

ALGEBRAIC EXPRESSIONS AND IDENTITIES

2 CHAPTER

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- Constant & Variable
- Algebraic Expressions
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- Identities

➤ CONSTANT & VARIABLE

◆ **Constant** : A symbol having a fixed numerical value is called a constant.

◆ **Variable** : A symbol which takes various numerical values is called a variable.

Eg. We know that the perimeter P of a square of side s is given by $P = 4 \times s$. Here, 4 is a constant and P and s are variables.

Eg. The perimeter P of a rectangle of sides l and b is given by $P = 2(l + b)$. Here, 2 is a constant and l and b are variables.

➤ ALGEBRAIC EXPRESSIONS

A combination of constants and variables connected by the signs of fundamental operation of addition, subtraction, multiplication and division is called an algebraic expression.

◆ **Terms** : Various parts of an algebraic expression which are separated by the signs of $+$ or $-$ are called the 'terms' of the expression.

Eg. $2x^2 - 3xy + 5y^2$ is an algebraic expression consisting of three terms, namely, $2x^2$, $-3xy$ and $5y^2$.

Eg. The expression $2x^3 - 3x^2 + 4x - 7$ is an algebraic expression consisting of four terms, namely, $2x^3$, $-3x^2$, $4x$ and -7 .

◆ **Monomial** : An algebraic expression containing only one term is called a monomial.

Eg. -5 , $3y$, $7xy$, $\frac{2}{3}x^2yz$, $\frac{5}{3}a^2bc^3$ etc. are all monomials.

◆ **Binomial** : An algebraic expression containing two terms is called a binomial.

Eg. The expression $2x - 3$, $3x + 2y$, $xyz - 5$ etc. are all binomials.

◆ **Trinomial** : An algebraic expression containing three terms is called a trinomial.

Eg. The expressions $a - b + 2$, $x^2 + y^2 - xy$, $x^3 - 2y^3 - 3x^2y^2z$ etc. are trinomial.

◆ **Factors** : Each terms in an algebraic expression is a product of one or more number(s) and / or literal(s). These number(s) and literal(s) are known as the factors of that terms.

A constant factor is called a numerical factor, while a variable factor is known as a literal factor.

◆ **Coefficient** : In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the other factors.

Eg. In $-5xy$, the coefficient of x is $-5y$; the coefficient of y is $-5x$ and the coefficient of xy is -5 .

Eg. In $-x$, the coefficient of x is -1 .

◆ **Constant Term** : A term of the expression having no literal factor is called a constant term.

Eg. In the algebraic expression $x^2 - xy + yz - 4$, the constant term is -4 .

◆ **Like and Unlike Terms** : The terms having the same literal factors are called like or similar terms, otherwise they are called unlike terms.

Eg. In the algebraic expression $2a^2b + 3ab^2 - 7ab - 4ba^2$, we have $2a^2b$ and $-4ba^2$ as like terms, whereas $3ab^2$ and $-7ab$ are unlike terms.

❖ EXAMPLES ❖

Ex.1 Add : $7x^2 - 4x + 5$, $-3x^2 + 2x - 1$ and $5x^2 - x + 9$.

Sol. We have,

Required sum

$$= (7x^2 - 4x + 5) + (-3x^2 + 2x - 1) + (5x^2 - x + 9)$$

$$= 7x^2 - 3x^2 + 5x^2 - 4x + 2x - x + 5 - 1 + 9$$

[Collecting like terms]

$$= (7 - 3 + 5)x^2 + (-4 + 2 - 1)x + (5 - 1 + 9)$$

[Adding like terms]

$$= 9x^2 - 3x + 13$$

Ex.2 Add : $5x^2 - \frac{1}{3}x + \frac{5}{2}$, $-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}$ and $-2x^2 + \frac{1}{5}x - \frac{1}{6}$.

Sol. Required sum

$$= \left(5x^2 - \frac{1}{3}x + \frac{5}{2}\right) + \left(-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}\right) + \left(-2x^2 + \frac{1}{5}x - \frac{1}{6}\right)$$

$$= 5x^2 - \frac{1}{2}x^2 - 2x^2 - \frac{1}{3}x + \frac{1}{2}x + \frac{1}{5}x + \frac{5}{2}$$

$$- \frac{1}{3} - \frac{1}{6} \text{ [Collecting like terms]}$$

$$= \left(5 - \frac{1}{2} - 2\right)x^2 + \left(-\frac{1}{3} + \frac{1}{2} + \frac{1}{5}\right)x + \left(\frac{5}{2} - \frac{1}{3} - \frac{1}{6}\right)$$

[Adding like term]

$$= \left(\frac{10-1-4}{2}\right)x^2 + \left(\frac{-10+15+6}{30}\right)x + \left(\frac{15-2-1}{6}\right)$$

$$= \frac{5}{2}x^2 + \frac{11}{30}x + 2$$

➤ MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Multiplication Of Algebraic Expressions

- (i) The product of two factors with like signs is positive and the product of two factors with unlike signs is negative

i.e., (a) $(+) \times (+) = +$

(b) $(+) \times (-) = -$

(c) $(-) \times (+) = -$

and, (d) $(-) \times (-) = +$

- (ii) If a is any variable and m, n are positive integers, then

$$a^m \times a^n = a^{m+n}$$

For example, $a^3 \times a^5 = a^{3+5} = a^8$,

$$y^4 \times y = y^{4+1} = y^5 \text{ etc.}$$

Ex.3 Find the product of the following pairs of polynomials :

(i) $4, 7x$

(ii) $-4a, 7a$

(iii) $-4x, 7xy$

(iv) $4x^3, -3xy$

(v) $4x, 0$

Sol. We have,

(i) $4 \times 7x = (4 \times 7) \times x = 28 \times x = 28x$

(ii) $(-4a) \times (7a) = (-4 \times 7) \times (a \times a) = -28a^2$

(iii) $(-4x) \times (7xy) = (-4 \times 7) \times (x \times xy) = -28x^{1+1}y$
 $= -28x^2y$

(iv) $(4x^3) \times (-3xy) = (4 \times -3) \times (x^3 \times xy)$
 $= -12(x^{3+1}y) = -12x^4y$

(v) $4x \times 0 = (4 \times 0) \times x = 0 \times x = 0$

Ex.4 Find the areas of rectangles with the following pairs of monomials as their length and breadth respectively :

(i) (x, y)

(ii) $(10x, 5y)$

(iii) $(2x^2, 5y^2)$

(iv) $(4a, 3a^2)$

(v) $(3mn, 4np)$

Sol. We know that the area of a rectangle is the product of its length and breadth.

Length Breadth Length $\times$ Breadth = Area

(i) $x \quad y \quad x \times y = xy$

(ii) $10x \quad 5y \quad 10x \times 5y = 50xy$

(iii) $2x^2 \quad 5y^2 \quad 2x^2 \times 5y^2 = (2 \times 5) \times (x^2 \times y^2) = 10x^2y^2$

(iv) $4a \quad 3a^2 \quad 4a \times 3a^2 = (4 \times 3) \times (a \times a^2) = 12a^3$

(v) $3mn \quad 4np \quad 3mn \times 4np = (3 \times 4) \times (m \times n \times n \times p) = 12mn^2p$

Ex.5 Multiply :

(i) $3ab^2c^3$ by $5a^3b^2c$

(ii) $4x^2yz$ by $-\frac{3}{2}x^2yz^2$

(iii) $-\frac{8}{5}x^2yz^3$ by $-\frac{3}{4}xy^2z$

(iv) $\frac{3}{14}x^2y$ by $\frac{7}{2}x^4y$

(v) $2.1a^2bc$ by $4ab^2$

Sol.

