Total Marks : 30

➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. If a + b + c = 0 then $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)=?$

(A) 1 (B) 0 (C) -1 (D) 3

Ans. :

d. 3

Solution:

a + b + c = 0 ⇒ $a^3 + b^3 + c^3 = 3abc$

Thus, we have:

$\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)=\frac{a^3+b^3+c^3}{abc}$

$=\frac{3abc}{abc}$

$=3$

2. The factors of a^2 - 1 - 2x - x^2 are:

(A) (a - x + 1)(a - x - 1) (B) (a + x - 1)(a - x + 1) (C) (a + x +1)(a - x + 1) (D) None of these.

Ans. :

c. (a + x +1)(a - x + 1)

Solution:

$a^2 - 1 - 2x - x^2$

$= a^2 - (1 + 2x + x^2)$

$= a^2 - (1 + x)^2$

= [a - (1 + x)][a + (1 + x)]

= (a - x - 1)(a + x + 1)

Hence, correct option is (c).

3. $(4x^2 + 4x - 3)$ = ?

(A) (2x - 1) (2x - 3) (B) None of these. (C) (2x + 3) (2x - 1) (D) (2x + 1) (2x - 3)

Ans. :

c. (2x + 3) (2x - 1)

Solution:

$(4x^2) + (4x - 3) = 4x^2 + 6x - 2x - 3$

= 2x(2x + 3) - 1(2x + 3)

= (2x + 3) (2x - 1)

4. If a + b + c = 9 and ab + bc + ca = 23, than $a^3 + b^3 + c^3 - 3abc$ =

(A) 729 (B) 207 (C) 669 (D) 108

Ans. :

d. 108

Solution:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

⇒ $(9)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

⇒ $(9)^2 = a^2 + b^2 + c^2 + 2(23)$

⇒ $a^2 + b^2 + c^2 = 81 - 46 = 35$

as we know that $a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

⇒ $a^3 + b^3 + c^3 - 3abc = 9 \times (35 - 23)$

⇒ $a^3 + b^3 + c^3 - 3abc = 108$

5. The zero of the polynomial p(x) = 5x - 2 is:

(A) $\frac{-2}{5}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{-5}{2}$

Ans. :

b. $\frac{2}{5}$

Solution:

p(x) = 5x - 2

To find of the polynomial, we write 5x - 2 = 0

$\Rightarrow 5x = 2$

$\Rightarrow x = \frac{2}{5}$

6. Which of the following is a polynomial in one variable?

(A) $x^2 + x^{-2}$ (B) $\sqrt{2} - x^2 + 3x$ (C) $\sqrt{2x} + 9$ (D) $x^2 + y^8 + 9$

Ans. :

b. $\sqrt{2} - x^2 + 3x$

Solution:

$\sqrt{2} - x^2 + 3x$ is a polynomial in one variable x and also the powers of each term is a whole number.

7. The factorization of $9x^2$ - 3x - 20 is:

(A) (3x - 4) (3x - 5) (B) (3x + 4) (3x - 5) (C) (3x + 4) (3x + 5) (D) (3x - 4) (3x + 5)

Ans. :

b. (3x + 4) (3x - 5)

Solution:

$9x^2$ - 3x - 20

= $9x^2$ - 15x + 12x - 20

= 3x(3x - 5) + 4(3x - 5)

= (3x + 4) (3x - 5)

8. If $x^4 + \frac{1}{x^4} = 194,$ then $x^3 + \frac{1}{x^3} =$

(A) 76 (B) 52 (C) 64 (D) None of these

Ans. :

b. 52

Solution:

$x^4 + \frac{1}{x^4} = 194$

Now $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 194 + 2 = 196$

$\Rightarrow x^2 + \frac{1}{x^2} = 14 \ldots (1)$

Now $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \quad \left\{x^2 + \frac{1}{x^2} = 14\right\}$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 14 + 2 = 16$ [From (1)]

$\Rightarrow x + \frac{1}{x} = \sqrt{16}$

$\Rightarrow x + \frac{1}{x} = 4 \ldots (3)$

By identity $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

$= (4)(14 - 1)$

$= 4 \times 13$

$= 52$

Hence, correct option is (b).

9. If $3x + \frac{2}{x} = 7,$ then $\left(9x^2 - \frac{4}{x^2}\right) =$

(A) 25 (B) 35 (C) 49 (D) 30

Ans. :

b. 35

Solution:

$\left(3x+\frac{2}{x}\right)^2=9x^2+\frac{4}{x^2}+12\ldots(1)$

$\left(3x-\frac{2}{x}\right)^2=9x^2+\frac{4}{x^2}-12\ldots(2)$

Subtracting eq. (1) from eq. (2). we get

$\left(3x-\frac{2}{x}\right)^2-\left(3x+\frac{2}{x}\right)^2=-24$

$\Rightarrow\left(3x-\frac{2}{x}\right)^2=(7)^2-24=25$

$\Rightarrow 3x-\frac{2}{x}=5$

Now $\left(3x+\frac{2}{x}\right)-\left(3x-\frac{2}{x}\right)=7\times 5$

$\left(9x^2-\frac{4}{x^2}\right)=35$

Hence, correct option is (b).

