➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. Choose the rational number which does not lie between $-\frac{2}{3}$ and $-\frac{1}{5}$.

(A) $-\frac{1}{4}$ (B) $-\frac{7}{20}$ (C) $\frac{3}{10}$ (D) $-\frac{3}{10}$

Ans. :

c. $\frac{3}{10}$

Solution:

Since $\frac{3}{10} > -\frac{2}{3}$ and $\frac{3}{10} > -\frac{1}{5}$

2. If $8 = t^{\frac{2}{3}} + 4t^{-\frac{1}{2}}$, What is the value of g when t = 64?

(A) $\frac{31}{2}$ (B) $\frac{33}{2}$ (C) 16 (D) $\frac{257}{16}$

Ans. :

b. $\frac{33}{2}$

Solution:

Given $t = 64,\ g = t^{\frac{2}{3}} + 4t^{\frac{1}{2}}$. We have to find the value of g

So,

$g = t^{\frac{2}{3}} + 4t^{-\frac{1}{2}}$

$g = 64^{\frac{2}{3}} + 4 \times 64^{\frac{1}{2}}$

$g = (64)^{\frac{2}{3}} + 4 \times \frac{1}{64^{\frac{1}{2}}}$

$g = 2^{6\times\frac{2}{3}} + 4 \times \frac{1}{2^{6\times\frac{1}{2}}}$

$g = 2^{2\times 2} + 4 \times \frac{1}{2^3}$

$g = 2^4 + 4 \times \frac{1}{8}$

$g = 16 + \frac{1}{2}$

$g = \frac{16\times 2}{1\times 2} + \frac{1}{2}$

$g = \frac{32}{2} + \frac{1}{2}$

$g = \frac{32+1}{2}$

$g = \frac{33}{2}$

The value of g is $\frac{33}{2}$

Hence the correct choice is b.

3. The value of $(\frac{81}{16})^{\frac{-3}{4}} \times \{(\frac{25}{9})^{\frac{-3}{2}} \div (\frac{5}{2})^{-3}\}$ is:

(A) 2 (B) 3 (C) 1 (D) 4

Ans. :

c. 1

Solution:

$(\frac{81}{16})^{\frac{-3}{4}} \times \{(\frac{25}{9})^{\frac{-3}{2}} \div (\frac{5}{2})^{-3}\}$

$\Rightarrow (\frac{3}{2})^{4\times\frac{-3}{4}} \times \{(\frac{5}{3})^{2\times\frac{-3}{2}} \div (\frac{5}{2})^{-3}\}$

$\Rightarrow (\frac{3}{2})^{-3} \times \{(\frac{5}{3})^{-3} \div (\frac{5}{2})^{-3}\}$

$\Rightarrow (\frac{3}{2})^{-3} \times (\frac{5}{3} \times \frac{2}{5})^{-3}$

$\Rightarrow (\frac{3}{2})^{-3} \times (\frac{2}{3})^{-3}$

$\Rightarrow (\frac{3}{2} \times \frac{2}{3})^{-3}$

$\Rightarrow (1)^{-3} = 1$

4. If a, b, c are positive real numbers, then $\sqrt{a^{-1}b} \times \sqrt{b^{-1}c} \times \sqrt{c^{-1}a}$ is equal to

(A) 1 (B) abc (C) $\sqrt{abc}$ (D) $\frac{1}{abc}$

Ans. :

a. 1

Solution:

We have to find the value of $\sqrt{a^{-1}b}\times\sqrt{b^{-1}c}\times\sqrt{c^{-1}a}$ when a, b, c are positive real numbers.

So,

$\sqrt{a^{-1}b}\times\sqrt{b^{-1}c}\times\sqrt{c^{-1}a}$

$=\sqrt{\frac{1}{a}\times b}\times\sqrt{\frac{1}{b}\times c}\times\sqrt{\frac{1}{c}\times a}$

$=\sqrt{\frac{b}{a}}\times\sqrt{\frac{c}{b}}\times\sqrt{\frac{a}{c}}$

Taking square root as common we get

$\sqrt{a^{-1}b}\times\sqrt{b^{-1}c}\times\sqrt{c^{-1}a}=\sqrt{\frac{b}{a}\times\frac{c}{b}\times\frac{a}{c}}$

$\sqrt{a^{-1}b}\times\sqrt{b^{-1}c}\times\sqrt{c^{-1}a}=1$

Hence the correct alternative is a.