(i) We have,

$$\begin{aligned} & (3ab^2c^3) \times (5a^3b^2c) \\ &= (3 \times 5) \times (a \times a^3 \times b^2 \times b^2 \times c^3 \times c) \\ &= 15a^{1+3}b^{2+2}c^{3+1} \\ &= 15a^4b^4c^4 \end{aligned}$$

(ii) We have,

$$\begin{aligned} & (4x^2yz) \times \left(-\frac{3}{2}x^2yz^2\right) \\ &= \left(4 \times -\frac{3}{2}\right) \times (x^2 \times x^2 \times y \times y \times z \times z^2) \\ &= -6x^{2+2}y^{1+1}z^{1+2} = -6x^4y^2z^3 \end{aligned}$$

(iii) We have,

$$\begin{aligned} & \left(-\frac{8}{5}x^2yz^3\right) \times \left(-\frac{3}{4}xy^2z\right) \\ &= \left(-\frac{8}{5} \times -\frac{3}{4}\right) \times (x^2 \times x \times y \times y^2 \times z^3 \times z) \end{aligned}$$

$$= \frac{6}{5}x^{2+1}y^{1+2}z^{3+1} = \frac{6}{5}x^3y^3z^4$$

(iv) We have,

$$\begin{aligned} & \left(\frac{3}{14}x^2y\right) \times \left(\frac{7}{2}x^4y\right) \\ &= \left(\frac{3}{14} \times \frac{7}{2}\right) \times (x^2 \times x^4 \times y \times y) \\ &= \frac{3}{4}x^{2+4}y^{1+1} = \frac{3}{4}x^6y^2 \end{aligned}$$

(v) We have, $(2.1a^2bc) \times (4ab^2)$

$$\begin{aligned} &= (2.1 \times 4) \times (a^2 \times a \times b \times b^2 \times c) \\ &= 8.4a^{2+1}b^{1+2}c = 8.4a^3b^3c \end{aligned}$$

Ex.6 Multiply :

(i) $-6a^2bc$, $2a^2b$ and $-\frac{1}{4}$

(ii) $\frac{4}{9}a^5b^2$, $10a^3b$ and 6

(iii) $3.15x$ and $-23x^2y$

(iv) $-x$, x^2yz and $-\frac{3}{7}xyz^2$

Sol.

(i) We have,

$$\begin{aligned} & (-6a^2bc) \times (2a^2b) \times \left(-\frac{1}{4}\right) \\ &= \left(-6 \times 2 \times -\frac{1}{4}\right) \times (a^2 \times a^2 \times b \times b \times c) \\ &= 3a^{2+2}b^{1+1}c = 3a^4b^2c \end{aligned}$$

(ii) We have,

$$\begin{aligned} & \left(\frac{4}{9}a^5b^2\right) \times (10a^3b) \times (6) \\ &= \left(\frac{4}{9} \times 10 \times 6\right) \times (a^5 \times a^3 \times b^2 \times b) \\ &= \frac{80}{3}a^{5+3}b^{2+1} = \frac{80}{3}a^8b^3 \end{aligned}$$

(iii) We have, $(3) \times (15x) \times (-23x^2y)$

$$\begin{aligned} &= (3 \times 15 \times -23) \times (x \times x^2 \times y) \\ &= -1035x^{1+2}y = -1035x^3y. \end{aligned}$$

(iv) We have,

$$\begin{aligned}
 & (-x) \times (x^2yz) \times \left(\frac{-3}{7}xyz^2\right) \\
 &= \left(-1 \times \frac{-3}{7}\right) \times (x \times x^2 \times x \times y \times y \times z \times z^2) \\
 &= \frac{3}{7}x^{1+2+1}y^{1+1}z^{1+2} = \frac{3}{7}x^4y^2z^3
 \end{aligned}$$

Ex.7 Multiply each of the following monomials :

(i) $3xyz$, $5x$, 0 (ii) $\frac{6}{5}ab$, $\frac{5}{6}bc$, $\frac{12}{9}abc$

(iii) $\frac{3}{4}x^2yz^2$, $0.5xy^2z^2$, $1.16x^2yz^3$, $2xyz$

(vi) $20x^{10}y^{20}z^{30}$, $(10xyz)^2$

(v) $(-3x^2y)$, $(4xy^2z)$, $(-xy^2z^2)$ and $\left(\frac{4}{5}z\right)$

Sol. (i) We have,

$$\begin{aligned}
 & (3xyz) \times (5x) \times 0 \\
 &= (3 \times 5 \times 0) \times (x \times x \times y \times z) \\
 &= 0 \times x^2yz = 0
 \end{aligned}$$

(ii) We have,

$$\begin{aligned}
 & \left(\frac{6}{5}ab\right) \times \left(\frac{5}{6}bc\right) \times \left(\frac{12}{9}abc\right) \\
 & \left(\frac{6}{5} \times \frac{5}{6} \times \frac{12}{9}\right) \times (a \times a \times b \times b \times b \times c \times c) \\
 &= \frac{12}{9}a^{1+1}b^{1+1+1}c^{1+1} = \frac{4}{3}a^2b^3c^2
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 & \left(\frac{3}{4}x^2yz^2\right) \times (0.5xy^2z^2) \times (1.16x^2yz^3) \times (2xyz) \\
 &= \left(\frac{3}{4} \times 0.5 \times 1.16 \times 2\right) \times (x^2 \times x \times x^2 \times x \times y \times y^2 \times y \times y \times z^2 \times z^2 \times z^3 \times z) \\
 &= \left(\frac{3}{4} \times \frac{5}{10} \times \frac{116}{100} \times 2\right) \times (x^{2+1+2+1} \times y^{1+2+1+1} \times z^{2+2+3+1}) \\
 &= \frac{87}{100}x^6y^5z^8
 \end{aligned}$$

(iv) We have,

$$\begin{aligned}
 & (20x^{10}y^{20}z^{30}) \times (10xyz)^2 \\
 &= (20x^{10}y^{20}z^{30}) \times (10xyz) \times (10xyz) \\
 &= (20 \times 10 \times 10) \times (x^{10} \times x \times x \times y^{20} \times y \times y \times z^{30} \times z \times z) \\
 &= 2000x^{10+1+1}y^{20+1+1}z^{30+1+1} \\
 &= 2000x^{12}y^{22}z^{32}
 \end{aligned}$$

(v) We have,

$$\begin{aligned}
 & (-3x^2y) \times (4xy^2z) \times (-xy^2z^2) \times \left(\frac{4}{5}z\right) \\
 &= \left(-3 \times 4 \times -1 \times \frac{4}{5}\right) \times (x^2 \times x \times x \times y \times y^2 \times y^2 \times z \times z^2 \times z) \\
 &= \frac{48}{5}x^{2+1+1}y^{1+2+2}z^{1+2+1} = \frac{48}{5}x^4y^5z^4
 \end{aligned}$$

Ex.8 Express the following product as a monomial:

$$(x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$$

Verify the product for $x = 1$

Sol. We have,

$$\begin{aligned}
 & (x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4) \\
 &= \left(1 \times 7 \times \frac{1}{5} \times -6\right) \times (x^3 \times x^5 \times x^2 \times x^4) \\
 &= -\frac{42}{5}x^{3+5+2+4} = -\frac{42}{5}x^{14}
 \end{aligned}$$

Verification : For $x = 1$, we have

$$\begin{aligned}
 \text{L.H.S.} &= (x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4) \\
 &= (1)^3 \times \{7 \times (1^5)\} \times \left\{\frac{1}{5} \times (1)^2\right\} \times \{-6 \times (1)^4\} \\
 &= 1 \times 7 \times \frac{1}{5} \times -6 = -\frac{42}{5} \\
 \text{and, R.H.S.} &= -\frac{42}{5} \times (1)^{14} = -\frac{42}{5} \\
 \therefore \text{L.H.S.} &= \text{R.H.S.}
 \end{aligned}$$

Ex.9 Find the value of $(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$
for $a = 1$ and $b = \frac{1}{2}$

Sol. We have,

$$\begin{aligned} & (5a^6) \times (-10ab^2) \times (-2.1a^2b^3) \\ &= (5 \times -10 \times -2.1) \times (a^6 \times a \times a^2 \times b^2 \times b^3) \\ &= \left(5 \times -10 \times -\frac{21}{10}\right) \times (a^6 \times a \times a^2 \times b^2 \times b^3) \\ &= 105 a^{6+1+2} b^{2+3} = 105a^9b^5 \end{aligned}$$

Putting $a = 1$ and $b = \frac{1}{2}$, we have

$$\begin{aligned} 105a^9b^5 &= 105 \times (1)^9 \times \left(\frac{1}{2}\right)^5 \\ &= 105 \times 1 \times \frac{1}{32} = \frac{105}{32} \end{aligned}$$

