UNDERSTANDING QUADRILATERALS

1
CHAPTER

CONTENTS

➢ POLYGONS

A simple closed figure made up of only line segments is called a polygon. If number of sides is n then number of angles is also n.

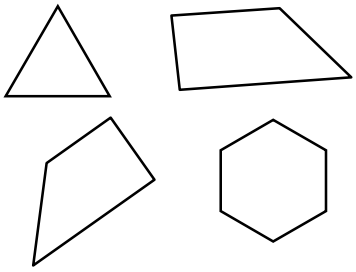
(n is natural no. ≥ 3)

Types : There are two types of polygon.

(1) Convex polygon (2) Concave polygon

◈ Convex polygon

If each angle of a polygon is less than 180° then these are called convex polygon.



If n is number of sides, then

n = 3  triangle

n = 4  quadrilateral

n = 5  pentagon

n = 6  Hexagon

n = 7  Heptagon

n = 8  Octagon

for n = 9, n = 10 polygon are called nonagon & decagon respectively.

If sides of polygon are different with each other then interior (as well as exterior) angles are different and polygon is said to be irregular but if all sides are equal then polygon is said to be regular polygon, like equilateral triangle, square etc. In these, all angles are equal and value of each interior angle is $\frac{(n-2)\,180^\circ}{n}$, n = number of sides.

also sum of all interior angles = (n – 2) 180°

❖ EXAMPLES ❖

Ex.1 Find the sum of all interior angles for a decagon.

Sol. Θ n = 10

$\therefore$ sum of angles = (n – 2) 180°

= (10 – 2) 180 = 8 × 180 = 1440°

Ex.2 If the sum of all angles of a polygon is 720° then find number of sides.

Sol. Θ sum of angles = (n – 2) 180°

$\Rightarrow$ 720° = (n – 2) 180°

$\Rightarrow n - 2 = \frac{720}{180} = 4 \Rightarrow n = 6$

Ex.3 If all sides are equal of a polygon of 15 sided then find value of each interior angle.

Sol. Θ n = 15

$\therefore$ each interior angle = $\frac{(n-2)180^\circ}{n}$

$= \left(\frac{15-2}{15}\right)180^\circ = 13 \times 12 = 156^\circ$

Note :

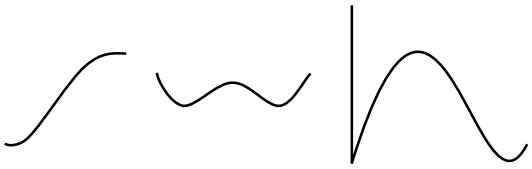
(1) Sum of all exterior angles is equal to 360°

(2) Value of each exterior angle of regular polygon is $\frac{360°}{n}$.

(3) Number of Diagonals in polygon = $\frac{n(n-3)}{2}$

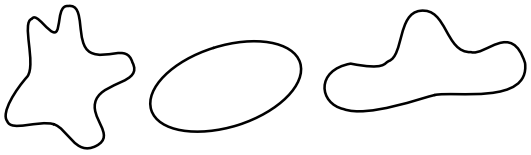
(4) Simple curve can be bound or not

Eg.



(5) Simple closed curve is not always circle

Eg.



Ex.4 Check whether 115° can an exterior angle of a regular polygon ?

Sol. Θ each exterior angle = $\frac{360°}{n}$

$= \frac{360°}{115}$

$\neq$ natural number

$\therefore$ 115° can not be an exterior angle of a regular polygon.

Ex.5 Find the number of diagonals if the sum of all interior angles is 900°.

Sol. Θ sum of all interior angles

$= (n-2)\ 180°$

$900 = (n-2)\ 180°$

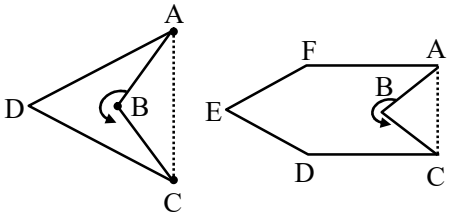
$\Rightarrow n - 2 = \frac{900}{180} = 5$

$\Rightarrow n = 7$

Now no. of diagonals = $\frac{7(7-3)}{2} = \frac{7(4)}{2} = 14$

◈ **Concave polygon**

If value of one angle of a polygon is more than 180° then these polygon are called concave polygon. In these, one diagonal is in exterior of polygon.



