

* Choose the right answer from the given options. [1 Marks Each]

[10]

1. On simplification, the expression $\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}}$ equals :

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $-\frac{3}{5}$

Ans. :

b. $-\frac{5}{3}$

Solution :

$$\begin{aligned} & \frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}} \\ &= \frac{5^{n+1}(5-6)}{5^n(13-2 \times 5)} \\ &= \frac{5^n \times 5 \times (-1)}{5^n(13-10)} \\ &= -\frac{5}{3} \end{aligned}$$

Hence, the correct option is (b).

2. $\frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}}$ is equal to :

(A) $\sqrt{2}$

(B) 4

(C) 8

(D) 2

Ans. :

4. 2

Solution :

$$\begin{aligned} & \frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}} \\ & \Rightarrow \frac{\sqrt{16 \times 2} + \sqrt{16 \times 3}}{\sqrt{4 \times 2} + \sqrt{4 \times 3}} \\ & \Rightarrow \frac{4\sqrt{2} + 4\sqrt{3}}{2\sqrt{2} + 2\sqrt{3}} \\ & \Rightarrow \frac{4}{2} \left(\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \right) \\ & \Rightarrow 2 \end{aligned}$$

3. If $x = (7 + 4\sqrt{3})$ then $\left(x + \frac{1}{x}\right) = ?$

(A) 49

(B) 14

(C) 48

(D) $8\sqrt{3}$

Ans. : b. 14

Solution :

$$\begin{aligned} x &= (7 + 4\sqrt{3}) \\ \frac{1}{x} &= \frac{1}{7 + 4\sqrt{3}} = (7 - 4\sqrt{3}) \\ x + \frac{1}{x} &= (7 + 4\sqrt{3}) + (7 - 4\sqrt{3}) \\ &= 14 \end{aligned}$$

4. Write the correct answer in the following:

$\sqrt[4]{\sqrt[3]{2^2}}$ equals.

(A) $2^{-\frac{1}{6}}$

(B) 2^{-6}

(C) $2^{\frac{1}{6}}$

(D) 2^6

Ans. :

c. $2^{\frac{1}{6}}$

Solution:

$$\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2^2)^{\frac{1}{3}}} = \left(2^{\frac{2}{3}}\right)^{\frac{1}{4}} = 2^{\frac{2}{3} \times \frac{1}{4}} = 2^{\frac{1}{6}}$$

Hence, (c) is the correct answer.

5. If $\sqrt{7} = 2.646$ then $\frac{1}{\sqrt{7}} = ?$

- (A) None of these. (B) 0.378 (C) 0.441 (D) 0.375

Ans. :

b. 0.378

Solution:

$$\begin{aligned}\frac{1}{\sqrt{7}} &= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{\sqrt{7}}{7} \\ &= \frac{1}{7} \times \sqrt{7} \\ &= \frac{1}{7} \times 2.646 \\ &= 0.378\end{aligned}$$

6. If $x = \sqrt{6} + \sqrt{5}$, then $x^2 + \frac{1}{x^2} - 2 =$

- (A) $2\sqrt{6}$ (B) $2\sqrt{5}$ (C) 24 (D) 20

Ans. :

d. 20

Solution :

$$\begin{aligned}x^2 + \frac{1}{x^2} - 2 &= \left(x - \frac{1}{x}\right)^2 \\ x &= \sqrt{6} + \sqrt{5} \\ \Rightarrow \frac{1}{x} &= \frac{1}{\sqrt{6} + \sqrt{5}} = \frac{1}{\sqrt{6} + \sqrt{5}} \times \frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} - \sqrt{5}} \\ &= \frac{\sqrt{6} - \sqrt{5}}{1} = \sqrt{6} - \sqrt{5}\end{aligned}$$

Now,

$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= [\sqrt{6} + \sqrt{5} - (\sqrt{6} - \sqrt{5})]^2 \\ &= (2\sqrt{5})^2 = 4 \times 5 = 20\end{aligned}$$

Hence, correct option is (d).

7. If $(3^3)^2 = 9^x$ then $5^x = ?$

- (A) 5 (B) 1 (C) 125 (D) 25

Ans. :

c. 125

Solution :

$$\begin{aligned}(3^3)^2 &= 9^x \\ (3^3)^3 &= 9^x \\ 9^3 &= 9^x \\ \Rightarrow x &= 3 \\ \therefore 5^3 &= 125\end{aligned}$$

8. The value of $\left(\frac{x^1}{x^m}\right)^{\frac{1}{lm}} \times \left(\frac{x^m}{x^n}\right)^{\frac{1}{mn}} \times \left(\frac{x^n}{x^l}\right)^{\frac{1}{nl}}$ is :

- (A) 4 (B) 1 (C) 2 (D) 0

Ans. :

b. 1

Solution :

$$\left(\frac{x^1}{x^m}\right)^{\frac{1}{lm}} \times \left(\frac{x^m}{x^n}\right)^{\frac{1}{mn}} \times \left(\frac{x^n}{x^l}\right)^{\frac{1}{nl}}$$

$$\Rightarrow x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$\Rightarrow x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$\Rightarrow x^0 = 1$$

9. The value of $\{5(8^{\frac{1}{4}} + 27^{\frac{1}{3}})^3\}^{\frac{1}{4}}$ is :

(A) 6

(B) 3

(C) 5

(D) 4

Ans. : c. 5

Solution :

$$\{5(8^{\frac{1}{4}} + 27^{\frac{1}{3}})^3\}^{\frac{1}{4}}$$

$$\Rightarrow \{5(2^{3 \times \frac{1}{3}} + 3^{3 \times \frac{1}{3}})^3\}^{\frac{1}{4}}$$

$$\Rightarrow \{5(3 + 2)^3\}^{\frac{1}{4}}$$

$$\Rightarrow \{5 \times 5^3\}^{\frac{1}{4}}$$

$$\Rightarrow 5^{4 \times \frac{1}{4}}$$

$$\Rightarrow 5$$

10. Which of the following is an irrational number ?

(A) 3.14

(B) 3.141414...

