 (C) 0 (D) 2

Ans. :

c. 0

Solution:

$(x - 2)^2 - (x + 2)^2$
$= (x - 2 + x + 2) (x - 2 - x - 2)$ [Using identity $a^2 - b^2 = (a + b) (a - b)$]
$= (2x) (-4)$
$= -8x$
Then the zero is,
$-8 = 0$
$\Rightarrow x = 0$

7. If $x + 2$ and $x - 1$ are the factor of $x^3 + 10x^2 + mx + n$, then the values of m and n are respectively.

(A) 5 and -3 (B) 7 and -18 (C) 23 and -19 (D) 17 and -8

Ans. :

b. 7 and -18

Solution:

It is given $(x + 2)$ and $(x - 1)$ are the factors of the polynomial $f(x)= x^3 + 10x^2 + mx + n$
i.e., $f(-2) = 0$ and $f(1) = 0$
New,
$f(-2) = (-2)^3 + 10(-2)^2 + m(-2) + n = 0$
$-8 + 40 - 2m + n =0$
$\Rightarrow -2m + n = -32$
$\Rightarrow 2m - n = 32$ ....(i)
$f(1) = (1)^3 + 10(1)^2 + m(1) + n = 0$
$1 + 10 + m + n = 0$
$m + n = -11$ .....(ii)
Solving equation (i) and (ii) we get,
$m = 7$ and $n = -18$

8. If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then k =

(A) 1 (B) 2 (C) 4 (D) 8

Ans. :

d. 8

Solution:

Let x + y = A and x - y = B

Now, $(A - B)^3 = A^3 - B^3 - 3AB(A - B)$

$\Rightarrow [(x + y) - (x - y)]^3 = (x + y)^3 - (x - y)^3 - 3(x + y)(x - y)[(x + y) - (x - y)]$

$= (x + y)^3 - (x - y)^3 - 3(x^2 - y^2)(2y)$

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

But, $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$

$\Rightarrow [(x + y) - (x - y)]^3 = (2y)^3 = k8y^3$

$\Rightarrow (2y)^3 = ky^3$

$\Rightarrow 8y^3 = ky^3$

$\Rightarrow k = 8$

Hence, correct option is (d).

9. The value of the polynomial $5x - 4x^2 + 3$, when x = -1 is:

(A) 6 (B) -6 (C) 1 (D) -1

Ans. :

b. -6

Solution:

$5x - 4x^2 + 3$

$= -4x^2 + 5x + 3$

Putting x = -1 in the given polynomial, we get

$-4(-1)^2 + 5(-1) + 3$

= -4 - 5 + 3

= -9 + 3

= -6

10. If $x + \frac{1}{x} = 3$, then $x^6 + \frac{1}{x^6} =$

(A) 927 (B) 414 (C) 364 (D) 322

Ans. :

d. 322

Solution:

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$x + \frac{1}{x} = 3$ (given)

$\Rightarrow x^2 + \frac{1}{x^2} = (3)^2 - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 7 \ldots (1)$

Cubing both side of equation (1). we have

$\left(x^2 + \frac{1}{x^2}\right)^3 = (7)^3$

$\Rightarrow (x^2)^3 + \left(\frac{1}{x^2}\right)^3 + 3(x^2)\frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right) = 7^3$

$\Rightarrow x^6 + \frac{1}{x^6} + 3(7) = 7^3$

$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 21$

$\Rightarrow x^6 + \frac{1}{x^6} = 322$

Hence, correct option is (d).

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

11. Factorize:

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Ans. : $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

The expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ can also be written as

$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2$
$\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$.

We can observe that, we can apply the identity $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$ with respect to the expression

$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2$,to get
$\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$

$\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ we get $\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$.

12. Write the cube in expanded form : $\left(x-\frac{2}{3}y\right)^3$

Ans. : $\left(x-\frac{2}{3}y\right)^3$ = $x^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

(Using Identity $(a - b)^3 = a^3 - b^3 - 3ab (a - b)$)

= $x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$ = $x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

= $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$

13. Factorise : $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$.

Ans. : $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

= $(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)\left(\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

= $\left(3p-\frac{1}{6}\right)^3$

(Using Identity $(a - b)^3 = a^3 - b^3 - 3ab (a - b)$)

= $\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

14. Verify : $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Ans. : We know that

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ {Using Identity $(a + b)^3 = a^3 + b^3 + 3ab (a + b)$}

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)\{(x + y)^2 - 3xy\}$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$ {Using Identity $(a + b)^2 = a^2 + 2ab + b^2$ }

$\Rightarrow x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

➤ **Answer the following questions. [3 Marks Each]** **[12]**

15. Use suitable identity to find the product:
$(x+4)(x+10)$

Ans. : $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$

Here a = 4 and b = 10

We need to apply the above identity to find the product

$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$

$=x^2+14x+40.$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2+14x+40$

16. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$.

Ans. : L.H.S = $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
(Using Identity $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$)
$= \frac{1}{2}(x + y + z)\{2(x^2 + y^2 + z^2 - xy - yz - zx)\}$ (Multiplying and Dividing by 2)
$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$
$= \frac{1}{2}(x + y + z)\{(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2)\}$
$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$. (Using Identity $(a - b)^2 = a^2 - 2ab + b^2$)

17. Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.

Ans. : The zero of $x + 2$ is -2.
Let $p(x) = x^3 + 3x^2 + 5x + 6$ and $s(x) = 2x + 4$
Then, $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$
$= -8 + 12 - 10 + 6$
$= 0$
So, by the Factor Theorem, $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$.
Again, $s(-2) = 2(-2) + 4 = 0$
So, $x + 2$ is a factor of $2x + 4$.

18. Factorise : $x^3 - 23x^2 + 142x - 120$

Ans. : Let $p(x) = x^3 - 23x^2 + 142x - 120$
We shall now look for all the factors of -120. Some of these are $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm8, \pm10, \pm12, \pm15, \pm20, \pm24, \pm30, \pm60$.
By hit and trial, we find that $p(1) = 0$. Therefore, $x - 1$ is a factor of $p(x)$.
Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$
$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$
$= (x - 1)(x^2 - 22x + 120)$ [Taking $(x - 1)$ common]
Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have: $x^2 - 22x + 120 = x^2 - 12x - 10x + 120$
$= x(x - 12) - 10(x - 12)$
$= (x - 12)(x - 10)$
Therefore, $x^3 - 23x^2 - 142x - 120 = (x - 1)(x - 10)(x - 12)$

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