Multiplication of a Monomial & a Binomial

Ex.10 Multiply : $2x$ by $(3x + 5y)$

Sol. We have,

$$2x \times (3x + 5y) = 2x \times 3x + 2x \times 5y = 6x^2 + 10xy$$

Ex.11 Multiply : $(7xy + 5y)$ by $3xy$

Sol. We have,

$$\begin{aligned} & (7xy + 5y) \times 3xy \\ &= 7xy \times 3xy + 5y \times 3xy \\ &= 21x^{1+1}y^{1+1} + 15xy^{1+1} = 21x^2y^2 + 15xy^2 \end{aligned}$$

Ex.12 Multiply : $-\frac{3ab^2}{5}$ by $\left(\frac{2a}{3} - b\right)$

Sol. We have,

$$\begin{aligned} & \left(-\frac{3ab^2}{5}\right) \times \left(\frac{2a}{3} - b\right) \\ &= \left(-\frac{3ab^2}{5}\right) \times \frac{2a}{3} - \left(-\frac{3ab^2}{5}\right) \times b \\ &= -\frac{3}{5} \times \frac{2}{3} a^{1+1}b^2 + \frac{3}{5} ab^{2+1} = -\frac{2}{5} a^2b^2 + \frac{3}{5} ab^3 \end{aligned}$$

Ex.13 Multiply : $\left(3x - \frac{4}{5}y^2x\right)$ by $\frac{1}{2}xy$.

Sol. Horizontal method

We have,

$$\left(3x - \frac{4}{5}y^2x\right) \times \frac{1}{2}xy$$

$$= 3x \times \frac{1}{2}xy - \frac{4}{5}y^2x \times \frac{1}{2}xy$$

$$= \left(3 \times \frac{1}{2}\right) \times x \times x \times y - \frac{3}{2}x^2y - \frac{2}{5}x^2y^3$$

$$\left(\frac{4}{5} \times \frac{1}{2}\right) \times y^2 \times y \times x \times x$$

$$= \frac{3}{2}x^2y - \frac{2}{5}y^3x^2 = \frac{3}{2}x^2y - \frac{2}{5}x^2y^3$$

Column method

We have,

$$3x - \frac{4}{5}y^2x$$

$$\times \frac{1}{2}xy$$

Ex.14 Determine each of the following products and find the value of each for $x = 2$, $y = 1.15$, $z = 0.01$.

(i) $27x^2(1 - 3x)$ (ii) $xz(x^2 + y^2)$

(iii) $z^2(x - y)$ (iv) $(2z - 3x) \times (-4y)$

Sol. (i) We have,

$$\begin{aligned} & 27x^2(1 - 3x) \\ &= 27x^2 \times (1 - 3x) \\ &= 27x^2 \times 1 - 27x^2 \times 3x \end{aligned}$$

[Expanding the bracket]

$$= 27x^2 - 81x^3$$

Putting $x = 2$, we have

$$\begin{aligned} & 27x^2(1 - 3x) \\ &= 27 \times (2)^2 \times (1 - 3 \times 2) = 27 \times 4 \times (1 - 6) \\ &= 27 \times 4 \times -5 = -540 \end{aligned}$$

(ii) We have, $xz(x^2 + y^2)$

$$\begin{aligned} &= xz \times (x^2 + y^2) \\ &= xz \times x^2 + xz \times y^2 = x^3z + xy^2z \end{aligned}$$

Putting $x = 2$, $y = 1.15$ and $z = 0.01$, we get

$$\begin{aligned} & xz(x^2 + y^2) \\ &= 2 \times 0.01 \times \{(2)^2 + (1.15)^2\} \\ &= 0.02 \times (4 + 1.3225) = 0.02 \times 5.3225 \\ &= 0.106450 \end{aligned}$$

(iii) We have,

$$\begin{aligned} z^2(x-y) \\ = z^2 \times (x-y) \\ = z^2 \times x - z^2 \times y = z^2x - z^2y \end{aligned}$$

Putting $x = 2$, $y = 1.15$ and $z = 0.01$, we get

$$\begin{aligned} z^2(x-y) \\ = (0.01)^2 \times (2 - 1.15) \\ = (0.0001) \times (0.85) = 0.000085 \end{aligned}$$

(vi) We have,

$$\begin{aligned} (2z - 3x) \times (-4y) \\ = (2z) \times (-4y) - 3x \times (-4y) = -8zy + 12xy \\ \text{Putting } x = 2, y = 1.15 \text{ and } z = 0.01, \text{ we have} \\ (2z - 3x) \times -4y \\ = [(2 \times 0.01) - (3 \times 2)] \times (-4 \times 1.15) \\ = (0.02 - 6) \times (-4.6) = -5.98 \times -4.6 = 27.508 \end{aligned}$$

Ex.15 Simplify the expression and evaluate them as directed :

(i) $x(x - 3) + 2$ for $x = 1$

(ii) $3y(2y - 7) - 3(y - 4) - 63$ for $y = -2$

Sol.

(i) We have,

$$\begin{aligned} x(x - 3) + 2 &= x^2 - 3x + 2 \\ \text{For } x = 1, \text{ we have} \\ x^2 - 3x + 2 &= (1)^2 - 3 \times 1 + 2 = 1 - 3 + 2 \\ &= 3 - 3 = 0 \end{aligned}$$

(ii) We have,

$$\begin{aligned} 3y(2y - 7) - 3(y - 4) - 63 \\ = (6y^2 - 21y) - (3y - 12) - 63 \\ = 6y^2 - 21y - 3y + 12 - 63 \\ = 6y^2 - 24y - 51 \\ \text{For } y = -2, \text{ we have} \\ 6y^2 - 24y - 51 &= 6 \times (-2)^2 - 24(-2) - 51 \\ &= 6 \times 4 + 24 \times 2 - 51 = 24 + 48 - 51 \\ &= 72 - 51 = 21 \end{aligned}$$

Ex.16 Subtract $3pq(p - q)$ from $2pq(p + q)$

Sol.

(i) We have,

$$\begin{aligned} 3pq(p - q) &= 3p^2q - 3pq^2 \\ \text{and, } 2pq(p + q) &= 2p^2q + 2pq^2 \end{aligned}$$

Subtraction :

$$\begin{array}{r} 2p^2q + 2pq^2 \\ 3p^2q - 3pq^2 \\ \hline -p^2q + 5pq^2 \end{array}$$

Ex.17 Add : (i) $p(p - q)$, $q(q - r)$ and $r(r - p)$

(ii) $2x(z - x - y)$ and $2y(z - y - x)$

Sol.

(i) We have,

$$\begin{aligned} p(p - q) + q(q - r) + r(r - p) \\ = p^2 - pq + q^2 - qr + r^2 - rp \\ = p^2 + q^2 + r^2 - pq - qr - rp \end{aligned}$$

(ii) We have,

$$\begin{aligned} 2x(z - x - y) + 2y(z - y - x) \\ = 2xz - 2x^2 - 2xy + 2yz - 2y^2 - 2xy \\ = 2xz - 2x^2 - 4xy + 2yz - 2y^2 \end{aligned}$$

Ex.18 Simplify each of the following expressions :

(i) $15a^2 - 6a(a - 2) + a(3 + 7a)$

(ii) $x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y)$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

Sol.