5. $\left(\frac{125}{216}\right)^{\frac{-1}{3}}=$

(A) $\frac{6}{5}$ (B) $\frac{5}{6}$ (C) 125 (D) 216

Ans. :

a. $\frac{6}{5}$

Solution:

$\left(\frac{125}{216}\right)^{\frac{-1}{3}}$

$\Rightarrow\left(\frac{5}{6}\right)^{3\times\frac{-1}{3}}$

$\Rightarrow\left(\frac{5}{6}\right)^{-1}$

$\Rightarrow\frac{6}{5}$

6. The value of $\sqrt{5+2\sqrt{6}}$, is:

(A) $\sqrt{3}-\sqrt{2}$ (B) $\sqrt{3}+\sqrt{2}$ (C) $\sqrt{5}+\sqrt{6}$ (D) None of these

Ans. :

b. $\sqrt{3}+\sqrt{2}$

Solution:

$\sqrt{5+2\sqrt{6}}$

$=\sqrt{3+2+2(\sqrt{3})(\sqrt{2})}$

$=\sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})}$

$=\sqrt{(\sqrt{2}+\sqrt{2})^2}$

$=\sqrt{3}+\sqrt{2}$

Hence, correct option is (b).

7. The decimal expansion that a rational number cannot have is:

(A) $0.\overline{2528}$ (B) 0.25 (C) $0.25\overline{28}$ (D) 0.5030030003

Ans. :

d. 0.5030030003

Solution:

0.5030030003

The decimal expansion that a rational number cannot have is non terminating, non repeating.

8. The value of $\frac{1}{\sqrt{8}-3\sqrt{2}}$ is:

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $-\frac{1}{\sqrt{2}}$ (D) $-\sqrt{2}$

Ans. :

c. $-\frac{1}{\sqrt{2}}$

Solution:

$\frac{1}{\sqrt{8}-3\sqrt{2}}$

$\Rightarrow \frac{1}{2\sqrt{2}-3\sqrt{2}}$

$\Rightarrow \frac{1}{-\sqrt{2}} \Leftrightarrow \frac{-1}{\sqrt{2}}$

9. The value of $\frac{9^{\frac{1}{3}}\times 27^{\frac{1}{2}}}{3^{\frac{-1}{6}}\times 3^{\frac{1}{3}}}$ is:

(A) 27 (B) 1 (C) 3 (D) 9

Ans. :

d. 9

Solution:

$\frac{9^{\frac{1}{3}}\times 27^{\frac{1}{2}}}{3^{\frac{-1}{6}}\times 3^{\frac{1}{3}}}$

$\Rightarrow \frac{3^{\frac{2}{3}}\times 3^{\frac{3}{2}}}{3^{\frac{-1}{6}}\times 3^{\frac{1}{3}}}$

$\Rightarrow \frac{3^{\frac{2}{3}+\frac{3}{2}}}{3^{\frac{-1}{6}+\frac{1}{3}}}$

$\Rightarrow \frac{3^{\frac{4+9}{6}}}{3^{\frac{-1+2}{6}}}$

$\Rightarrow \frac{3^{\frac{13}{6}}}{3^{\frac{1}{6}}} = 3^{\frac{13}{6}-\frac{1}{6}}$

$\Rightarrow 3^{\frac{12}{6}} = 9$

$= (\frac{1}{5})^{-1} = 25$

10. The value of $\sqrt{p^{-1}q}.\sqrt{q^{-1}r}.\sqrt{r^{-1}p}$ is:

(A) -1 (B) 0 (C) 1 (D) 2

Ans. :

c. 1

Solution:

$\sqrt{p^{-1}q}.\sqrt{q^{-1}r}.\sqrt{r^{-1}p}$

$= \sqrt{\frac{q}{p}}.\sqrt{\frac{r}{q}}.\sqrt{\frac{p}{r}}$

$= \sqrt{\frac{q}{p}\times\frac{r}{q}\times\frac{p}{r}}$

$= \sqrt{1}$

$= 1$

Hence, the correct option is (c).

