*** Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. If $x = \sqrt{6} + \sqrt{5}$, then $x^2 + \frac{1}{x^2} - 2 =$

(A) $2\sqrt{6}$ (B) $2\sqrt{5}$ (C) 24 (D) 20

Ans. :

d. 20

Solution :

$x^2 + \frac{1}{x^2} - 2 = \left(x - \frac{1}{x}\right)^2$

$x = \sqrt{6} + \sqrt{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{6}+\sqrt{5}} = \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

$= \frac{\sqrt{6}-\sqrt{5}}{1} = \sqrt{6} - \sqrt{5}$

Now,

$\left(x - \frac{1}{x}\right)^2 = \left[\sqrt{6} + \sqrt{5} - \left(\sqrt{6} - \sqrt{5}\right]^2\right.$

$= \left(2\sqrt{5}\right)^2 = 4 \times 5 = 20$

Hence, correct option is (d).

2. If $\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}}$ then x =

(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Ans. :

b. -3

Solution:

We have to find the value of x provided $\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$

So,

$\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$

By using law of rational exponents we get

$3^{5x+8+8-2x} = 3^7$

By equating exponents we get

$5x + 88 - 2x = 7$

$3x + 16 = 7$

$3x = 7 - 16$

$3x = -9$

$x = \frac{-9}{3}$

$x = -3$

Hence the correct choice is b.

3. $\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to :

(A) $\frac{1}{3+2\sqrt{2}}$ (B) $\frac{1}{2}(3 - 2\sqrt{2})$ (C) $3 + 2\sqrt{2}$ (D) $3 - 2\sqrt{2}$

Ans. :

c. $3+2\sqrt{2}$

Solution:

After rationalising

$\frac{1}{\sqrt{9}-\sqrt{8}}=\frac{1}{\sqrt{9}-\sqrt{8}}\times\frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}$

$=\frac{\sqrt{9}+\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}$

$=\frac{\sqrt{3\times3}+\sqrt{2\times2\times2}}{9-8}$

$=\frac{3+2\sqrt{2}}{1}$

$=3+2\sqrt{2}$

4. $(\frac{125}{216})^{\frac{-1}{3}}=$

(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) 125 (D) 216

Ans. :

b. $\frac{6}{5}$

Solution:

$(\frac{125}{216})^{\frac{-1}{3}}$

$\Rightarrow(\frac{5}{6})^3\times\frac{-1}{3}$

$\Rightarrow(\frac{5}{6})^{-1}$

$\Rightarrow\frac{6}{5}$

5. The number $0.\overline{32}$ when expressed in the form $\frac{p}{q}$ $(p, q \text{ are integers and } q\neq0),$ is:

(A) $\frac{8}{25}$ (B) $\frac{29}{90}$ (C) $\frac{32}{99}$ (D) $\frac{32}{199}$

Ans. :

b. $\frac{29}{90}$

Solution:

Let $x=0.\overline{32}=0.32222..(1)$

Now, $10x=3.2222=3.\overline{2}...(2)$,

Subtracting equation (2) and (3), we get

$90x=29$

$\Rightarrow x=\frac{29}{90}$

Hence, option (b) is correct.

6. If $\frac{2+\sqrt{3}}{2-\sqrt{3}}=a+b\sqrt{3}$, then,

(A) a = 7 and b = 4 (B) a = -7 and b = -4 (C) a = -7 and b = 4 (D) a = 7 and b = -4

Ans. :

a. a = 7 and b = 4

Solution:

$\frac{2+\sqrt{3}}{2-\sqrt{3}}$

Multiplying numerator and denominator by $2\div\sqrt{3}$

So, $\frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$

$=\frac{4+3+4\sqrt{3}}{4-3}$,

$=7\div4\sqrt{3}$

Now equating $7 \div 4\sqrt{3}$ and $a \div b\sqrt{3}$
we get,
a = 7 and b = 4

7. If $x = \frac{2}{3+\sqrt{7}}$, then $(x - 3)^2$=

(A) 1 (B) 3 (C) 6 (D) 7

Ans. :

d. 7

Solution:

$$x = \frac{2}{3+\sqrt{7}} = \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{2(3-\sqrt{7})}{3-\sqrt{7}}$$
$$= \frac{6-2\sqrt{7}}{9-7} = \frac{6-2\sqrt{7}}{2} = 3 - 2\sqrt{7}$$

Now $(x - 3)^2 = (\not{3} - \sqrt{7} - \not{3})^2 = \left(-\sqrt{7}\right)^2 = 7$

Hence, correct option is (d).