(i) We have,

$$\begin{aligned} 15a^2 - 6a(a - 2) + a(3 + 7a) \\ = 15a^2 - 6a^2 + 12a + 3a + 7a^2 \\ = 15a^2 - 6a^2 + 7a^2 + 12a + 3a = 16a^2 + 15a \end{aligned}$$

(ii) We have,

$$\begin{aligned} x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y) \\ = x^2 \times 1 - 3y^2 \times x^2 + x \times xy^2 - x \times 2x - 3y \\ \times y + 3y \times 4x^2y \\ = x^2 - 3x^2y^2 + x^2y^2 - 2x^2 - 3y^2 + 12x^2y^2 \\ = (x^2 - 2x^2) + (-3x^2y^2 + x^2y^2 + 12x^2y^2) - 3y^2 \\ = -x^2 + 10x^2y^2 - 3y^2 \end{aligned}$$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

$$\begin{aligned} &= 4st \times s - 4st \times t - 6s^2 \times t + 6s^2 \times t^2 \\ &\quad - 3t^2 \times 2s^2 + 3t^2 \times s + 2st \times s - 2st \times t \\ &= 4s^2t - 4st^2 - 6s^2t + 6s^2t^2 - 6s^2t^2 \\ &\quad + 3st^2 + 2s^2t - 2st^2 \\ &= (4s^2t - 6s^2t + 2s^2t) + (-4st^2 + 3st^2 - 2st^2) \\ &\quad + (6s^2t^2 - 6s^2t^2) \\ &= -3st^2 \end{aligned}$$

Multiplication of Two Binomials

Ex.19 Multiply $(3x + 2y)$ and $(5x + 3y)$.

Sol. We have,

$$\begin{aligned}(3x + 2y) \times (5x + 3y) &= 3x \times (5x + 3y) + 2y \times (5x + 3y) \\ &= (3x \times 5x + 3x \times 3y) + (2y \times 5x + 2y \times 3y) \\ &= (15x^2 + 9xy) + (10xy + 6y^2) \\ &= 15x^2 + 9xy + 10xy + 6y^2 \\ &= 15x^2 + 19xy + 6y^2\end{aligned}$$

Ex.20 Multiply $(2x + 3y)$ and $(4x - 5y)$

Sol. We have,

$$\begin{aligned}(2x + 3y) \times (4x - 5y) &= 2x \times (4x - 5y) + 3y \times (4x - 5y) \\ &= (2x \times 4x - 2x \times 5y) + (3y \times 4x - 3y \times 5y) \\ &= (8x^2 - 10xy) + (12xy - 15y^2) \\ &= 8x^2 - 10xy + 12xy - 15y^2 \\ &= 8x^2 + 2xy - 15y^2\end{aligned}$$

Ex.21 Multiply $(7a + 3b)$ and $(2a + 3b)$ by column method.

Sol. We have,

$$\begin{array}{r} 7a + 3b \\ \times 2a + 3b \\ \hline 14a^2 + 6ab \\ + 21ab + 9b^2 \\ \hline 14a^2 + 27ab + 9b^2 \end{array}$$

Multiplying $7a + 3b$ by $2a$
Multiplying $7a + 3b$ by $3b$
Adding the like term

Ex.22 Multiply $(7x - 3y)$ by $(4x - 5y)$ by column method.

Sol. We have,

$$\begin{array}{r} 7x - 3y \\ \times 4x - 5y \\ \hline 28x^2 - 12xy \\ - 35xy + 15y^2 \\ \hline 28x^2 - 47xy + 15y^2 \end{array}$$

Multiplying $7x - 3y$ by $4x$
Multiplying $7x - 3y$ by $-5y$
Adding the like terms

Ex.23 Multiply $(0.5x - y)$ by $(0.5x + y)$

Sol. Horizontal Method :

We have,

$$\begin{aligned}(0.5x - y) \times (0.5x + y) &= 0.5x (0.5x + y) - y (0.5x + y) \\ &= 0.5x \times 0.5x + 0.5x \times y - y \times 0.5x - y \times y \\ &= 0.25x^2 + 0.5xy - 0.5xy - y^2 \\ &= 0.25x^2 - y^2\end{aligned}$$

Column method:

We have,

$$\begin{array}{r} 0.5x - y \\ \times 0.5x + y \\ \hline 0.25x^2 - 0.5xy \\ + 0.5xy - y^2 \\ \hline 0.25x^2 - y^2 \end{array}$$

Multiplying $0.5x - y$ by $0.5x$
Multiplying $0.5x - y$ by y
Adding the like terms

Ex.24 Multiplying $\left(4x + \frac{3y}{5}\right)$ and $\left(3x - \frac{4y}{5}\right)$

Sol. Horizontal Method :

$$\begin{aligned}\left(4x + \frac{3y}{5}\right) \times \left(3x - \frac{4y}{5}\right) &= 4x \times \left(3x - \frac{4y}{5}\right) + \frac{3y}{5} \times \left(3x - \frac{4y}{5}\right) \\ &= 4x \times 3x - 4x \times \frac{4y}{5} + \frac{3y}{5} \times 3x - \frac{3y}{5} \times \frac{4y}{5} \\ &= 12x^2 - \frac{16}{5}xy + \frac{9}{5}xy - \frac{12}{25}y^2 \\ &= 12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2\end{aligned}$$

Column method:

We have,

$$\begin{array}{r} 4x + \frac{3y}{5} \\ \times 3x - \frac{4y}{5} \\ \hline 12x^2 + \frac{9}{5}xy \\ - \frac{16}{5}xy - \frac{12}{25}y^2 \\ \hline 12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2 \end{array}$$

Multiplying $4x + \frac{3y}{5}$ by $3x$.
Multiplying $4x + \frac{3y}{5}$ by $-\frac{4y}{5}$.
Adding the like terms

Ex.25 Find the value of the following products:

- (i) $(x + 2y)(x - 2y)$ at $x = 1, y = 0$
- (ii) $(3m - 2n)(2m - 3n)$ at $m = 1, n = -1$
- (iii) $(4a^2 + 3b)(4a^2 + 3b)$ at $a = 1, b = 2$

Sol. (i) We have,

$$\begin{aligned}(x+2y)(x-2y) &= x(x-2y) + 2y(x-2y) \\ &= x \times x - x \times 2y + 2y \times x - 2y \times 2y \\ &= x^2 - 2xy + 2yx - 4y^2 \\ &= x^2 - 4y^2\end{aligned}$$

When $x = 1$, $y = 0$, we get

$$\begin{aligned}(x+2y)(x-2y) &= x^2 - 4y^2 = (1)^2 - 4 \times (0)^2 = 1 - 0 = 1.\end{aligned}$$

(ii) We have,

$$\begin{aligned}(3m-2n)(2m-3n) &= 3m(2m-3n) - 2n(2m-3n) \\ &= 3m \times 2m - 3m \times 3n - 2n \times 2m + 2n \times 3n \\ &= 6m^2 - 9mn - 4mn + 6n^2 \\ &= 6m^2 - 13mn + 6n^2\end{aligned}$$

When $m = 1$, $n = -1$, we get

$$\begin{aligned}(3m-2n)(2m-3n) &= 6m^2 - 13mn + 6n^2 \\ &= 6 \times (1)^2 - 13 \times 1 \times (-1) + 6 \times (-1)^2 \\ &= 6 + 13 + 6 = 25\end{aligned}$$

(iii) We have

$$\begin{aligned}(4a^2+3b)(4a^2+3b) &= 4a^2 \times (4a^2+3b) + 3b \times (4a^2+3b) \\ &= 4a^2 \times 4a^2 + 4a^2 \times 3b + 3b \times 4a^2 + 3b \times 3b \\ &= 16a^4 + 12a^2b + 12a^2b + 9b^2 \\ &= 16a^4 + 24a^2b + 9b^2\end{aligned}$$

When, $a = 1$, $b = 2$, we get

$$\begin{aligned}(4a^2+3b)(4a^2+3b) &= 16a^4 + 24a^2b + 9b^2 \\ &= 16 \times (1)^4 + 24 \times (1)^2 \times 2 + 9 \times (2)^2 \\ &= 16 + 48 + 36 = 100\end{aligned}$$

Ex.26 Simplify the following :

- $(2x+5)(3x-2) + (x+2)(2x-3)$
- $(3x+2)(2x+3) - (4x-3)(2x-1)$
- $(2x+3y)(3x+4y) - (7x+3y)(x+2y)$

Sol. (i) We have,

$$(2x+5)(3x-2) + (x+2)(2x-3)$$

$$\begin{aligned}&= 2x(3x-2) + 5(3x-2) + x(2x-3) + 2(2x-3) \\ &= 6x^2 - 4x + 15x - 10 + 2x^2 - 3x + 4x - 6 \\ &= (6x^2 + 2x^2) + (-4x + 15x - 3x + 4x) + (-10 - 6) \\ &= 8x^2 + 12x - 16\end{aligned}$$

