

MENSURATION

5

CHAPTER

CONTENTS

- Area of Rectangle and Square
- Area of Quadrilaterals
- Area of Irregular Rectilinear Figures

Area :

A figure made up of straight line segments is called a **rectilinear figure**.

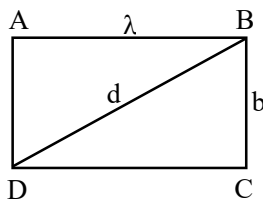
➤ AREA OF RECTANGLE AND SQUARE

◆ Rectangle :

Area = length \times breadth or $A = \lambda \times b$

Perimeter = 2 (length + breadth) or

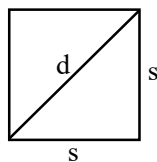
$P = 2(\lambda + b)$



◆ Square :

Area = (side)² or $A = s^2$

Perimeter = 4 \times side or $P = 4s$



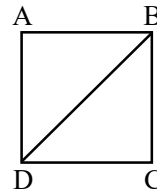
❖ EXAMPLES ❖

Ex.1 Show that area of a square = $\frac{1}{2} \times (\text{diagonal})^2$.

Find the area of a square whose diagonal = 2.5 cm.

Sol. In right triangle BCD

$$(\text{diagonal})^2 = DC^2 + CB^2 = s^2 + s^2 = 2s^2$$



But area of square = s^2

$$\therefore (\text{diagonal})^2 = 2 \times \text{area}$$

$$\text{or area} = \frac{1}{2} \times (\text{diagonal})^2$$

If diagonal = 2.5 cm

$$\text{area} = \frac{1}{2} \times (2.5)^2 \text{ cm}^2 = \frac{6.25}{2} \text{ cm}^2 = 3.125 \text{ cm}^2.$$

Ex.2 The area of a square is 42.25 m². Find the side of the square. If tiles measuring 13 cm \times 13 cm area paved on the square area. Find how many such tiles are used for paving it.

Sol.: The area of the square = 42.25 m²
= 422500 cm²

The side of the square = $\sqrt{\text{area}}$

$$= \sqrt{422500} \text{ cm} = 650 \text{ cm}$$

The area of 1 tile = 13 cm \times 13 cm = 169 cm²

Number of tiles required

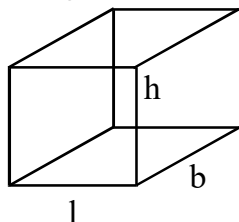
$$= 422500 \div 169 = 2500$$

Ex.3 A room is 5 metres long, 4 metres broad and 3 metres high. Find the area of the four walls. Also find the area of the ceiling and the area of the floor. If it costs ₹ 0.30 to whitewash 1 dm^3 of wall, find the cost of whitewashing the four walls and the ceiling.

Sol.: Area of four walls

$$= \lambda h + bh + \lambda h + bh = 2h(\lambda + b) \\ = 6 \times 9 \text{ m}^2 = 54 \text{ m}^2$$

$$\text{Area of ceiling} = \text{Area of floor} = 20 \text{ m}^2$$



Since $1 \text{ m}^2 = 100 \text{ dm}^2$,

$$\therefore 54 \text{ m}^2 = 5400 \text{ dm}^2 \text{ and } 20 \text{ m}^2 = 2000 \text{ dm}^2$$

Cost of whitewashing the four walls at the rate of ₹ 0.30 per dm^2

$$= ₹ (5400 \times 0.30) = ₹ 1620$$

Cost of whitewashing the ceiling at the rate of ₹ 0.30 per dm^2

$$= ₹ (2000 \times 0.30) = ₹ 600$$

Total cost of white washing

$$= ₹ 1620 + ₹ 600 = ₹ 2220$$

Ex.4 The length and breadth of a rectangular field is in the ratio 4 : 3. If the area is 3072 m^2 , find the cost of fencing the field at the rate of ₹ 4 per metre.

Sol.: Let the length and breadth of the field be $4x$ and $3x$ metres respectively. The area of the field

$$= 4x \times 3x = 12x^2 = 3072 \text{ m}^2$$

$$\text{Hence } x^2 = 3072 \div 12 = 256$$

$$\text{or } x = \sqrt{256} = 16$$

$$\text{Length} = 4x = 64 \text{ m; Breadth} = 3x = 48 \text{ m}$$

Length of fencing = Perimeter of the field

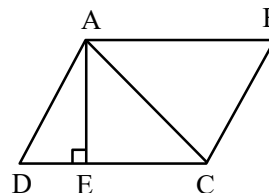
$$= 2 (64 + 48) \text{ m} = 224 \text{ m}$$

Cost of fencing at ₹ 4 per meter

$$= ₹ (224 \times 4) = ₹ 896$$

➤ AREA OF QUADRILATERALS

◆ Area of a Parallelogram :



Consider parallelogram ABCD.

