STD 9 Maths
Number system (DPP-3)

Total Marks : 30

* **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. The value of $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$.

(A) $\frac{12}{27}$ (B) $\frac{4}{9}$ (C) $\frac{2}{3}$ (D) None of these.

Ans. :

c. $\frac{2}{3}$

Solution:

$\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$.

$\Rightarrow \left(\frac{12}{27}\right)^{\frac{1}{5}\times\frac{5}{2}}$

$\Rightarrow \left(\frac{12}{27}\right)^{\frac{1}{2}}$

$\Rightarrow \frac{2\sqrt{3}}{3\sqrt{3}}$

$\Rightarrow \frac{2}{3}$

2. $\left(\frac{2}{3}\right)^{x}\left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$ then x =

(A) 2 (B) 3 (C) 4 (D) 1

Ans. :

c. 4

Solution:

We have to find value of x provided $\left(\frac{2}{3}\right)^{x}\left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

So,

$\left(\frac{2}{3}\right)^{x}\left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

$\left(\frac{2}{3}\right)^{x}\left(\frac{3}{2}\right)^{2x} = \frac{3^4}{2^4}$

$\frac{2^x}{3^x}\frac{3^{2x}}{2^{2x}} = \frac{3^4}{2^4}$

$\frac{3^{2x-x}}{2^{2x-x}} = \frac{3^4}{2^4}$

$\frac{3^x}{2^x} = \frac{3^4}{2^4}$

Equating exponents of power we get x = 5

Hence the correct alternative is c.

3. The value of $2.\overline{45} + 0.\overline{36}$ is:

(A) $\frac{67}{33}$ (B) $\frac{24}{11}$ (C) $\frac{31}{11}$ (D) $\frac{167}{110}$

Ans. :

c. $\frac{31}{11}$

Solution:

Let $x = 2.\overline{45}$

i.e., $x = 2.4545\ldots(i)$

$\Rightarrow 100x = 245.4545\ldots(ii)$

On subtracting (i) and (ii), we get

$99x = 243$

$\Rightarrow x = \frac{243}{99}$

Let $y = 0.\overline{36}$

i.e., $y = 0.3636\ldots(iii)$

$\Rightarrow 100y = 36.3636\ldots(iv)$

On subtracting (iii) and (iv), we get

$99y = 36$

$\Rightarrow y = \frac{36}{99}$

$\therefore 2.\overline{45} + 0.\overline{36} = x + y = \frac{243}{99} + \frac{36}{99} = \frac{279}{99} = \frac{31}{11}$

Hence, the correct answer is option (c).

4. If a, m, n are positive ingegers, then $\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn}$ is equal to

(A) a^{nm} (B) a (C) $a^{\frac{m}{n}}$ (D) 1

Ans. :

b. a

Solution:

Find the value of $\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn}$.

So,

$$\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn} = \left\{\sqrt[m]{a^{\frac{1}{n}}}\right\}^{mn}$$

$$= \left\{a^{\frac{1}{n}\times\frac{1}{m}}\right\}^{mn}$$

$$= \left\{a^{\frac{1}{n}\times\frac{1}{m}} \times m \times n\right\}$$

$$\Rightarrow \left\{\sqrt[m]{\sqrt[n]{a}}\right\} = \left\{a^{\frac{1}{n}\times\frac{1}{m}} \times m \times n\right\}$$

$$\Rightarrow \left\{\sqrt[m]{\sqrt[n]{a}}\right\} = a$$

Hence the correct choice is b.

5. Between any two rational numbers there.

(A) Is no irrational number. (B) Is no rational number. (C) Are many rational numbers. (D) Are exactly two rational numbers.

Ans. :

c. Are many rational numbers.

Solution:

Between any two rational number there are many rational number,

Example - 4 and 8

We have 5, 6, 7, 7.5……. and many more.