(ii) We have,

$$\begin{aligned}&(3x+2)(2x+3) - (4x-3)(2x-1) \\ &= \{3x(2x+3) + 2(2x+3)\} - \{4x(2x-1) - 3(2x-1)\} \\ &= (6x^2 + 9x + 4x + 6) - (8x^2 - 4x - 6x + 3) \\ &= (6x^2 + 13x + 6) - (8x^2 - 10x + 3) \\ &= 6x^2 + 13x + 6 - 8x^2 + 10x - 3 \\ &= -2x^2 + 23x + 3\end{aligned}$$

(iii) We have,

$$\begin{aligned}&(2x+3y)(3x+4y) - (7x+3y)(x+2y) \\ &= \{2x(3x+4y) + 3y(3x+4y) - 7x(x+2y) + 3y(x+2y)\} \\ &= (6x^2 + 8xy + 9xy + 12y^2) - (7x^2 + 14xy + 3xy + 6y^2) \\ &= (6x^2 + 17xy + 12y^2) - (7x^2 + 17xy + 6y^2) \\ &= 6x^2 + 17xy + 12y^2 - 7x^2 - 17xy - 6y^2 \\ &= 6x^2 - 7x^2 + 17xy - 17xy + 12y^2 - 6y^2 \\ &= -x^2 + 6y^2.\end{aligned}$$

Ex.27 Multiply : $(2x^2 - 3x + 5)$ by $(5x + 2)$.

Sol. Horizontal method:

We have,

$$\begin{aligned}&(2x^2 - 3x + 5) \times (5x + 2) \\ &= (2x^2 - 3x + 5) \times 5x + (2x^2 - 3x + 5) \times 2 \\ &= (10x^3 - 15x^2 + 25x) + (4x^2 - 6x + 10) \\ &= 10x^3 - 11x^2 + 19x + 10\end{aligned}$$

Column Method:

We have,

$$\begin{array}{r} 2x^2 - 3x + 5 \\ \times \quad 5x + 2 \\ \hline 10x^3 - 15x^2 + 25x \\ \quad + 4x^2 - 6x + 10 \\ \hline 10x^3 - 11x^2 + 19x + 10 \end{array}$$

Multiplying $2x^2 - 3x + 5$ by $5x$
 Multiplying $2x^2 - 3x + 5$ by 2
 Adding the like terms

Ex.28 Simplify :

- $(3x-2)(x-1)(3x+5)$
- $(5-x)(3-2x)(4-3x)$

Sol. (i) We have,

$$\begin{aligned}
 & (3x - 2)(x - 1)(3x + 5) \\
 &= \{(3x - 2)(x - 1)\} \times (3x + 5) \\
 & \quad \text{[By Associativity of Multiplication]} \\
 &= \{3x(x - 1) - 2(x - 1)\} \times (3x + 5) \\
 &= (3x^2 - 3x - 2x + 2) \times (3x + 5) \\
 &= (3x^2 - 5x + 2) \times (3x + 5) \\
 &= 3x^2 \times (3x + 5) - 5x(3x + 5) + 2 \times (3x + 5) \\
 &= (9x^3 + 15x^2) + (-15x^2 - 25x) + (6x + 10) \\
 &= 9x^3 + 15x^2 - 15x^2 - 25x + 6x + 10 \\
 &= 9x^3 - 19x + 10
 \end{aligned}$$

(ii) We have,

$$\begin{aligned}
 & (5 - x)(3 - 2x)(4 - 3x) \\
 &= \{(5 - x)(3 - 2x)\} \times (4 - 3x) \\
 &= \{5(3 - 2x) - x(3 - 2x)\} \times (4 - 3x) \\
 &= (15 - 10x - 3x + 2x^2) \times (4 - 3x) \\
 &= (2x^2 - 13x + 15) \times (4 - 3x) \\
 &= 2x^2 \times (4 - 3x) - 13x \times (4 - 3x) + 15 \times (4 - 3x) \\
 &= 8x^2 - 6x^3 - 52x + 39x^2 + 60 - 45x \\
 &= -6x^3 + 47x^2 - 97x + 60
 \end{aligned}$$

IDENTITIES

Identities

◆ **Identity** An identity is an equality which is true for all values of the variables (s).

◆ **Standard Identities :**

Identity 1. $(a + b)^2 = a^2 + 2ab + b^2$

Identity 2. $(a - b)^2 = a^2 - 2ab + b^2$

Identity 3. $(a + b)(a - b) = a^2 - b^2$

Other identity

$(x + a)(x + b) = x^2 + x(a + b) + ab$

Ex.29 Evaluate :

(i) $(2x + 3y)^2$

(ii) $(2x - 3y)^2$

(iii) $(2x + 3y)(2x - 3y)$

Sol. (i) We have,

$$\begin{aligned}
 (2x + 3y)^2 &= (2x)^2 + 2 \times (2x) \times (3y) + (3y)^2 \\
 & \quad \text{[Using: } (a + b)^2 = a^2 + 2ab + b^2\text{]} \\
 &= 4x^2 + 12xy + 9y^2
 \end{aligned}$$

(ii) We have,

$$\begin{aligned}
 (2x - 3y)^2 &= (2x)^2 - 2 \times (2x) \times (3y) + (3y)^2 \\
 & \quad \text{[Using: } (a - b)^2 = a^2 - 2ab + b^2\text{]} \\
 &= 4x^2 - 12xy + 9y^2
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 (2x + 3y)(2x - 3y) &= (2x)^2 - (3y)^2 \\
 & \quad \text{[Using: } (a + b)(a - b) = a^2 - b^2\text{]} \\
 &= 4x^2 - 9y^2.
 \end{aligned}$$

Ex.30 Write down the squares of each of the following binomials :

(i) $\left(x + \frac{a}{2}\right)$ (ii) $\left(5b - \frac{1}{2}\right)$ (iii) $\left(y + \frac{y^2}{2}\right)$

Sol. (i) We have,

$$\begin{aligned}
 \left(x + \frac{a}{2}\right)^2 &= x^2 + 2 \times x \times \frac{a}{2} + \left(\frac{a}{2}\right)^2 \\
 & \quad \text{[Using: } (a + b)^2 = a^2 + 2ab + b^2\text{]} \\
 &= x^2 + xa + \frac{a^2}{4}
 \end{aligned}$$

(ii) We have,

$$\begin{aligned}
 \left(5b - \frac{1}{2}\right)^2 &= (5b)^2 - 2 \times 5b \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 \\
 & \quad \text{[Using: } (a - b)^2 = a^2 - 2ab + b^2\text{]} \\
 &= 25b^2 - 5b + \frac{1}{4}
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 \left(y + \frac{y^2}{2}\right)^2 &= y^2 + 2 \times y \times \frac{y^2}{2} + \left(\frac{y^2}{2}\right)^2 \\
 &= y^2 + y^3 + \frac{y^4}{4}
 \end{aligned}$$

Ex.31 Find the product of the following binomials :

(i) $\left(\frac{4}{3}x^2 + 3\right) \left(\frac{4}{3}x^2 + 3\right)$

(ii) $\left(\frac{2}{3}x^2 + 5y^2\right) \left(\frac{2}{3}x^2 + 5y^2\right)$

Sol. (i) We have,

$$\begin{aligned} & \left(\frac{4}{3}x^2 + 3\right) \left(\frac{4}{3}x^2 + 3\right) \\ &= \left(\frac{4}{3}x^2 + 3\right)^2 \quad [\ominus a.a = a^2] \\ &= \left(\frac{4}{3}x^2\right)^2 + 2 \times \frac{4}{3}x^2 \times 3 + (3)^2 \\ & \quad [\text{Using : } (a + b)^2 = a^2 + 2ab + b^2] \\ &= \frac{16}{9}x^4 + 8x^2 + 9 \end{aligned}$$