Let AC be a diagonal

In $\triangle ADC$ and $\triangle CBA$

$$AD = CB, \quad CD = AB$$

AC is common

$$\therefore \triangle ADC \cong \triangle CBA$$

$$\therefore \text{Area of parallelogram ABCD}$$

$$= \text{Area of } \triangle ADC + \text{Area of } \triangle ABC$$

$$= 2 \times \text{Area of } \triangle ADC$$

$$= 2 \times \left(\frac{1}{2} CD \times AE \right) \text{ (where } AE \perp DC)$$

$$= DC \times AE$$

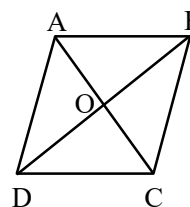
i.e. Area of parallelogram = base \times height

◆ Area of a Rhombus

Since a rhombus is also a parallelogram, its area is given by

$$\text{Area of rhombus} = \text{base} \times \text{height}$$

The area of a rhombus can also be found if the length of the diagonals are given. Let ABCD be a rhombus. We know that its diagonals AC and BD bisect each other at right angles.



Area of rhombus ABCD = area of $\triangle ABD$ + area of $\triangle CBD$

$$= \frac{1}{2} (BD \times AO) + \frac{1}{2} (BD \times CO)$$

(since $AO \perp BD$ and $CO \perp BD$)

$$= \frac{1}{2} BD (AO + CO) = \frac{1}{2} BD \times AC$$

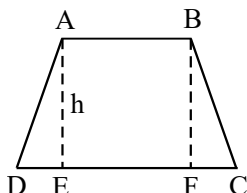
i.e. Area of rhombus = $\frac{1}{2} \times$ product of diagonals

◆ **Area of a Trapezium :**

Let ABCD be a trapezium with $AB \parallel DC$. Draw AE and BF perpendicular to DC.

Then $AE = BF = \text{height of trapezium} = h$

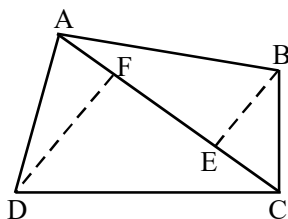
Area of trapezium ABCD = Area of $\triangle ADE$
 + Area of rectangle ABFE
 + Area of $\triangle BCF$



$$\begin{aligned}
 &= \frac{1}{2} \times DE \times h + EF \times h + \frac{1}{2} FC \times h \\
 &= \frac{1}{2} h (DE + 2EF + FC) \\
 &= \frac{1}{2} h (DE + EF + FC + EF) \\
 &= \frac{1}{2} h (DC + AB) \quad (\text{since } EF = AB) \\
 \text{i.e. Area of trapezium} &= \frac{1}{2} \times (\text{sum of parallel sides}) \\
 &\quad \times (\text{distance between parallel sides})
 \end{aligned}$$

◆ **Area of a Quadrilateral :**

Let ABCD be a quadrilateral, and AC be one of its diagonals. Draw perpendiculars BE and DF from B and D respectively to AC.



$$\begin{aligned}
 \text{Area of quadrilateral ABCD} &= \text{Area of } \triangle ABC + \text{Area of } \triangle ADC \\
 &= \frac{1}{2} AC \times BE + \frac{1}{2} AC \times DF \\
 &= \frac{1}{2} AC (BE + DF)
 \end{aligned}$$

If $AC = d$, $BE = h_1$ and $DF = h_2$ then

$$\text{Area of quadrilateral} = \frac{1}{2} d (h_1 + h_2)$$

◆ **EXAMPLES** ◆

Ex.5 A rectangle and a parallelogram have the same area of 72 cm^2 . The breadth of the rectangle is 8 cm. The height of the parallelogram is 9 cm. Find the base of the parallelogram and the length of the rectangle.

Sol. Area of rectangle $= \lambda \times b = \lambda \times 8 = 72$

$$\therefore \lambda = 9 \text{ cm}$$

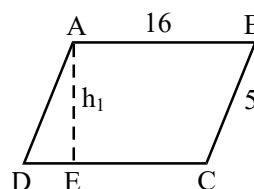
$$\begin{aligned}
 \text{Area of parallelogram} &= \text{base} \times \text{height} \\
 &= \text{base} \times 9 = 72
 \end{aligned}$$

$$\therefore \text{Base} = 8 \text{ cm}$$

Ex.6 The area of a parallelogram is 64 cm^2 . Its sides are 16 cm and 5 cm. Find the two heights of the parallelogram.

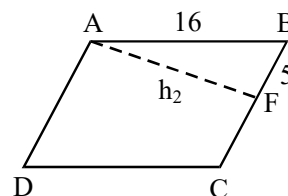
Sol.: (i) Area = base \times height $= 16 \times h_1 = 64$

$$\therefore h_1 = 4 \text{ cm}$$



(ii) Area = base \times height $= 5 \times h_2 = 64$

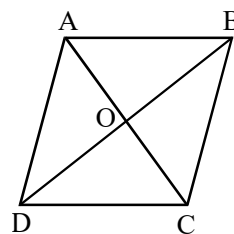
$$\therefore h_2 = 12.8 \text{ cm}$$



Ex.7 The diagonals of a rhombus measure 10 cm and 24 cm. Find its area. Also find the measure of its side.