6. The value of $\left\{\left(23+2^2\right)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2$, is:

(A) 196 (B) 289 (C) 324 (D) 400

Ans. :

d. 400

Solution:

We have to find the value of $\left\{\left(23+2^2\right)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2$,

$\left\{\left(23+2^2\right)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2$

$=\left\{(23+4)^{\frac{2}{3}}+(121)^{\frac{1}{2}}\right\}^2$

$=\left\{(27)^{\frac{2}{3}}+(121)^{\frac{1}{2}}\right\}^2$

$=\left\{\left(3^3\right)^{\frac{2}{3}}+\left(11^2\right)^{\frac{1}{2}}\right\}^2$

$\left\{\left(23+2^2\right)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2$

$=\left\{3^{3\times\frac{2}{3}}+11^{2\times\frac{2}{3}}\right\}$

$=\left\{3^2+11\right\}^2$

$\Rightarrow\left\{\left(23+2^2\right)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2=\{9+11\}^2$

By using the identity $(\mathrm{a}+\mathrm{b})^2=\mathrm{a}^2+2\mathrm{ab}+\mathrm{b}^2$ we get,

$=9\times9+2\times9\times11+11\times11$

$=81+198+121$

$=400$

Hence correct choice is d.

7. Which of the following is not equal to $\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}}$?

(A) $\left(\frac{5}{6}\right)^{\frac{1}{5}}\frac{1}{6}$ (B) $\frac{1}{\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{\frac{1}{6}}}$ (C) $\left(\frac{6}{5}\right)^{\frac{1}{30}}$ (D) $\left(\frac{5}{6}\right)^{\frac{1}{30}}$

Ans. :

c. $\left(\frac{6}{5}\right)^{\frac{1}{30}}$

Solution:

$\left(\frac{5}{6}\right)^{\frac{1}{5}}\frac{1}{6}$

$\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}}$

$=\left(\frac{5}{6}\right)^{\frac{1}{5}\times\left(-\frac{1}{6}\right)}$

$=\left(\frac{5}{6}\right)^{\frac{-1}{30}}$

$=\left(\frac{6}{5}\right)^{\frac{1}{30}}$

8. The decimal form of $\frac{2}{11}$ is:

(A) $0.0\overline{18}$ (B) 0.18 (C) $0.\overline{18}$ (D) 0.018

Ans. :

c. $0.\overline{18}$

Solution:

When we divide 2 by 11

We have value = 0.181818

Which is $0.\overline{18}$

9. If $\mathrm{x}+\sqrt{15}=4$, then $\mathrm{x}+\frac{1}{\mathrm{x}}=$

(A) 2 (B) 4 (C) 8 (D) 1

Ans. :

c. 8

Solution:

$\mathrm{x}+\sqrt{15}=4$

$=\mathrm{x}=4-\sqrt{15}\Rightarrow\frac{1}{\mathrm{x}}=\frac{1}{4-\sqrt{15}}$

$\frac{1}{\mathrm{x}}=\frac{1}{4-\sqrt{15}}\times\frac{4+\sqrt{15}}{4+\sqrt{15}}=\frac{4+\sqrt{15}}{(4)^2-\left(\sqrt{15}\right)^2}$

$=\frac{4+\sqrt{15}}{16-15}=4+\sqrt{15}$

Now, $\mathrm{x}+\frac{1}{\mathrm{x}}=4-\sqrt{15}+4+\sqrt{15}=8$

Hence, correct option is (c).

10. The value of m for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}}=7^{\mathrm{m}}$, is:

(A) $-\frac{1}{3}$ (B) $\frac{1}{4}$ (C) -3 (D) 2

Ans. :

a. $-\frac{1}{3}$

Solution:

We have to find the value of m for $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}} = 7^m$,

$\Rightarrow \left[\left\{\frac{1}{7^{2\times-2}}^{-2}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow \left[\left\{\frac{1}{7^{-4}}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow \left[\left\{\frac{1}{7^{-4\times\frac{-1}{3}}}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow \left[\left\{\frac{1}{7^{\frac{4}{3}}}\right\}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow \left[\frac{1}{7^{\frac{4}{3}\times\frac{1}{4}}}\right] = 7^m$

$\Rightarrow \left[\frac{1}{7^{\frac{1}{3}}}\right] = 7^m$

By using rational exponents $\frac{1}{a^n} = a^{-n}$

$7^{\frac{-1}{3}} = 7^m$

Equating power of exponents we get $-\frac{1}{3} = m$

Hence the correct choice is a.