(ii) We have,

$$\begin{aligned} & \left(\frac{2}{3}x^2 + 5y^2\right) \left(\frac{2}{3}x^2 + 5y^2\right) \\ &= \left(\frac{2}{3}x^2 + 5y^2\right)^2 \quad [\ominus a.a = a^2] \\ &= \left(\frac{2}{3}x^2\right)^2 + 2 \times \frac{2}{3}x^2 \times 5y^2 + (5y^2)^2 \\ & \quad [\text{Using : } (a + b)^2 = a^2 + 2ab + b^2] \\ &= \frac{4}{9}x^4 + \frac{20}{3}x^2y^2 + 25y^4 \end{aligned}$$

Ex.32 If $x + \frac{1}{x} = 4$, find the values of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Sol. (i) We have,

$$x + \frac{1}{x} = 4$$

On squaring both sides, we get

$$\begin{aligned} & \left(x + \frac{1}{x}\right)^2 = 4^2 \\ \Rightarrow & x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 16 \\ \Rightarrow & x^2 + 2 + \frac{1}{x^2} = 16 \\ \Rightarrow & x^2 + \frac{1}{x^2} = 16 - 2 \\ & \quad [\text{On transposing 2 on RHS}] \end{aligned}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 14$$

(ii) We have,

$$x^2 + \frac{1}{x^2} = 14$$

On squaring both sides, we get

$$\begin{aligned} & \left(x^2 + \frac{1}{x^2}\right)^2 = 14^2 \\ \Rightarrow & (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 196 \\ \Rightarrow & x^4 + \frac{1}{x^4} + 2 = 196 \\ \Rightarrow & x^4 + \frac{1}{x^4} = 196 - 2 \\ & \quad [\text{On transposing 2 on RHS}] \\ \Rightarrow & x^4 + \frac{1}{x^4} = 194 \end{aligned}$$

Ex.33 If $x - \frac{1}{x} = 9$, find the value of $x^2 + \frac{1}{x^2}$.

Sol. We have,

$$x - \frac{1}{x} = 9$$

On squaring both sides, we get

$$\begin{aligned} & \left(x - \frac{1}{x}\right)^2 = 81 \\ \Rightarrow & x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 81 \\ \Rightarrow & x^2 - 2 + \frac{1}{x^2} = 81 \\ \Rightarrow & x^2 + \frac{1}{x^2} = 81 + 2 \\ & \quad [\text{On transposing } -2 \text{ on RHS}] \\ \Rightarrow & x^2 + \frac{1}{x^2} = 83 \end{aligned}$$

Ex.34 If $x^2 + \frac{1}{x^2} = 27$, find the values of each of the following :

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$

Sol. (i) We have,

$$\begin{aligned}\left(x + \frac{1}{x}\right)^2 &= x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= x^2 + 2 + \frac{1}{x^2} \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} + 2 \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= 27 + 2 \\ &\left[\Theta x^2 + \frac{1}{x^2} = 27(\text{given})\right] \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= 29 \\ \Rightarrow x + \frac{1}{x} &= \pm \sqrt{29}\end{aligned}$$

[Taking square root of both sides]

(ii) We have,

$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 + \frac{1}{x^2} \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} - 2 \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= 27 - 2 \\ &\left[\Theta x^2 + \frac{1}{x^2} = 27(\text{given})\right] \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= 25 \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= 5^2 \\ \Rightarrow x - \frac{1}{x} &= \pm 5\end{aligned}$$

Ex.35 If $3x + 2y = 12$ and $xy = 6$, find the value of $9x^2 + 4y^2$.

Sol. We have,

$$\begin{aligned}(3x + 2y)^2 &= (3x)^2 + (2y)^2 + 2 \times 3x \times 2y \\ \Rightarrow (3x + 2y)^2 &= 9x^2 + 4y^2 + 12xy \\ \Rightarrow 12^2 &= 9x^2 + 4y^2 + 12 \times 6 \\ &[\text{Putting } 3x + 2y = 12 \text{ and } xy = 6] \\ \Rightarrow 144 &= 9x^2 + 4y^2 + 72 \\ \Rightarrow 144 - 72 &= 9x^2 + 4y^2 \\ \Rightarrow 9x^2 + 4y^2 &= 72\end{aligned}$$

Ex.36 If $4x^2 + y^2 = 40$ and $xy = 6$, find the value of $2x + y$.

Sol. We have,

$$\begin{aligned}(2x + y)^2 &= (2x)^2 + y^2 + 2 \times 2x \times y \\ \Rightarrow (2x + y)^2 &= (4x^2 + y^2) + 4xy \\ \Rightarrow (2x + y)^2 &= 40 + 4 \times 6 \\ &[\text{Using } 4x^2 + y^2 = 40 \text{ and } xy = 6] \\ \Rightarrow (2x + y)^2 &= 64 \Rightarrow 2x + y = \pm \sqrt{64} \\ \Rightarrow 2x + y &= \pm 8\end{aligned}$$

[Taking square root of both sides]

Ex.37 Find the continued product :

$$\begin{aligned}\text{(i)} & (x + 2)(x - 2)(x^2 + 4) \\ \text{(ii)} & (2x + 3y)(2x - 3y)(4x^2 + 9y^2) \\ \text{(iii)} & (x - 1)(x + 1)(x^2 + 1)(x^4 + 1) \\ \text{(iv)} & \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right) \\ \text{(v)} & \left(x - \frac{y}{5} - 1\right)\left(x + \frac{y}{5} + 1\right)\end{aligned}$$

Sol. (i) We have,

$$\begin{aligned}& (x + 2)(x - 2)(x^2 + 4) \\ &= \{(x + 2)(x - 2)\}(x^2 + 4) \\ &[\text{By associativity of multiplication}] \\ &= (x^2 - 2^2)(x^2 + 4) [\Theta (a + b)(a - b) = a^2 - b^2] \\ &= (x^2 - 4)(x^2 + 4) \\ &= (x^2)^2 - 4^2 [\Theta (a + b)(a - b) = a^2 - b^2] \\ &= x^4 - 16\end{aligned}$$

(ii) We have,

$$\begin{aligned}
 & (2x + 3y) (2x - 3y) (4x^2 + 9y^2) \\
 &= \{(2x + 3y) (2x - 3y)\} (4x^2 + 9y^2) \\
 &= \{(2x + 3y) (2x - 3y)\} (4x^2 + 9y^2) \\
 &= \{(2x)^2 - (3y)^2\} (4x^2 + 9y^2) \\
 &\quad [\text{Using : } (a + b) (a - b) = a^2 - b^2] \\
 &= (4x^2 - 9y^2) (4x^2 + 9y^2) \\
 &= (4x^2)^2 - (9y^2)^2 \\
 &\quad [\text{Using : } (a + b) (a - b) = a^2 - b^2] \\
 &= 16x^4 - 81y^4.
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 & (x - 1) (x + 1) (x^2 + 1) (x^4 + 1) \\
 &= \{(x - 1) (x + 1)\} (x^2 + 1) (x^4 + 1) \\
 &= (x^2 - 1) (x^2 + 1) (x^4 + 1) \\
 &= \{(x^2 - 1)(x^2 + 1)\} (x^4 + 1) \\
 &= \{(x^2)^2 - 1^2\} (x^4 + 1) \\
 &= (x^4 - 1) (x^4 + 1) \\
 &= \{(x^4)^2 - 1^2\} \\
 &= x^8 - 1
 \end{aligned}$$

(iv) We have

$$\begin{aligned}
 & \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x^4 + \frac{1}{x^4}\right) \\
 &= \left\{\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\right\} \left(x^2 + \frac{1}{x^2}\right) \left(x^4 + \frac{1}{x^4}\right) \\
 &= \left(x^2 - \frac{1}{x^2}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x^4 + \frac{1}{x^4}\right) \\
 &= \left\{(x^2)^2 - \left(\frac{1}{x^2}\right)^2\right\} \left(x^4 + \frac{1}{x^4}\right) \\
 &= \left(x^4 - \frac{1}{x^4}\right) \left(x^4 + \frac{1}{x^4}\right)
 \end{aligned}$$

$$\begin{aligned}
 &= (x^4)^2 - \left(\frac{1}{x^4}\right)^2 \\
 &= x^8 - \frac{1}{x^8}
 \end{aligned}$$