Sol.: $AC = 10 \text{ cm}$, $BD = 24 \text{ cm}$

$$\text{Area} = \frac{1}{2} (d_1 \times d_2) = \frac{1}{2} \times 10 \times 24 \text{ cm}^2 = 120 \text{ cm}^2$$



In $\triangle ABO$, $\angle AOB = 90^\circ$, $AO = \frac{1}{2} AC = 5 \text{ cm}$,

$$BO = \frac{1}{2} BD = 12 \text{ cm}.$$

$$\therefore AB^2 = AO^2 + OB^2 = 25 + 144 = 169 = 13 \times 13$$

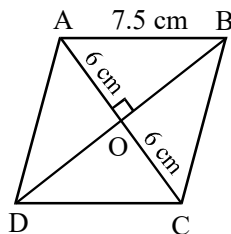
$$\therefore AB = 13 \text{ cm}$$

$$\therefore \text{Measure of } \lambda \text{ side} = 13 \text{ cm}$$

Ex.8 In rhombus ABCD, AB = 7.5 cm, and AC = 12 cm. Find the area of the rhombus.

Sol.: In $\triangle ABO$, $\angle AOB = 90^\circ$, $AO = \frac{1}{2} AC = 6$ cm,

$$AB = 7.5 \text{ cm}$$



$$\begin{aligned}\therefore OB^2 &= AB^2 - OA^2 \\ &= (7.5)^2 - 6^2 = 56.25 - 36 = 20.25\end{aligned}$$

$$\therefore OB = \sqrt{20.25} = 4.5 \text{ cm}$$

$$\therefore BD = 2 \times OB = 9 \text{ cm}$$

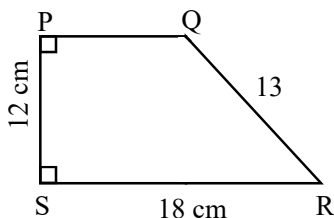
$$\text{Area of rhombus} = \frac{1}{2} d_1 \times d_2$$

$$= \frac{1}{2} \times 9 \times 12 \text{ cm}^2 = 54 \text{ cm}^2$$

Ex.9 In the trapezium PQRS, $\angle P = \angle S = 90^\circ$, PQ = QR = 13 cm, PS = 12 cm and SR = 18 cm. Find the area of the trapezium.

Sol.: The parallel sides are PQ and SR, and the distance between them is PS,

$$\text{since } \angle P = \angle S = 90^\circ$$



$$\begin{aligned}\therefore \text{Area} &= \frac{1}{2} \times \text{sum of parallel sides} \times \text{heights} \\ &= \frac{1}{2} \times (13 + 18) \times 12 \text{ cm}^2 \\ &= 186 \text{ cm}^2\end{aligned}$$

Ex.10 In trapezium ABCD, AB = AD = BC = 13 cm and CD = 23 cm. Find the area of the trapezium.

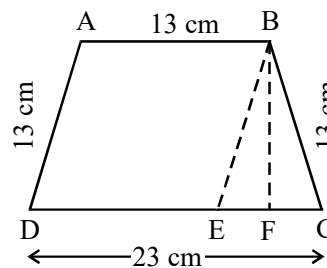
Sol.: From B draw BE \parallel AD, and BF \perp DC

Since ABED is a parallelogram, DE = 13 cm.

$$\therefore EC = 23 \text{ cm} - 13 \text{ cm} = 10 \text{ cm}$$

$$\text{Also } BE = 13 \text{ cm.}$$

Therefore BEC is an isosceles triangle.



Since $BF \perp EC$, therefore F is the midpoint of EC

$$\therefore FC = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$$

In the right triangle BFC

$$BF^2 = BC^2 - FC^2 = 13^2 - 5^2 = 144$$

$$\therefore BF = 12 \text{ cm}$$

Area of trapezium = $\frac{1}{2}$ sum of parallel sides \times height

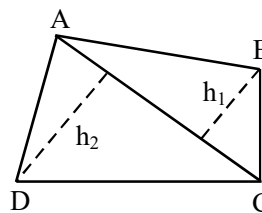
$$= \frac{1}{2} (13 + 23) \times 12 \text{ cm}^2$$

$$= 216 \text{ cm}^2$$

Note : We can also say : Area of ABCD = Area of \parallel^{gm} ABED + Area of $\triangle BCE$ (can be found by Hero's formula as all its sides are known).

Ex.11 In a quadrilateral ABCD, AC = 15 cm, The perpendiculars drawn from B and D respectively to AC measure 8.2 cm and 9.1 cm. Find the area of the quadrilateral.

Sol.:



$$\text{Area of quadrilateral} = \frac{1}{2} d (h_1 + h_2)$$

$$= \frac{1}{2} \times 15 \times (8.2 + 9.1) \text{ cm}^2$$

$$= \frac{1}{2} \times 15 \times 17.3 \text{ cm}^2$$

$$= 129.75 \text{ cm}^2$$

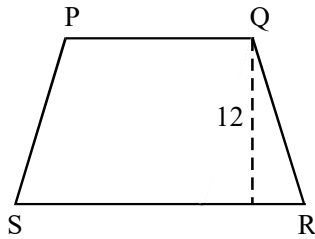
Ex.12 PQRS is a trapezium, in which $SR \parallel PQ$, and SR is 5 cm longer than PQ. If the area of the trapezium is 186 cm^2 and the height is 12 cm, find the lengths of the parallel sides.