*** Answer the following short questions. [2 Marks Each]** **[8]**

11. If a = 2, b = 3, find the values of:

$\left(a^a + b^b\right)^{-1}$

Ans. : Given, a = 2 and b = 3

$\left(a^a + b^b\right)^{-1} = (2^2 + 3^3)^{-1}$

$= (4 + 27)^{-1}$

$= (31)^{-1}$

$= \frac{1}{31}$

12. Find the value of x in the following:

$\sqrt[5]{5x + 2} = 2$

Ans. : $\sqrt[5]{5x + 2} = 2$

$\Rightarrow (5x + 2)^{\frac{1}{5}} = 2$

$\Rightarrow \left[(5x + 2)^{\frac{1}{5}}\right] = 2^5$

$\Rightarrow 5x + 2 = 32$

$\Rightarrow 5x = 30$

$\Rightarrow x = 6$

13. Examine whether the following number are rational or irrational:

$\sqrt{8} + 4\sqrt{32} - 6\sqrt{2}$

Ans. : $\sqrt{8} + 4\sqrt{32} - 6\sqrt{2}$

$= \sqrt{4 \times 2} + 4\sqrt{16 \times 2} - 6\sqrt{2}$

$= 2\sqrt{2} + 16\sqrt{2} - 6\sqrt{2}$

$= 12\sqrt{2}$

Thus, the given number is irrational.

14. Prove that:

$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$

Ans. : $\text{L.H.S} = \frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$

$= \frac{X^{ab-ac}}{X^{ab-bc}} \div \frac{X^{bc}}{X^{ac}}$

$= X^{ab-ac-ab+bc} \div X^{bc-ac}$

$= X^{bc-ac} \div X^{bc-ac}$

$= 1$

$= \text{R.H.S}$

*** Answer the following questions. [3 Marks Each]** **[12]**

15. Give two rational numbers lying between 0.232332333233332 and 0.212112111211112.

Ans. : Let a = 0.212112111211112

And, b = 0.232332333233332...

Clearly, a < b because in the second decimal place a has digit 1 and b has digit 3. If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let x = 0.22

y = 0.22112211... Then a < x < y < b

Hence, x and y are required rational numbers.

16. Evaluate:

$\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

Ans. : $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5\left(2^{3 \times \frac{1}{3}} + 3^{3 \times \frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5(2+3)^3\right]^{\frac{1}{4}}$

$= \left[5(5)^3\right]^{\frac{1}{4}}$

$= \left[5^4\right]^{\frac{1}{4}}$

$= 5$

17. Prove that:

$\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)} = 1$

Ans. : $\text{L.H.S} = \frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)} = 1$

$= \frac{(X^{2a+2b})(X^{2b+2c})(X^{2c+2a})}{X^{4a} X^{4b} X^{4c}}$

$= \frac{X^{2a+2b+2b+2c+2c+2a}}{X^{4a+4b+4c}}$

$= \frac{X^{4a+4b+4c}}{X^{4a+4b+4c}}$

$= 1$

$= \text{R.H.S}$

18. Evaluate:

$\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

Ans. : $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$= \frac{4}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{-\frac{3}{4}}} + \frac{2}{(3^5)^{-\frac{1}{5}}}$

$= \frac{4}{6^{3\times\left(-\frac{2}{3}\right)}} + \frac{1}{4^{4\times\left(-\frac{3}{4}\right)}} + \frac{2}{3^{5\times\left(-\frac{1}{5}\right)}}$

$= \frac{4}{6^{-2}} + \frac{1}{4^{-3}} + \frac{2}{3^{-1}}$

$= 4 \times 6^2 + 1 \times 4^3 + 2 \times 3$

$= 4 \times 36 + 64 + 6$

$= 144 + 70$

$= 214$

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