(v) We have,

$$\begin{aligned}
 & \left(x - \frac{y}{5} - 1\right) \left(x + \frac{y}{5} + 1\right) \\
 &= \left\{x - \left(\frac{y}{5} + 1\right)\right\} \left\{x + \left(\frac{y}{5} + 1\right)\right\} \\
 &= x^2 - \left(\frac{y}{5} + 1\right)^2 \\
 &= x^2 - \left(\frac{y^2}{25} + \frac{2y}{5} + 1\right) \\
 &= x^2 - \frac{y^2}{25} - \frac{2y}{5} - 1
 \end{aligned}$$

Ex.38 Using the formulae for squaring a binomial, evaluate the following :

$$(i) (101)^2 \quad (ii) (99)^2 \quad (iii) (93)^2$$

Sol. We have,

$$\begin{aligned}
 (i) \quad (101)^2 &= (100 + 1)^2 \\
 &= (100)^2 + 2 \times 100 \times 1 + (1)^2 \\
 &\quad [\text{Using : } (a + b)^2 = a^2 + 2ab + b^2] \\
 &= 10000 + 200 + 1 \\
 &= 10201 \\
 (ii) \quad (99)^2 &= (100 - 1)^2 \\
 &= (100)^2 - 2 \times 100 \times 1 + (1)^2 \\
 &\quad [\text{Using : } (a - b)^2 = a^2 - 2ab + b^2] \\
 &= 10000 - 200 + 1 \\
 &= 9801 \\
 (iii) \quad (93)^2 &= (90 + 3)^2 \\
 &= (90)^2 + 2 \times 90 \times 3 + (3)^2 \\
 &= 8100 + 540 + 9 = 8649
 \end{aligned}$$

Ex.39 Find the value of x, if

$$\begin{aligned}
 (i) \quad 6x &= 23^2 - 17^2 \quad (ii) \quad 4x = 98^2 - 88^2 \\
 (iii) \quad 25x &= 536^2 - 136^2
 \end{aligned}$$

Sol. (i) We have,

$$6x = 23^2 - 17^2$$

$$\Rightarrow 6x = (23 + 17) \times (23 - 17)$$

$$\quad \quad \quad [\text{Using : } a^2 - b^2 = (a + b)(a - b)]$$

$$\Rightarrow 6x = 40 \times 6$$

$$\Rightarrow \frac{6x}{6} = \frac{40 \times 6}{6} \quad [\text{Dividing both sides by 6}]$$

$$\Rightarrow x = 40$$

(ii) We have,

$$4x = 98^2 - 88^2$$

$$\Rightarrow 4x = (98 + 88) \times (98 - 88)$$

$$\quad \quad \quad [\text{Using : } a^2 - b^2 = (a + b)(a - b)]$$

$$\Rightarrow 4x = 186 \times 10$$

$$\Rightarrow \frac{4x}{4} = \frac{186 \times 10}{4} \quad [\text{Dividing both sides by 4}]$$

$$\Rightarrow x = \frac{1860}{4}$$

$$\Rightarrow x = 465$$

(iii) We have,

$$25x = 536^2 - 136^2$$

$$\Rightarrow 25x = (536 + 136) \times (536 - 136)$$

$$\quad \quad \quad [\text{Using : } (a^2 - b^2) = (a + b)(a - b)]$$

$$\Rightarrow 25x = 672 \times 400$$

$$\Rightarrow \frac{25x}{25} = \frac{672 \times 400}{25} \quad [\text{Dividing both sides by 25}]$$

$$\Rightarrow x = 672 \times 16$$

$$\Rightarrow x = 10752$$

Ex.40 What must be added to $9x^2 - 24x + 10$ to make it a whole square ?

Sol. We have,

$$9x^2 - 24x + 10 = (3x)^2 - 2 \times 3x \times 4 + 10$$

It is evident from the above expression that

First term = $3x$ and, Second term = 4

To make the given expression a whole square, we must have $(4)^2 = 16$ in place of 10 .

Hence, we must add 6 to it to make a perfect square.

Adding and subtracting 6 , we get

$$9x^2 - 24x + 10 + 6 - 6 = 9x^2 - 24x + 16 - 6$$

$$= (3x - 4)^2 - 6$$

Ex.41 Find the following products:

(i) $(x + 2)(x + 3)$

(ii) $(x + 7)(x - 2)$

(iii) $(y - 4)(y - 3)$

(iv) $(y - 7)(y + 3)$

(v) $(2x - 3)(2x + 5)$

(iv) $(3x + 4)(3x - 5)$

Sol. Using the identity :

$$(x + a)(x + b) = x^2 + (a + b)x + ab, \text{ we have}$$

(i) $(x + 2)(x + 3) = x^2 + (2 + 3)x + 2 \times 3$

$$= x^2 + 5x + 6$$

(ii) $(x + 7)(x - 2) = (x + 7)\{x + (-2)\}$

$$= x^2 + \{7 + (-2)\}x + 7 \times 2$$

$$= x^2 + 5x - 14$$

(iii) $(y - 4)(y - 3) = \{y + (-4)\}\{y + (-3)\}$

$$= y^2 + \{(-4) + (-3)\}y + (-4) \times (-3)$$

$$= y^2 - 7y + 12$$

(iv) $(y - 7)(y + 3) = \{y + (-7)\}(y + 3)$

$$= y^2 + \{(-7) + 3\}y + (-7) \times 3$$

$$= y^2 - 4y - 21$$

(v) $(2x - 3)(2x + 5) = (y - 3)(y + 5),$

where $y = 2x$

$$= \{y + (-3)\}(y + 5)$$

$$= y^2 + \{(-3) + 5\}y + (-3) \times 5$$

$$= y^2 + 2y - 15$$

Ex.42 Evaluate the following:

(i) 107×103 (ii) 56×48 (iii) 95×97

Sol. Using the identity :

$$(x + a)(x + b) = x^2 + (a + b)x + ab \text{ we have}$$

(i) $107 \times 103 = (100 + 7) \times (100 + 3)$

$$= (100)^2 + (7 + 3) \times 100 + 7 \times 3$$

$$= 10000 + 10 \times 100 + 21$$

$$= 10000 + 1000 + 21$$

$$= 11021$$

(ii) $56 \times 48 = (50 + 6) \times (50 - 2)$

$$= (50 + 6) \times \{50 + (-2)\}$$

$$= (50)^2 + \{6 + (-2)\} \times 50 + 6 \times (-2)$$

$$= 2500 + 4 \times 50 - 12$$

$$= 2500 + 200 - 12$$

$$= 2700 - 12 = 2688$$

(iii) $95 \times 97 = (100 - 5) \times (100 - 3)$

$$= \{100 + (-5)\} \times \{100 + (-3)\}$$

$$= (100)^2 + \{(-5) + (-3)\} \times 100 + (-5) \times (-3)$$

$$= 10000 - 8 \times 100 + 15$$

$$= 10000 - 800 + 15 = 9215$$

EXERCISE # 1

Q.1 Add the following algebraic expressions:

$$2, \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{2}, -\frac{4}{3} + \frac{2y^2}{3} - \frac{y}{2},$$

$$\frac{5y^3}{3} + 3y^2 + 3y + \frac{6}{5}$$

Q.2 Subtract : $\left(-2y^2 + \frac{1}{2}y - 3\right)$ from $7y^2 - 2y + 10$.

Q.3 Subtract: $\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{3}x^2yz$ from

$$\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y.$$

Q.4 Find the volume of the rectangular boxes with following length, breadth and height :

	Length	Breadth	Height
(i)	2ax	3by	5cz
(ii)	m ² n	n ² p	p ² m
(iii)	2q	4q ²	8q ³

Q.5 Find each of the following products:

- (i) $(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$
 (ii) $(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

Q.6 Multiply $-\frac{4}{3}xy^3$ by $\frac{6}{7}x^2y$ and verify your result for $x = 2$ and $y = 1$.

Q.7 Find the product of $-5x^2y$, $-\frac{2}{3}xy^2z$,

$\frac{8}{15}xyz^2$ and $-\frac{1}{4}$. Verify the result when $x = 1$,
 $y = 2$ and $z = q$.

Q.8 Find the product of $\frac{7}{2}s^2t$ and $s + t$. Verify the result for $s = \frac{1}{2}$ and $t = 5$.