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

11. Simplify $\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

Ans. : $\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$ $=\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}\times\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}-\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}\times\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$ $=\frac{\left(\sqrt{13}-\sqrt{11}\right)^2}{\left(\sqrt{13}\right)^2-\left(\sqrt{11}\right)^2}+\frac{\left(\sqrt{13}+\sqrt{11}\right)^2}{\left(\sqrt{13}\right)^2-\left(\sqrt{11}\right)^2}$

$=\frac{13+11-2\times\sqrt{13}\times\sqrt{11}}{13-11}+\frac{13+11+2\times\sqrt{13}\times\sqrt{11}}{13-11}$ $=\frac{24-2\sqrt{143}}{2}+\frac{24+2\sqrt{143}}{2}$ $=\frac{24-2\sqrt{143}+24+2\sqrt{143}}{2}$ $=\frac{48}{2}$ $=24$

12. Find rational numbers a and b such that:

$\frac{2-\sqrt{5}}{2+\sqrt{5}} = a\sqrt{5}+b$

Ans. : $\frac{2-\sqrt{5}}{2+\sqrt{5}}=a\sqrt{5}+b$ we have, $\frac{2-\sqrt{5}}{2+\sqrt{5}}$ $=\frac{2-\sqrt{5}}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$ $=\frac{\left(2-\sqrt{5}\right)^2}{(2)^2-\left(\sqrt{5}\right)^2}$ $=\frac{(2)^2-2\times2\sqrt{5}+\left(\sqrt{5}\right)^2}{4-5}$ $=\frac{4-4\sqrt{5}+5}{-1}$

$=-\left(-4\sqrt{5}+9\right)$ $=4\sqrt{5}-9$ Now, $\frac{2-\sqrt{5}}{2+\sqrt{5}}=a\sqrt{5}+b$ $\Rightarrow 4\sqrt{5}-9=a\sqrt{5}+b$ $\Rightarrow a=4$ and $b=-9$

13. If $a=3-2\sqrt{2}$, find the value of $a^2-\frac{1}{a^2}$.

Ans. : $a=3-2\sqrt{2}$ $\Rightarrow a^2=\left(3-2\sqrt{2}\right)^2$ $=3^2-2\times3\times2\sqrt{2}+\left(2\sqrt{2}\right)^2$ $=9-12\sqrt{2}+8$ $=17-12\sqrt{2}$ $\Rightarrow\frac{1}{a^2}=\frac{1}{17-12\sqrt{2}}$

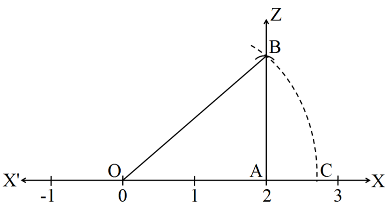
$=\frac{1}{17-12\sqrt{2}}\times\frac{17+12\sqrt{2}}{17+12\sqrt{2}}$ $=\frac{17+12\sqrt{2}}{17^2-\left(12\sqrt{2}\right)^2}$ $=\frac{17+12\sqrt{2}}{289-288}$ $=17+12\sqrt{2}$ $\Rightarrow a^2-\frac{1}{a^2}=\left(17-12\sqrt{2}\right)-\left(17+12\sqrt{2}\right)$

$=17-12\sqrt{2}-17-12\sqrt{2}$ $=-24\sqrt{2}$

14. Locate $\sqrt{8}$ on the number line.

Ans. : Draw a number line as shown. On the number line, take point O corresponding to zero. Now take point A on number line such that OA = 2 units. Draw perpendicular AZ at A on the number line and cut-off arc AB = 2 units. By Pythagoras Theorem, $OB^2 = OA^2 + AB^2 = 2^2 + 2^2 = 4 + 4 = 8$ $\Rightarrow OB=\sqrt{8}$ Taking O as centre and $OB=\sqrt{8}$ as radius draw an arc cutting real line at C. Clearly,

$OC=OB=\sqrt{8}$



Hence, C represents $\sqrt{8}$ on the number line.