8. The value of $\frac{x^{a(c-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$ is:

(A) 4 (B) 3 (C) 2 (D) 1

Ans. :

d. 1

Solution:

$\frac{x^{a(c-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$

$\Rightarrow \frac{x^{ab-ac}}{x^{ba-be}} \div \left(\frac{x^{bc}}{x^{ac}}\right)$

$\Rightarrow x^{ab-ac-ab+bc} \div x^{bc-ac}$

$\Rightarrow x^{bc-ac} \div x^{bc-ac}$

$\Rightarrow 1$

9. If $\sqrt{2} = 1.414$ than $\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}} = ?$

(A) 0.207 (B) 0.414 (C) 2.414 (D) 0.621

Ans. :

b. 0.414

Solution:

$\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}}$

$= \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$

$= \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}}$

$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}}$

$= \sqrt{(\sqrt{2}-1)^2}$

$= 1.414 - 1$

$= 0.414$

10. If $\sqrt{2^{n}} = 1024,$ then $3^{2\left(\frac{n}{4}-4\right)} =$

(A) 3 (B) 9 (C) 27 (D) 81

Ans. :

b. 9

Solution:

We have to find $3^{2\left(\frac{n}{4}-4\right)}$

Given $\sqrt{2^{n}} = 1024$

$\sqrt{2^{n}} = 2^{10}$

$2^{n\times\frac{1}{2}}$

Equating powers of rational exponents we get

$n \times \frac{1}{2} = 10$

$n = 10 \times 2$

$n = 20$

Substituting in $3^{2\left(\frac{n}{4}-4\right)}$ we get

$3^{2\left(\frac{n}{4}-4\right)} = 3^{2\left(\frac{20}{4}-4\right)}$

$= 3^{2(5-4)}$

$= 3^{2\times1}$

$= 9$

Hence the correct choice is b.

*** Answer the following short questions. [2 Marks Each]** **[8]**

11. Simplify:

$3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

Ans. : $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

$3\sqrt{9\times5} - \sqrt{25\times5} + \sqrt{100\times2} - \sqrt{25\times2}$

$= 3\times3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= 9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= 4\sqrt{5} + 5\sqrt{2}$

12. If a = 2, b = 3, find the values of:

$\left(a^{b} + b^{a}\right)^{-1}$

Ans. : Given, a = 2 and b = 3

$\therefore \left(a^{b} + b^{a}\right)^{-1} = \frac{1}{a^{b}+b^{a}}$

$= \frac{1}{2^{3}+3^{2}}$

$= \frac{1}{8+9}$

$= \frac{1}{17}$

13. Find two rational and two irrational number between 0.5 and 0.55.

Ans. : The two rational numbers between 0.5 and 0.55 are: 0.51 and 0.52

The two irrational numbers between 0.5 and 0.55 are: 0.505005000... and 0.5101100111000...

Disclaimer: There are infinite number of rational and irrational numbers between 0.5 and 0.55.

14. Simplify $\left[\left\{(256)^{-\frac{1}{2}}\right\}^{-\frac{1}{4}}\right]^2$.

Ans. : $\left[\left\{(256)^{-\frac{1}{2}}\right\}^{-\frac{1}{4}}\right]^2$

$=\left[\left\{\left(16^2\right)^{-\frac{1}{2}}\right\}^{-\frac{1}{4}}\right]^2$

$=\left[\left\{16^{-1}\right\}^{-\frac{1}{4}}\right]^2$

$=\left[16^{-1\times\left(-\frac{1}{4}\right)}\right]^2$

$=\left[16^{\frac{1}{4}}\right]^2$

$=\left[2^{4\times\frac{1}{4}}\right]^2$

$=2^2$

$=4$

*** Answer the following questions. [3 Marks Each]** **[12]**

15. If x is a positive real number and exponents are rational numbers, simplify

$\left(\frac{x^b}{x^c}\right)^{b+c-a}\times\left(\frac{x^c}{x^a}\right)^{c+a-b}\times\left(\frac{x^a}{x^b}\right)^{a+b-c}$.