Q.9 Find the following products:

- (i) $100x \times (0.01x^4 - 0.01x^2)$
 (ii) $121.5ab \times \left(ac + \frac{b}{10}\right)$
 (iii) $0.1a \times (0.01a \times 0.001b)$

Q.10 Add:

- (i) $5m(3 - m)$ and $6m^2 - 13m$
 (ii) $4y(3y^2 + 5y - 7)$ and $2(y^3 - 4y^2 + 5)$

Q.11 (i) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$
 (ii) Subtract : $3a(a + b + c) - 2b(a + b + c)$ from $4c(-a + b + c)$

Q.12 Multiply $\left(\frac{1}{5}x - \frac{1}{4}y\right)$ and $(5x^2 - 4y^2)$

Q.13 Multiply $(3x^2 + y^2)$ by $(x^2 + 2y^2)$.

Q.14 Multiply: $\{2m + (-n)\}$ by $\{-3m + (-5)\}$

Q.15 Find the product of $\left(y + \frac{2}{7}y^2\right)$ and $(7y - y^2)$ and verify the result for $y = 3$.

Q.16 Simplify the following:

- (i) $\frac{1}{3}(6x^2 + 15y^2)(6x^2 - 15y^2)$
 (ii) $9x^4(2x^3 - 5x^4) \times 5x^6(x^4 - 3x^2)$

Q.17 Multiply: $(2x^2 - 4x + 5)$ by $(x^2 + 3x - 7)$

Q.18 Find the product of the following binomials:

- (i) $(6x^2 - 7y^2)(6x^2 - 7y^2)$
 (ii) $\left(\frac{1}{2}x - \frac{1}{5}y\right)\left(\frac{1}{2}x - \frac{1}{5}y\right)$

Q.19 Find the product of the following binomials:

- (i) $\left(\frac{3}{4}x + \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right)$
 (ii) $\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$
 (iii) $(a^2 + b^2)(-a^2 + b^2)$
 (iv) $(-a + c)(-a - c)$

Q.20 If $x + \frac{1}{x} = 9$ and $x^2 + \frac{1}{x^2} = 53$, find the value of $x - \frac{1}{x}$.

Q.21 If $x + y = 12$ and $xy = 14$, find the value of $x^2 + y^2$.

Q.22 Simplify the following products:

(i) $(x^2 + x + 1)(x^2 - x + 1)$

(ii) $(x^2 + 2x + 2)(x^2 - 2x + 2)$

Q.23 Simplify the following by using:

$(a + b)(a - b) = a^2 - b^2$.

(i) 68×72 (ii) 101×99

(iii) 67×73 (iv) $128^2 - 77^2$

Q.24 Find the greatest common factor of the monomials $6x^3a^2b^2c$, $8x^2ab^3c^3$ and $12a^3b^2c^2$.

ANSWER KEY

EXERCISE # 1

1. $\frac{28}{15} + \frac{19}{6}y + 2y^2 + \frac{25}{6}y^3$
2. $9y^2 - \frac{5}{2}y + 13$
3. $\frac{41}{15}x^2yz - \frac{5}{6}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$
4. (i) $30\text{ }abcxyz$ (ii) $m^3n^3p^3$ (iii) $64q^6$
5. (i) $-84x^{14}a^7$ (ii) $-48s^6t^8$ (iii) $1000x^{14}y^{11}$
6. $-\frac{8}{7}x^3y^4$
7. $-\frac{4}{9}x^4y^4z^4$
8. $\frac{7}{2}s^3t + \frac{7}{2}s^2t^2$
9. (i) $x^5 - x^3$
(ii) $121.5a^2bc + 12.15ab^2$
(iii) $0.001a^2 + 0.0001ab$
10. (i) $2m + m^2$
11. (i) $25ln + 5l^2$
(ii) $-7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$
12. $x^3 - \frac{4}{5}xy^2 - \frac{5}{4}x^2y + y^3$
13. $3x^4 + 7x^2y^2 + 2y^4$
15. $7y^2 + y^3 - \frac{2}{7}y^4$
16. (i) $12x^4 - 75y^4$
(ii) $-225x^{18} + 90x^{17} + 675x^{16} - 270x^{15}$
17. $2x^4 + 2x^3 - 21x^2 + 43x - 35$
18. (i) $36x^4 - 84^2y^2 + 49y^4$ (ii) $\frac{1}{4}x^4 - \frac{xy}{5} + \frac{1}{25}y^2$
19. (i) $\frac{9}{16}x^2 - \frac{25}{36}y^2$ (ii) $4a^2 - \frac{9}{b^2}$
(iii) $b^4 - a^4$ (iv) $a^2 - c^2$
20. ± 5
21. 116
22. (i) $x^4 + x^2 + 1$, (ii) $x^4 - 2x^2 + 4$
23. (i) 4896 (ii) 9999 (iii) 4891 (iv) 10455

EXERCISE # 2

- Q.1** If $\left(x + \frac{1}{x}\right) = 3$, then find value of $\left(x^2 + \frac{1}{x^2}\right)$.
- Q.2** If $\left(x - \frac{1}{x}\right) = \frac{1}{2}$, then find value of $\left(4x^2 + \frac{4}{x^2}\right)$.
- Q.3** If $\left(x + \frac{1}{x}\right) = 4$, then find value of $\left(x^4 + \frac{1}{x^4}\right)$.
- Q.4** If $\left(x + \frac{1}{x}\right) = \sqrt{3}$, then find the value of $\left(x^3 + \frac{1}{x^3}\right)$.
- Q.5** If $\left(x + \frac{1}{x}\right) = 2$, then find the value of $\left(x^3 + \frac{1}{x^3}\right)$.
- Q.6** If $\left(x^2 + \frac{1}{x^2}\right) = 102$, then find the value of $\left(x - \frac{1}{x}\right)$.
- Q.7** If $\left(x^4 + \frac{1}{x^4}\right) = 322$, then find the value of $\left(x - \frac{1}{x}\right)$.
- Q.8** If $\left(x^3 + \frac{1}{x^3}\right) = 52$, then find the value of $\left(x + \frac{1}{x}\right)$.
- Q.9** If $\left(x^3 - \frac{1}{x^3}\right) = 14$, then find the value of $\left(x - \frac{1}{x}\right)$.
- Q.10** If x is an integer such that $\left(x + \frac{1}{x}\right) = \left(\frac{17}{4}\right)$, then find the value of $\left(x - \frac{1}{x}\right)$.
- Q.11** If $\left(x^4 + \frac{1}{x^4}\right) = 727$, then find the value of $\left(x^3 - \frac{1}{x^3}\right)$.
- Q.12** If $\left(2x - \frac{3}{x}\right) = 5$, then find the value of $\left(4x^2 - \frac{9}{x^2}\right)$.
- Q.13** If $x + y = 7$ and $xy = 12$, then find the value of $(x^2 + y^2)$.
- Q.14** If $\frac{5^x}{125} = 1$, then find the value of x .
- Q.15** Find the values of -
(i) 998^2 (ii) 5.2^2
(iii) 78×82 (iv) 1.05×9.5
(v) $51^2 - 49^2$ (vi) $(1.02)^2 - (0.98)^2$
(vii) $12.1^2 - 7.9^2$ (viii) 103×104
(ix) 5.1×5.2 (x) 9.7×9.8
- Q.16** Show that
(i) $(3x + 7)^2 - 84x = (3x - 7)^2$
(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$
(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$
(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

ANSWER KEY

EXERCISE # 2

- | | | | | | | |
|----------------|------------|------------|------------|---------|-----------|----------|
| 1. 7 | 2. 9 | | | | | |
| 3. 194 | 4. 0 | | | | | |
| 5. 2 | 6. 10 | | | | | |
| 7. 4 | 8. 4 | | | | | |
| 9. 2 | 10. $15/4$ | | | | | |
| 11. 140 | 12. 35 | | | | | |
| 13. 25 | 14. 3 | | | | | |
| 15. (i) 996004 | (ii) 27.04 | (iii) 6396 | (iv) 9.975 | (v) 200 | (vi) 0.08 | (vii) 84 |
| (viii) 10712 | (ix) 26.52 | (x) 95.06 | | | | |