➤ **Answer the following questions. [3 Marks Each]** **[12]**

15. If $x=\frac{5-\sqrt{3}}{5+\sqrt{3}}$ and $y=\frac{5+\sqrt{3}}{5-\sqrt{3}}$, show that $x-y=-\frac{10\sqrt{3}}{11}$.

Ans. :

$x=\frac{5-\sqrt{3}}{5+\sqrt{3}}$

$=\frac{5-\sqrt{3}}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}$

$=\frac{\left(5-\sqrt{3}\right)^2}{5^2-\left(\sqrt{3}\right)^2}$

$=\frac{25-10\sqrt{3}+3}{25-3}$

$=\frac{28-10\sqrt{3}}{22}$

$=\frac{14-5\sqrt{3}}{11}$

$y=\frac{5+\sqrt{3}}{5-\sqrt{3}}$

$=\frac{5+\sqrt{3}}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}$

$=\frac{\left(5+\sqrt{3}\right)^2}{5^2-\left(\sqrt{3}\right)^2}$

$=\frac{25+10\sqrt{3}+3}{25-3}$

$=\frac{28+10\sqrt{3}}{22}$

$=\frac{14+5\sqrt{3}}{11}$

$\therefore x-y=\frac{14-5\sqrt{3}}{11}-\frac{14+5\sqrt{3}}{11}$

$=\frac{14-5\sqrt{3}-14-5\sqrt{3}}{11}$

$=-\frac{10\sqrt{3}}{11}$

16. Find the values of a and b if:

$\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=a+b\sqrt{5}$

Ans. : $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$ $=\frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$ $=\frac{7(3-\sqrt{5})+3\sqrt{5}(3-\sqrt{5})}{3^2-(\sqrt{5})^2}-\frac{7(3+\sqrt{5})-3\sqrt{5}(3+\sqrt{5})}{3^2-(\sqrt{5})^2}$

$=\frac{21-7\sqrt{5}+9\sqrt{5}-15}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-15}{9-5}$ $=\frac{6+2\sqrt{5}}{4}-\frac{6-2\sqrt{5}}{4}$ $=\frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$ $=\frac{4\sqrt{5}}{4}$ $=\sqrt{5}$ $\therefore\ \frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=0+1\times\sqrt{5}$

Comparing with the given expression, we get a = 0 and b = 1 Thus, the values of a and b are 0 and 1, respectively.

17. If $x=\frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

Ans. : $x=\frac{1}{2-\sqrt{3}}=\frac{1}{2-\sqrt{3}}\times=\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2}$

$=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$

Now, x^3-2x^2-7x+5

$=(2+\sqrt{3})^3-2(2+\sqrt{3})^2-7(2+\sqrt{3})+5$ $=\left[2^3+(\sqrt{3})^3+3\times2\sqrt{3}(2+\sqrt{3})\right]-2\left[2^2+2\times2\sqrt{3}+(\sqrt{3})^2\right]-7\times2-7\sqrt{3}+5$

$=\left[8+3\sqrt{3}+6\sqrt{3}(2+\sqrt{3})\right]-2\left[4+4\sqrt{3}+3\right]-14-7\sqrt{3}+5$ $=\left[8+3\sqrt{3}+12\sqrt{3}+18\right]-2\left[7+4\sqrt{3}\right]-9-7\sqrt{3}$

$=26+15\sqrt{3}-14-8\sqrt{3}-9-7\sqrt{3}$ $=3$

18. Prove that:

$\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}=1$

Ans. : $\text{L.H.S}=\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}$ $=\frac{1}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}\times\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$=\frac{3-\sqrt{7}}{3^2-(\sqrt{7})^2}+\frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7})^2-(\sqrt{5})^2}+\frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{\sqrt{3}-1}{(\sqrt{3})^2-1}$ $=\frac{3-\sqrt{7}}{9-7}+\frac{\sqrt{7}-\sqrt{5}}{7-5}+\frac{\sqrt{5}-\sqrt{3}}{5-3}+\frac{\sqrt{3}-1}{3-1}$ $=\frac{3-\sqrt{7}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}$

$=\frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}$ $=\frac{2}{2}$ $=1$ $=\text{R.H.S.}$

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