$\angle B > 180°$ also dia. (AC) is in exterior

◈ **Angle sum property**

The sum of measure of all interior angles of a polygon is called angle sum property (a. s. p.)

Eg. A. S. P. for triangle, quadrilateral……are 180°, 360°…….

Ex.6 If two angles of a triangle are 40° & 58° then find the third angle.

Sol. Θ The sum of all angles = 180° (A.S.P.)

$40° + 58° +$ Third angle $= 180°$

$\therefore$ Third angle $= 180° - 98° = 102°$

Ex.7 If two angles of a hexagon are right angles & rest angles are same to each other then find the value of one of the other angles.

Sol. Let the other each angle = x°

$\therefore 90° + 90° + x + x + x + x = (n-2)\ 180°$

$\Rightarrow 180 + 4x = (6-2)180°$

$\Rightarrow 4x = 720 - 180°$

$\Rightarrow x = \frac{540}{4} = 135°.$

Ex.8 Find the maximum exterior and minimum interior angle of regular polygon.

Sol. Θ Minimum number of sides in a regular polygon is 3 (equilateral Δ)

$\therefore$ each angle = x° (Let)

$\therefore 3x = 180 \Rightarrow x = 60°$

$\therefore$ minimum value of interior angle = 60°

$\therefore$ maximum exterior angle = 120°

(by linear pair).

Ex.9 The angles of a quadrilateral are in ratio 1 : 3 : 7 : 9 find the measure of each angle.

Sol. Let angles are x°, 3x°, 7x°, 9x°

$\therefore$ x + 3x + 7x + 9x = 360° (A.S.P.)

$\Rightarrow$ 20x = 360

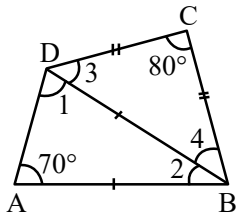
x = 18

$\therefore$ angles are 18, 18 × 3, 18 × 7, 18 × 9

= 18°, 54°, 126°, 162°

Verification 18° + 54° + 126° + 162° = 360°

Ex.10 Find the angles of quadrilateral ABCD, in given figure.



Sol. Θ AB = BD (in ΔADB)

$\therefore$ $\angle 1 = 70°$

$\therefore$ $\angle 1 + 70° + \angle 2 = 180°$ (A.S.P.)

$\Rightarrow$ $70 + 70 + \angle 2 = 180°$

$\Rightarrow$ $\angle 2 = 180° - 140° = 40°$

also in ΔDCB

DC = CB

$\therefore$ $\angle 3 = \angle 4 = y$

$\therefore$ y + y + 80 = 180° (A.S.P.)

$\Rightarrow$ 2y = 180 – 80

$\Rightarrow$ $y = \frac{100}{2} = 50°$

$\therefore$ $\angle 3 = \angle 4 = 50°$

$\therefore$ $\angle ABC = \angle 2 + \angle 4 = 40° + 50° = 90°$

& $\angle ADC = \angle 1 + \angle 3 = 70° + 50° = 120°$

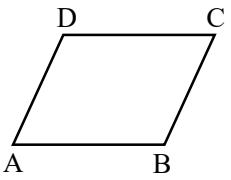
$\therefore$ angles are 70°, 90°, 80°, 120°.

◈ **Types of quadrilateral :**

(1) Parallelogram (2) Rhombus (3) Rectangle

(4) Square (5) Trapezium (6) Kite

(1) If opposite sides are equal and parallel then quadrilateral is called parallelogram ($||^{gm}$)



Properties :

(i) Opposite sides are equal i.e. AB = CD, AD = BC

(ii) Opposite sides are parallel i.e. AB || CD, AD || BC

(iii) Opposite angles are equal i.e. $\angle A = \angle C$; $\angle B = \angle D$

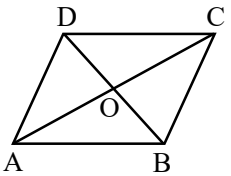
(iv) Sum of adjacent angles is 180° or adjacent angles are supplementary i.e. $\angle A + \angle C = 180°$ or $\angle A + \angle D = 180°$ etc.