10. For what value of k is the polynomial p(x) = $2x^3 - kx^2 + 3x + 10$ exactly divisible by (x + 2)?

(A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ (C) 3 (D) -3

Ans. :

d. -3

Solution:

p(x) = $2x^3 - kx^2 + 3x + 10$

x + 2 = 0 ⇒ x = -2

By the factor theorem, we know that when p(x) is divided by (x + 2), the remainder is p(-2).

Now, p(-2) = $2(-2)^3 + k(-2)^2 + 3(-2) + 10$

⇒ 0 = -16 - 4k - 6 + 10

⇒ 0 = -12 - 4k

⇒ 4k = -12

⇒ k = -3

11. If $p(x)=x^2-2\sqrt{2}x+1$ then $p(2\sqrt{2})=?$

a. 0

b. 1

c. $4\sqrt{2}$

d. -1

Ans. :

b. 1

Solution:

$p(x)=x^2-2\sqrt{2}x+1$

$p(2\sqrt{2})=(2\sqrt{2})^2-2\sqrt{2}(2\sqrt{2})+1$

$=8-8+1$

$=1$

12. The zeros of the polynomial p(x) = $x^2 + x - 6$ are:

a. 2, 3

b. -2, 3

c. 2, -3

d. -2, -3

Ans. :

c. 2, -3

Solution:

Let p(x) be a polynomial. If $p(\alpha)=0,$ then we say that α is a zero of a polynomial.

p(x) = $x^2 + x - 6$

Now, p(x) = 0

⇒ $x^2 + x - 6$

⇒ $x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x + 3) - 2(x + 3) = 0$
$\Rightarrow (x - 2)(x + 3) = 0$
$\Rightarrow (x - 2) = 0$ or $(x + 3) = 0$
$\Rightarrow x = 2$ or $x = -3$
$\therefore$ 2 and -3 are the zeroes of the polynomial p(x).

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

13. If $p(x) = 5 - 4x + 2x^2$, find:
    i. p(0)
    ii. p(3)
    iii. p(-2)

Ans. :

i. $p(x) = 5 - 4x + 2x^2$
$\Rightarrow p(0) = (5 - 4 \times 0 + 2 \times 0^2)$
$= (5 - 0 + 0)$
$= 5$

ii. $p(x) = 5 - 4x + 2x^2$
$p(3) = (5 - 4 \times 3 + 2 \times 3^2)$
$= (5 - 12 + 18)$
$= 11$

iii. $p(x) = 5 - 4x + 2x^2$
$\Rightarrow p(-2) = [(5 - 4 \times (-2) + 2 \times (-2)^2]$
$= (5 + 8 + 8)$
$= 21$

14. Using factor theorem, show that g(x) is a factor of p(x), when
$p(x) = 2x^4 + x^3 - 8x^2 - x + 6$, $g(x) = 2x - 3$

Ans. : $f(x) = (2x^4 + x^3 - 8x^2 - x + 6)$ By the Factor Theorem, (x - a) will be a factor of f(x) if f(a) = 0. Here, 2x - 3 = 0 $x = \frac{3}{2}$ $f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$ $= 2 \times \frac{81}{16} + \frac{27}{8} - 8 \times \frac{9}{4} - \frac{3}{2} + 6$ $= \frac{81}{8} + \frac{27}{8} - 18 - \frac{3}{2} + 6$ $= \frac{81+27-144-12+48}{8}$ $= \frac{156-156}{8} = 0$ $\therefore (2x - 3)$ is a factor of $(2x^4 + x^3 - 8x^2 - x + 6)$.

15. Evaluate:
$(28)^3 + (-15)^3 + (-13)^3$

Ans. : $(28)^3 + (-15)^3 + (-13)^3$ We know: $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ $x^3 + y^3 + z^3 = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$ Here, x = (-28), y = -15, z = -13 $(28)^3 + (-15)^3 + (-13)^3 = (28 - 15 - 13)[(28)^2 + (-15)^2 + (-13)^2 - 28(-15) - (-15)(-13) - 28(-13)] + 3 \times 28(-15)(-13) = 0 + 16380 = 16380$