Sol.: Let PQ = x cm; then SR = (x + 5)

Area of PQRS

$$= \frac{1}{2} \times 12 \times (x + x + 5) \text{ cm}^2$$

$$= 186 \text{ cm}^2$$



$$\therefore 6(2x + 5) = 186$$

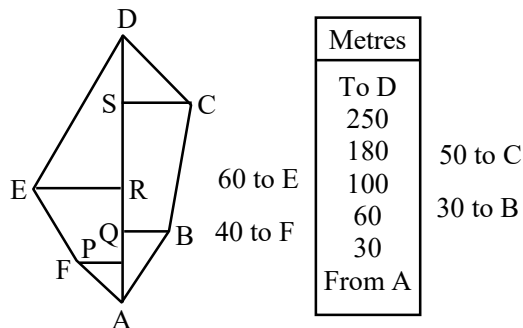
$$\text{or } 2x + 5 = 31 \quad \therefore x = 13$$

$$\therefore PQ = 13 \text{ cm}, SR = 13 \text{ cm} + 5 \text{ cm} = 18 \text{ cm}$$

➤ AREA OF IRREGULAR RECTILINEAR FIGURES

For field ABCDEF, to find its area, we proceed as follows :

1. Select two farthest corners (A and D) such that the line joining them does not intersect any of the sides. Join the corners. The line joining them is called the **base line**. In this case the base line is AD.
2. From each corner draw perpendiculars FP, BQ, ER and CS to AD. These are called **offsets**.
3. Measure and record the following lengths: AP and PF, AQ and QB, AR and RE, AS and SC.
4. Record these measurements as shown.



The field has been divided into four right triangles and two trapezia. In the trapezia, the parallel sides are perpendicular to the base line.

The area of the field is the sum of the areas of the triangles and trapezia.

$$\text{Area of } \triangle APF = \frac{1}{2} \times AP \times FP = \frac{1}{2} \times 30 \times 40 \text{ m}^2$$

$$= 600 \text{ m}^2$$

$$\text{Area of } \triangle AQB = \frac{1}{2} \times AQ \times QB = \frac{1}{2} \times 60 \times 30 \text{ m}^2$$

$$= 900 \text{ m}^2$$

$$\text{Area of trapezium PREF} = \frac{1}{2} \times PR (PF + RE)$$

$$= \frac{1}{2} \times 70 \times 100 \text{ m}^2 = 3500 \text{ m}^2$$

$$\text{Area of trapezium BQSC} = \frac{1}{2} \times QS (BQ + SC)$$

$$= \frac{1}{2} \times 120 \times 80 \text{ m}^2 = 4800 \text{ m}^2$$

$$\text{Area of } \triangle SCD = \frac{1}{2} \times SD \times SC$$

$$= \frac{1}{2} \times 70 \times 50 \text{ m}^2 = 1750 \text{ m}^2$$

$$\text{Area of } \triangle ERD = \frac{1}{2} \times RD \times ER$$

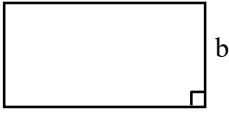
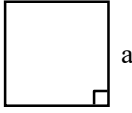
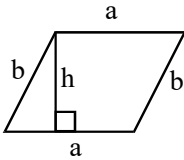
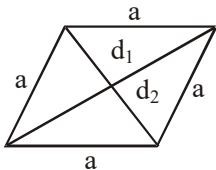
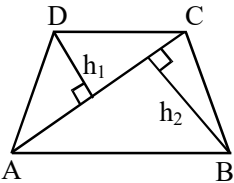
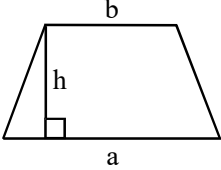
$$= \frac{1}{2} \times 150 \times 60 \text{ m}^2 = 4500 \text{ m}^2$$

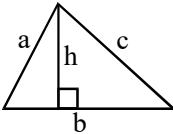
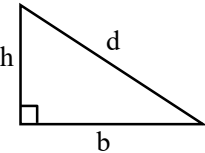
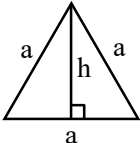
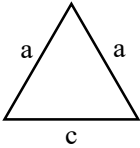
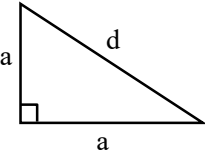
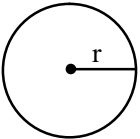
$$\text{Total area} = (600 + 900 + 3500 + 4800$$

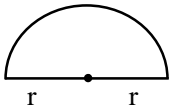
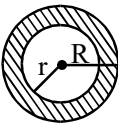
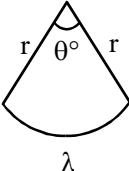
$$+ 1750 + 4500) \text{ m}^2$$

$$= 16050 \text{ m}^2$$

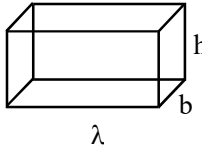
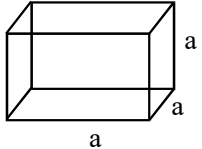
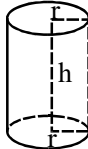
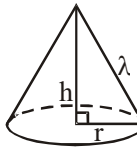
◆ Formulae to calculate area of some geometrical figures :