(C) 3.14444

(D) 3.141141114...

Ans. :

4. 3.141141114...

Solution :

The decimal expansion of an irrational number is non-terminating recurring non-recurring.

Hence, 3.141141114... is an irrational number.

Hence, the correct option is (d).

*** Answer the following short questions. [2 Marks Each]**

[6]

11. Find six rational numbers between 3 and 4.

Ans. : We know that there are infinite rational numbers between any two numbers. A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. We know that the numbers all lie between 3 and 4. We need to rewrite the numbers in $\frac{p}{q}$ form to get the rational numbers between 3 and 4. So, we cover it in $\frac{p}{q}$

$$\frac{3}{1} = \frac{3}{1} \times \frac{10}{10} = \frac{30}{10}$$

$$\frac{4}{1} = \frac{4}{1} \times \frac{10}{10} = \frac{40}{10}$$

So any six number between $\frac{30}{10}, \frac{40}{10}$ will be the answer example $\frac{31}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}, \frac{37}{10}, \frac{38}{10}$

12. Show that $0.2353535... = 0.2\overline{35}$, can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Ans. : Let $x = 0.235... \text{ ---- (i)}$

Multiplying both sides by 10

$$10x = 2.35... \text{ ----(ii)}$$

Multiplying both sides by 100

$$1000x = 235.35... \text{ ----(iii)}$$

Subtracting (ii) from (iii)

$$1000x - 10x = 235.35... - 2.35...$$

$$990x = 233$$

$$x = \frac{233}{990}$$

13. Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$

Ans. : Let $y = \frac{5}{\sqrt{3}-\sqrt{5}}$ and its denominator = $\sqrt{3}-\sqrt{5}$

Here, the conjugate of denominator ($\sqrt{3}-\sqrt{5}$) is ($\sqrt{3}+\sqrt{5}$).

$$y = \frac{5}{\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} \text{ [by rationalising]}$$

$$= \frac{5(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2} [\because (a-b)(a+b)=a^2-b^2]$$

$$= \frac{5(\sqrt{3}+\sqrt{5})}{3-5} = -\frac{5}{2}(\sqrt{3}+\sqrt{5})$$

[9]

* Answer the following questions. [3 Marks Each]

14. It being given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$, find the value of three places of decimals, of the following:

$$\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$$

$$\text{Ans. : } \frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$$

$$= \frac{\sqrt{5 \times 2} - \sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{5} - \sqrt{10}}{(\sqrt{2})^2}$$

$$= \frac{2\sqrt{5} - \sqrt{10}}{2}$$

$$= \frac{2 \times 2.236 - 3.162}{2}$$

$$= \frac{4.472 - 3.162}{2}$$

$$= \frac{1.31}{2}$$

$$= 0.655 \text{ (three places of decimal)}$$

15. Evaluate:

$$\frac{(25)^{\frac{5}{2}} \times (729)^{\frac{1}{3}}}{(125)^{\frac{2}{3}} \times (27)^{\frac{2}{3}} \times 8^{\frac{4}{3}}}$$

$$\text{Ans. : } \frac{(25)^{\frac{5}{2}} \times (729)^{\frac{1}{3}}}{(125)^{\frac{2}{3}} \times (27)^{\frac{2}{3}} \times 8^{\frac{4}{3}}}$$

$$= \frac{(5^2)^{\frac{5}{2}} \times (9^3)^{\frac{1}{3}}}{(5^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{5^{2 \times \frac{5}{2}} \times 9^{3 \times \frac{1}{3}}}{5^{3 \times \frac{2}{3}} \times 3^{3 \times \frac{2}{3}} \times 2^{3 \times \frac{4}{3}}}$$

$$= \frac{5^5 \times 9}{5^2 \times 3^2 \times 2^4}$$

$$= \frac{5^3}{2^4}$$

$$= \frac{125}{16}$$

16. Prove that:

$$\left[8^{-\frac{2}{3}} \times 2^{\frac{1}{2}} \times 25^{-\frac{5}{4}} \right] \div \left[32^{-\frac{2}{5}} \times 125^{-\frac{5}{6}} \right] = \sqrt{2}$$

$$\text{Ans. : L.H.S.} = \left[8^{-\frac{2}{3}} \times 2^{\frac{1}{2}} \times 25^{-\frac{5}{4}} \right] \div \left[32^{-\frac{2}{5}} \times 125^{-\frac{5}{6}} \right]$$

$$= \left[2^{3 \times \left(-\frac{2}{3}\right)} \times \sqrt{2} \times 5^{2 \times \left(-\frac{5}{4}\right)} \right] \div \left[2^{5 \times \left(-\frac{2}{5}\right)} \times 5^{3 \times \left(-\frac{5}{6}\right)} \right]$$

$$= \left[2^{-2} \times \sqrt{2} \times 5^{-\frac{5}{2}} \right] \div \left[2^{-2} \times 5^{-\frac{5}{2}} \right]$$

$$= \frac{2^{-2} \times \sqrt{2} \times 5^{-\frac{5}{2}}}{2^{-2} \times 5^{-\frac{5}{2}}}$$

$$= \sqrt{2}$$

$$= \text{R.H.S.}$$