16. Factorise:
$6\left(2x - \frac{3}{x}\right)^2 + 7\left(2x - \frac{3}{x}\right) - 20$

Ans. : Given equation: $6\left(2x - \frac{3}{x}\right)^2 + 7\left(2x - \frac{3}{x}\right) - 20$ Let $2x - \frac{3}{x} = a$ Then, we have $= 6a^2 + 7a - 20 = 6a^2 + 15a - 8a - 20 = 3a(2a + 5) - 4(2a + 5) = (2a + 5)(3a - 4)$ $= \left[2\left(2x - \frac{3}{x}\right) + 5\right]\left[3\left(2x - \frac{3}{x}\right) - 4\right]$ $= \left(4x - \frac{6}{x} + 5\right)\left(6x - \frac{9}{x} - 4\right)$

➤ **Answer the following questions. [3 Marks Each]** **[12]**

17. If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$.

Ans. : In the given problem, we have to find the value of $x^2+\frac{1}{x^2},\ x^3+\frac{1}{x^3},\ x^4+\frac{1}{x^4}$ Given $x+\frac{1}{x}=3,$ We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$ Here putting $x+\frac{1}{x}=3,$ $\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$ $(3)^2=x^2+\frac{1}{x^2}+2\times\not{x}\times\frac{1}{\not{x}}$ $9=x^2+\frac{1}{x^2}+2$ $9-2=x^2+\frac{1}{x^2}$ $7=x^2+\frac{1}{x^2}$

Again squaring on both sides we get, $\left(x^2+\frac{1}{x^2}\right)^2=(7)^2$ We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$ $\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$ $(7)^2=x^4+\frac{1}{x^4}+2\times\not{x}^2\times\frac{1}{\not{x}^2}$ $49=x^4+\frac{1}{x^4}+2$

$49-2=x^4+\frac{1}{x^4}$ $47=x^4+\frac{1}{x^4}$ Again cubing on both sides we get, $\left(x+\frac{1}{x}\right)^3=(3)^3$ We shall use the identity $(a + b)^3 = a^3 + b^3 + 2ab$ $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

$(3)^3=x^3+\frac{1}{x^3}+3\times\not{x}\times\frac{1}{\not{x}}\times 3$ $27=x^3+\frac{1}{x^3}+9$ $27-9=x^3+\frac{1}{x^3}$ $18=x^3+\frac{1}{x^3}$ Hence the value of $x^2+\frac{1}{x^2},\ x^3+\frac{1}{x^3},\ x^4+\frac{1}{x^4}$ is 7, 18, 47 respectively.

18. Factorize:

$\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$

Ans. : $\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$ $=x^2+\frac{1}{x^2}-4x-\frac{4}{x}+4+2$ $=x^2+\frac{1}{x^2}+4+2-4x-4x$

$=\left(x^2\right)+\left(\frac{1}{x}\right)^2+(-2)^2+2\times x\times\frac{1}{x}+2\times\frac{1}{x}\times(-2)+2(-2)x$ Using identity $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$ We get, $=\left[x+\frac{1}{x}+(-2)\right]^2$ $=\left[x+\frac{1}{x}-2\right]^2$ $=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

$\therefore\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6$ $=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

19. Factorize:

$x^2+\frac{12}{35}x+\frac{1}{35}$

Ans. : $x^2+\frac{12}{35}x+\frac{1}{35}$ Splitting the middle term, $=x^2+\frac{5}{35}x+\frac{7}{35}x+\frac{1}{35}$ $\left[\because\frac{12}{35}=\frac{5}{35}+\frac{7}{35}\text{ and }\frac{5}{35}\times\frac{7}{35}=\frac{1}{35}\right]$ $=x^2+\frac{x}{7}+\frac{x}{5}+\frac{1}{35}$ $=x\left(x+\frac{1}{7}\right)+\frac{1}{5}\left(x+\frac{1}{7}\right)$ $=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

$\therefore x^2+\frac{12}{35}x+\frac{1}{35}=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

20. In the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division:

$f(x) = 9x^3 - 3x^2 + x - 5$, $g(x)=x-\frac{2}{3}$

Ans. : Here,

$f(x) = 9x^3 - 3x^2 + x - 5,$

$g(x)=x-\frac{2}{3}$

From, the remainder theorem when f(x) is divided by $g(x)=x-\frac{2}{3}$ the remainder will be equal to $f\left(\frac{2}{3}\right)$

Let, g(x) = 0

$\Rightarrow x-\frac{2}{3}=0$

$\Rightarrow x=\frac{2}{3}$

Substitute the value of x in f(x)

$$f\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right) - 3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) - 5$$

$$= 9\left(\frac{8}{27}\right) - 3\left(\frac{4}{9}\right) + \frac{2}{3} - 5$$

$$= \left(\frac{8}{3}\right) - \left(\frac{4}{3}\right) + \frac{2}{3} - 5$$

$$= \frac{8-4+2-15}{3}$$

$$= \frac{10-19}{3}$$

$$= \frac{-9}{3}$$

$$= -3$$

Therefore, the remainder is -3.

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