S. No.	Name	Figure	Perimeter in units of length	Area in square units
1.	Rectangle	 <p>a = length, b = breadth</p>	$2(a + b)$	ab
2.	Square	 <p>a = side</p>	$4a$	a^2 $\frac{1}{2}(\text{diagonal})^2$
3.	Parallelogram	 <p>a = side b = side adjacent to a h = distance between the opp. parallel sides</p>	$2(a + b)$	ah
4.	Rhombus	 <p>a = side of rhombus $d_1 d_2$ are the two diagonals</p>	$4a$	$\frac{1}{2} d_1 d_2$
5.	Quadrilateral	 <p>AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.</p>	Sum of its four sides	$\frac{1}{2} (AC) (h_1 + h_2)$
6.	Trapezium	 <p>a, b, are parallel sides and h is the distance between parallel sides</p>	Sum of its four sides	$\frac{1}{2} h (a + b)$

S. No.	Name	Figure	Perimeter in units of length	Area in square units
7.	Triangle	 <p>b is the base and h is the altitude a, b, c are three sides of Δ.</p>	$a + b + c = 2s$ where s is the semi perimeter	$\frac{1}{2} b \times h$ or $\sqrt{s(s-a)(s-b)(s-c)}$
8.	Right triangle	 <p>d(hypotenuse) $= \sqrt{b^2 + h^2}$</p>	$b + h + d$	$\frac{1}{2} bh$
9.	Equilateral triangle	 <p>a = side $h = \text{altitude} = \frac{\sqrt{3}}{2} a$</p>	$3a$	(i) $\frac{1}{2} ah$ (ii) $\frac{\sqrt{3}}{4} a^2$
10.	Isosceles triangle	 <p>c = unequal side a = equal side</p>	$2a + c$	$\frac{c\sqrt{4a^2 - c^2}}{4}$
11.	Isosceles right triangle	 <p>d(hypotenuse) $= a\sqrt{2}$, a = Each of equal sides, The angles are $90^\circ, 45^\circ, 45^\circ$.</p>	$2a + d$	$\frac{1}{2} a^2$
12.	Circle	 <p>r = radius of the circle $\pi = \frac{22}{7}$ or 3.1416</p>	$2\pi r$	πr^2

S. No.	Name	Figure	Perimeter in units of length	Area in square units
13.	Semicircle	 r r r = radius of the circle	$\pi r + 2r$	$\frac{1}{2} \pi r^2$
14.	Ring (shaded region)	 R = outer radius r = inner radius	$\pi(R^2 - r^2)$
15.	Sector of a circle	 θ° λ θ° = central angle of The sector, r = radius of the sector λ = length of the arc	$\lambda + 2r$ where $\lambda = \frac{\theta}{360} \times 2\pi r$	$\frac{\theta}{360} \times \pi r^2$

◆ Volume of some solid figures :

S. No.	Nature of the solid	Shape of the solid	Lateral/curved surface area	Total surface area	Volume	Abbreviations used
1.	Cuboid	 λ b h	$2h(\lambda + b)$	$2(\lambda b + bh + \lambda h)$	λbh	λ = length b = breadth h = height
2.	Cube	 a a a	$4a^2$	$6a^2$	a^3	a = length of edge
3.	Right circular cylinder	 r h	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height of the cylinder
4.	Right circular cone	 h r λ	$\pi r \lambda$ where $\lambda = \sqrt{r^2 + h^2}$	$\pi r(\lambda + r)$	$\frac{1}{3} \pi r^2 h$	h = height r = radius λ = slant height

❖ EXAMPLES ❖

Ex.13 Find the volume and surface area of a cuboid of $\lambda = 10$ cm, $b = 8$ cm and $h = 6$ cm.

Sol.: $V = \lambda \times b \times h = 10 \text{ cm} \times 8 \text{ cm} \times 6 \text{ cm} = 480 \text{ cm}^3$

$$\begin{aligned}\text{Surface area} &= 2(\lambda b + \lambda h + bh) \\ &= 2(10 \text{ cm} \times 8 \text{ cm} + 10 \text{ cm} \times 6 \text{ cm} + 8 \text{ cm} \times 6 \text{ cm}) \\ &= 2(80 + 60 + 48) \text{ cm}^2 = 376 \text{ cm}^2\end{aligned}$$

Ex.14 How many matchboxes of size $4 \text{ cm} \times 3 \text{ cm} \times 1.5 \text{ cm}$ can be packed in a cardboard box of size $30 \text{ cm} \times 30 \text{ cm} \times 20 \text{ cm}$?

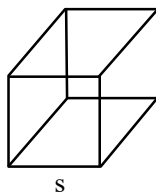
Sol.: Volume of cardboard box $= 30 \text{ cm} \times 30 \text{ cm} \times 20 \text{ cm} = 18000 \text{ cm}^3$

$$\begin{aligned}\text{Volume of each matchbox} &= 4 \text{ cm} \times 3 \text{ cm} \times 1.5 \text{ cm} \\ &= 18 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Number of matchboxes that can fit in the cardboard box} &= 18000 \text{ cm}^3 \div 18 \text{ cm}^3 = 1000\end{aligned}$$

Ex.15 The dimensions of a cube are doubled. By how many times will its volume and surface area increase?