Ans. : $\left(\frac{x^b}{x^c}\right)^{b+c-a}\times\left(\frac{x^c}{x^a}\right)^{c+a-b}\times\left(\frac{x^a}{x^b}\right)^{a+b-c}$

$=\left(\frac{X^{b^2+bc-ab}}{X^{bc+c^2-ac}}\right)\times\left(\frac{X^{c^2+ac-bc}}{X^{ac+a^2-ab}}\right)\times\left(\frac{X^{a^2+ab-ac}}{X^{ab+b^2-bc}}\right)$

$=\left(X^{b^2+bc-ab-bc-c^2+ac}\right)\left(X^{c^2+ac-bc-ac-a^2+ab}\right)\left(X^{a^2+ab-ac-ab-b^2+bc}\right)$

$=\left(X^{b^2-ab-c^2+ac}\right)\left(X^{c^2-bc-a^2+ab}\right)\left(X^{a^2-ac-b^2+bc}\right)$

$=X^{b^2-ab-c^2+ac+c^2-bc-a^2+ab+a^2-ac-b^2+bc}$

$=X^0$

$=1$

16. Prove that:

$\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}}+\frac{\sqrt{25}}{\sqrt[3]{64}}=\frac{65}{16}$

Ans. : $\text{L.H.S.}=\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}}+\frac{\sqrt{25}}{\sqrt[3]{64}}$

$=\left(\frac{125}{64}\right)^{\frac{2}{3}}+\left(\frac{625}{256}\right)^{\frac{1}{4}}+\frac{\sqrt{5^2}}{\sqrt[3]{4^3}}$

$= \frac{5^{3\times\frac{2}{3}}}{4^{3\times\frac{2}{3}}} + \frac{5^{4\times\frac{1}{4}}}{4^{4\times\frac{1}{4}}} + \frac{5}{4}$

$= \frac{24}{16} + \frac{5}{4} + \frac{5}{4}$

$= \frac{25+20+20}{16}$

$= \frac{65}{16}$

$= \text{R.H.S.}$

17. Prove that:

$\left[8^{-\frac{2}{3}} \times 2^{\frac{1}{2}} \times 25^{-\frac{5}{4}}\right] \div \left[32^{-\frac{2}{5}} \times 125^{-\frac{5}{6}}\right] = \sqrt{2}$

Ans. : $\text{L.H.S.} = \left[8^{-\frac{2}{3}} \times 2^{\frac{1}{2}} \times 25^{-\frac{5}{4}}\right] \div \left[32^{-\frac{2}{5}} \times 125^{-\frac{5}{6}}\right]$

$= \left[2^{3\times\left(-\frac{2}{3}\right)} \times \sqrt{2} \times 5^{2\times\left(-\frac{5}{4}\right)}\right] \div \left[2^{5\times\left(-\frac{2}{5}\right)} \times 5^{3\times\left(-\frac{5}{6}\right)}\right]$

$= \left[2^{-2} \times \sqrt{2} \times 5^{-\frac{5}{2}}\right] \div \left[2^{-2} \times 5^{-\frac{5}{2}}\right]$

$= \frac{2^{-2}\times\sqrt{2}\times 5^{-\frac{5}{2}}}{2^{-2}\times 5^{-\frac{5}{2}}}$

$= \sqrt{2}$

$= \text{R.H.S.}$

18. Simplify:

$\frac{1}{\sqrt{3}+\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}}$

Ans. : $\frac{1}{\sqrt{3}+\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}}$

$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$

$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} - \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} - \frac{3\left(\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2}$

$= \frac{\sqrt{3}+\sqrt{2}}{3-2} - \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3} - \frac{3\left(\sqrt{2}+\sqrt{5}\right)}{2-5}$

$= \frac{\sqrt{3}+\sqrt{2}}{1} - \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2} - \frac{3\left(\sqrt{2}+\sqrt{5}\right)}{-3}$

$= \left(\sqrt{3}+\sqrt{2}\right) - \left(\sqrt{5}+\sqrt{3}\right) + \left(\sqrt{2}+\sqrt{5}\right)$

$= \sqrt{3}+\sqrt{2}-\sqrt{5}-\sqrt{3}+\sqrt{2}+\sqrt{5}$

$= 2\sqrt{2}$

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