Sol.: Let the side of the original cube be s
Then side of the new cube $= 2s$

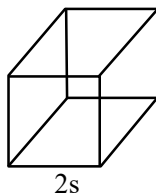


$$\begin{aligned}\text{(i) Volume of original cube} &= s \times s \times s \\ &= s^3 \text{ cubic units}\end{aligned}$$

$$\begin{aligned}\text{Volume of new cube} &= 2s \times 2s \times 2s \\ &= 8s^3 \text{ cubic units}\end{aligned}$$

\therefore Volume increases eight times if the side is doubled.

$$\begin{aligned}\text{(ii) Surface area of original cube} &= 6s^2 \\ \text{Surface area of new cube} &= 6(2s)^2 = 24s^2 \\ &= 4(6s^2)\end{aligned}$$



\therefore Surface area increases four times.

Ex.16 The outer surface of a cube of edge 5 m is painted. if the cost of painting is $\text{₹ } 1$ per 100 cm^2 , find the total cost of painting the cube.

Sol.: Surface area of cube $= 6s^2 = 6 \times 5 \text{ m} \times 5 \text{ m} = 150 \text{ m}^2$

$$= 150 \times 10000 \text{ cm}^2$$

Cost of painting 100 cm^2 is $\text{₹ } 1$.

\therefore Cost of painting $150 \times 10000 \text{ cm}^2$ is

$$\begin{aligned}&\text{₹ } \frac{1}{100} \times 150 \times 10000 \\ &= \text{₹ } 15,000\end{aligned}$$

Ex.17 A right circular cylinder has a height of 1 m and a radius of 35 cm . Find its volume, area of curved surface and total area.

Sol. $h = 1 \text{ m}$, $r = 35 \text{ cm} = 0.35 \text{ m}$

$$\begin{aligned}\text{Volume} &= \pi r^2 h = \frac{22}{7} \times 0.35 \times 0.35 \times 1 \text{ m}^3 \\ &= 0.385 \text{ m}^3\end{aligned}$$

Area of curved surface

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.35 \times 1 \text{ m}^2 = 2.2 \text{ m}^2$$

Total surface area $= 2\pi r(h + r)$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times 0.35 (1 + 0.35) \text{ m}^2 \\ &= \frac{2 \times 22 \times 0.35 \times 1.35}{7} \text{ m}^2 = 2.97 \text{ m}^2\end{aligned}$$

Ex.18 An open cylindrical tank is of radius 2.8 m and height 3.5 m . What is the capacity of the tank?

Sol. Capacity = volume of cylinder

$$\begin{aligned}&= \pi r^2 h = \frac{22}{7} \times 2.8 \times 2.8 \times 3.5 \text{ m}^3 \\ &= 86.24 \text{ m}^3\end{aligned}$$

Ex.19 A metal pipe 154 cm long, has an outer radius equal to 5.5 cm and an inner radius of 4.5 cm . what is the volume of metal used to make the pipe?

Sol. Outer volume $= \pi r^2 h = \frac{22}{7} \times (5.5)^2 \times 154 \text{ cm}^3$

$$\text{Inner volume} = \frac{22}{7} \times (4.5)^2 \times 154 \text{ cm}^3$$

\therefore Volume of metal = outer volume – inner volume

$$\begin{aligned}&= \frac{22}{7} \times 154 \times (5.5)^2 - \frac{22}{7} \times 154 \times (4.5)^2 \\ &= \frac{22}{7} \times 154 [(5.5)^2 - (4.5)^2] \\ &= \frac{22}{7} \times 154 (5.5 + 4.5) (5.5 - 4.5) \\ &= \frac{22}{7} \times 154 \times 10 \times 1 = 4840 \text{ cm}^2\end{aligned}$$

Ex.20 A cylindrical roller is used to level a rectangular playground. The length of the roller is 3.5 m and its diameter is 2.8 m . if the roller rolls over 200 times to completely cover the playground, find the area of the playground.

Sol.: When the roller rolls over the ground once completely, It covers a ground area equal to its curved surface area.

$$\text{Area of curved surface} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 1.4 \times 3.5 \text{ m}^2$$

$$\therefore \text{Area of ground} = \frac{200 \times 2 \times 22 \times 1.4 \times 3.5}{7} \text{ m}^2$$

$$= 6160 \text{ m}^2$$

Ex.21 A cylindrical pipe has an outer diameter of 1.4m and an inner diameter of 1.12m. Its length is 10m. It has to be painted on the outer and inner surfaces as well as on the rims at the top and bottom. If the rate of painting is 0.01 per cm^2 , find the cost of painting the pipe.

Sol. Outer surface area

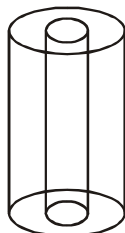
$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.7 \times 10 \text{ m}^2$$

$$= 44 \text{ m}^2$$

Inner surface area

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.56 \times 10 \text{ m}^2$$

$$= 35.2 \text{ m}^2$$



$$\text{Area of two rims} = 2 \times \frac{22}{7} \times (0.7^2 - 0.56^2)$$

$$= 1.1088 \text{ m}^2$$

\therefore Total area to be painted

$$= 44 \text{ m}^2 + 35.2 \text{ m}^2 + 1.1088 \text{ m}^2$$

$$= 80.3088 \text{ m}^2$$

Rate of painting = ₹ 0.01 per cm^2

$$= ₹ 0.01 \times 10000 \text{ per m}^2$$

$$= ₹ 100 \text{ per m}^2$$

\therefore Total cost = ₹ 80.3088 \times 100

$$= ₹ 8030.88$$

Ex.22 A rectangular piece of paper of width 20 cm and length 44 cm is rolled along its width to form a cylinder. What is the volume of the cylinder so formed?

Sol. The length of the rectangle becomes the circumference of the base of the cylinder.

$\therefore 2\pi r = 44$, where r is the radius of the cylinder.

$$\therefore r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

The width of the rectangle becomes the height of the cylinder.

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 \text{ cm}^3$$

$$= 3080 \text{ cm}^3$$

EXERCISE # 1

- Q.1** One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.
- Q.2** A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. the area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.
- Q.3** Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of ₹ 12.40 per square metre.
- Q.4** If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.
- Q.5** The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.
- Q.6** In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.
- Q.7** A rectangular grassy plot 110 m by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.
- Q.8** The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.
- Q.9** A room 5 m 55 cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.
- Q.10** Find the area of a square, one of whose diagonals is 3.8 m long.
- Q.11** The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.
- Q.12** If each side of a square is increased by 25%, find the percentage change in its area.
- Q.13** If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.
- Q.14** A room is half as long again as it is broad. The cost of carpeting the room at ₹ 5 per sq. m is ₹ 270 and the cost of papering the four walls at ₹ 10 per m² is ₹ 1720. If a door and 2 windows occupy 8 sq.m, find the dimensions of the room.
- Q.15** Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.
- Q.16** Find the area of a right-angled triangle whose base is 12 cm and hypotenuse 13 cm.
- Q.17** The base of a triangular field is three times its altitude. If the cost of cultivating the field at ₹ 24.68 per hectare be ₹ 333.18, find its base and height.
- Q.18** The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.
- Q.19** Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.
- Q.20** In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

- Q.21** The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.
- Q.22** Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.
- Q.23** The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm^2 , find the lengths of the parallel sides.
- Q.24** Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.
- Q.25** The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of ₹ 4.40 per metre.
- Q.26** The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph ?
- Q.27** A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.
- Q.28** The inner circumference of a circular race track, 14 m wide, is 440 m. Find the radius of the outer circle.
- Q.29** A sector of 120° , cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.
- Q.30** Find the ratio of the areas of the incircle and circumcircle of a square.
- Q.31** Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.
- Q.32** Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.
- Q.33** The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.
- Q.34** Find the number of bricks, each measuring $24 \text{ cm} \times 12 \text{ cm} \times 8 \text{ cm}$, required to construct a wall 24 m long, 8 m high and 60 cm thick, if 10% of the wall is filled with mortar ?
- Q.35** Water flows into a tank $200 \text{ m} \times 150 \text{ m}$ through a rectangular pipe $1.5 \text{ m} \times 1.25 \text{ m}$ @ kmph. In what time (in minutes) will the water rise by 2 metres ?
- Q.36** The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.
- Q.37** The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume and surface area.
- Q.38** The surface area of a cube is 1734 sq. cm. Find its volume.
- Q.39** A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.
- Q.40** A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are $20 \text{ cm} \times 15 \text{ cm}$, find the rise in water level.
- Q.41** Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.
- Q.42** If each edge of a cube is increased by 50%, find the percentage increase in its surface area.

- Q.43** Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.
- Q.44** Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.
- Q.45** If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.
- Q.46** 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. Find the length of the wire in metres.
- Q.47** How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron ?
- Q.48** The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.
- Q.49** If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.
- Q.50** Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.
- Q.51** Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.
- Q.52** The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.
- Q.53** The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.
- Q.54** A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

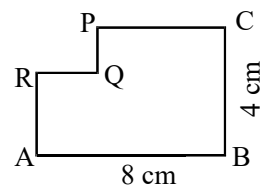
ANSWER KEY

EXERCISE # 1

- | | | | |
|---|---------------------------|--|-----------------------|
| 1. 120 m^2 | 2. 50 m. | 3. 1934.4 | 4. 120 cm^2 |
| 5. 20 cm | 6. 0.8% | 7. 680 | 8. 24 cm |
| 9. 176 | 10. 7.22 m^2 | 11. 4 : 5 | 12. 56.25% |
| 13. 50 cm | 14. $L = 9, B = 6, H = 6$ | 15. 84 cm^2 | 16. 30 cm^2 |
| 17. $B = 900 \text{ m}, H = 300 \text{ m}$. | 18. 60 cm^2 | 19. 4.5 cm | 20. 16 : 9 |
| 21. 6 cm | 22. 384 cm^2 | 23. 27 cm, 23 cm | 24. 56 m |
| 25. 5808 | 26. 250 | 27. 14 m | 28. 84 m |
| 29. 3 cm | 30. 1 : 2 | 31. 868 m^2 | 32. 17 m |
| 33. 40 cm | 34. 45000 | 35. 96 min. | 36. 8.04 kg. |
| 37. $216 \text{ cm}^3, 216 \text{ cm}^2$ | 38. 4913 cm^3 | 39. 40 | 40. 11.25 cm |
| 41. 486 cm^2 | 42. 125% | 43. 1 : 9 | |
| 44. $1540 \text{ cm}^3, 880 \text{ cm}^2, 957 \text{ cm}^2$ | 45. 12 m | 46. 112 m | 47. 400 |
| 48. 2 : 5 | 49. 26.4 kg | 50. $12936 \text{ cm}^3, 2310 \text{ cm}^2, 3696 \text{ cm}^2$ | |
| 51. 440 m | 52. 9 : 32 | 53. 9 : 8 | 54. 24 cm |

EXERCISE # 2

- Q.1** The length of a room is 5.5 m and width is 3.75 m. Find the cost of paving the floor by slabs at the rate of ₹ 800 per sq. metre.
- Q.2** The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same?
- Q.3** A rectangular plot measuring 90 meters by 50 meters is to be enclosed by wire fencing. If the poles of the fence are kept 5 metres apart, how many poles will be needed?
- Q.4** A length of a rectangular plot is 60% more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle?
- Q.5** A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
- Q.6** The difference between the length and breadth of a rectangle is 23m. If its perimeter is 206 m then find its area.
- Q.7** The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot @ ₹ 26.50 per metre is ₹ 5300, what is the length of the plot in meters?
- Q.8** The breadth of a rectangular field is 60% of its length. If the perimeter of the field is 800 m., what is the area of the field?
- Q.9** The ratio between the length and the perimeter of a rectangular plot is 1 : 3. What is the ratio between the length and breadth of the plot ?
- Q.10** The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hr completes one round in 8 minutes, then find the area of the park (in sq.).
- Q.11** The length of a rectangular hall is 5m more than its breadth. The area of the hall is 750 m². Find the length of the hall.
- Q.12** The area of a rectangle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field?
- Q.13** A rectangular field is to be fenced on three sides leaving a side of 20 feet uncovered. If the area of the field is 680 sq. feet. How many feet of fencing will be required?
- Q.14** The ratio between the perimeter and the breadth of a rectangular is 5 : 1. If the area of the rectangle is 216 sq. cm, what is the length of the rectangle?
- Q.15** A Farmer wishes to start a 100 sq. m rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. Find the dimension of the garden.
- Q.16** The sides of a rectangular field are in the ratio 3 : 4. If The area of the field is 7500 sq. m. Find the cost of fencing the field @ 25 paise per metre.
- Q.17** A rectangle of certain dimensions is chopped off from one corner of a larger rectangle as shown. AB = 8 cm and BC = 4 cm. Find the perimeter of the figure ABCPQRA (in cm).



- Q.18** A large field of 700 hectares is divided into two parts. The difference of the areas of the two parts is one-fifth of the average of the two areas. What is the area of the smaller part in hectares?
- Q.19** A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?
- Q.20** A rectangular plot is half as long again as it is broad and its area is $\frac{2}{3}$ hectares. Then find its length.
- Q.21** The areas of two circular fields are in the ratio 16 : 49. If the radius of the latter is 14 m, then what is the radius of the former.
- Q.22** If the ratio of areas of two circles is 4 : 9, then find the ratio of their circumferences.
- Q.23** The perimeter of a circle is equal to the perimeter of a square. Then find the ratio of their areas.
- Q.24** The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions?
- Q.25** Find the number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m.
- Q.26** The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.
- Q.27** The wheel of an engine, $7\frac{1}{2}$ meters in circumference makes 7 revolutions in 9 seconds. Find the speed of the train in km per hour.
- Q.28** The wheel of a motorcycle, 70 cm in diameter, makes 40 revolutions in every 10 seconds. What is the speed of the motorcycle in km/hr?
- Q.29** Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds find the speed of the smaller wheel.
- Q.30** A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions?

ANSWER KEY

EXERCISE # 2

- | | | | | |
|------------------------|-------------------|---------------|-------------------------|----------------|
| 1. j- 16,500 | 2. 7.2 cm | 3. 56 | 4. 2560 sq. cm | 5. 126 |
| 6. 2520 m ² | 7. none of these | 8. 37500sq. m | 9. 2 : 1 | 10. 153600 |
| 11. 30 m | 12. None of these | 13. 88 | 14. 18 cm | 15. 20 m × 5 m |
| 16. j- 87.50 | 17. 24 | 18. 315 | 19. 140 cm ² | 20. 100 m |
| 21. 8 m | 22. 2 : 3 | 23. 14 : 11 | 24. 1980 m | 25. 140 |
| 26. 7000 | 27. 132 | 28. 31.68 | 29. 66 cm/sec | 30. 25 |