(v) Length of both diagonals are different.

(vi) Diagonal bisect each other at same point.

(vii) One diagonal divides $||^{gm}$ into two congruent triangles i.e. $\Delta ABC \cong \Delta ADC$.

(2) A quadrilateral whose all sides are equal, is called rhombus. It is $||^{gm}$ also Θ opposite sides are equal and parallel.



Properties :

(i) All sides are equal i.e. AB = BC = CD = DA

(ii) Opposite sides are parallel i.e. AB || CD, AD || BC

(iii) Opposite angles are equal i.e. $\angle A = \angle C$; $\angle B = \angle D$

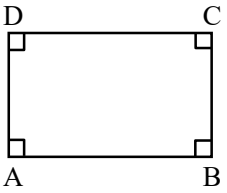
(iv) Sum of adjacent angles is 180° or adjacent angles are supplementary i.e. $\angle A + \angle C = 180°$ or $\angle A + \angle D = 180°$ etc.

(v) Length of both diagonals are different.

(vi) Diagonals bisect each other at right angle. i.e. AO = OC, OB = OD and BD $\perp$ AC

(vii) A diagonal divides a rhombus into two congruent Δs.

(3) **Rectangle :** A rectangle is a $\|^{gm}$ with all equal angles and value of each angle is 90°

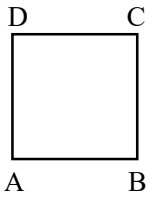


Properties :

(i) Diagonals are equal.

Other properties are same as parallelogram.

(4) **Square :** A square is a rectangle with all sides are equal or a square is a rhombus with all angles are equal (each 90°)

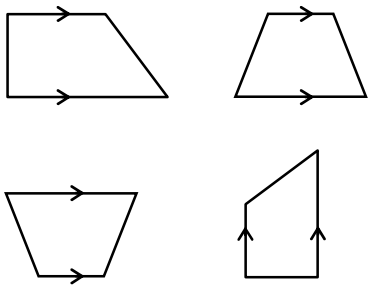


Properties :

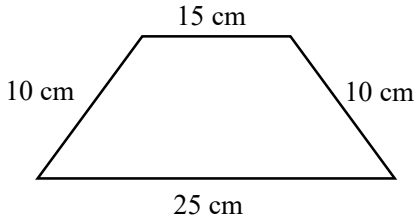
(i) Diagonals are of same length.

Other properties are same as rhombus.

(5) **Trapezium :** If opposite sides of one pair of quadrilateral are parallel & other two sides are non parallel then quadrilateral is called trapezium. The parallel sides are different in lengths.



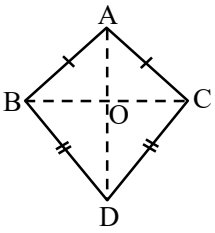
If non parallel sides are equal then it is called isosceles trapezium.



(6) **Kite :** This a is special type of a quadrilateral. In this adjacent sides are equal pair wise.

So there are exactly two distinct consecutive pairs of sides of equal length.

i.e. AB = AC and BD = DC



AD is longer diagonal & BC is smaller

AD bisects BC at right angle

i.e. BO = OC {not AO = OD}

also $\angle BOD = \angle DOC = \angle AOC = \angle AOB = 90°$ or $AD \perp BC$.

IMPORTANT POINTS TO BE REMEMBERED

1. (i) A quadrilateral which has exactly one pair of parallel sides is called a trapezium.

(ii) A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.

(iii) A parallelogram in which all the sides are equal is called a rhombus.

(iv) A parallelogram in which each angle is a right angle is called a rectangle.

(v) A parallelogram in which all the sides are equal and each angle is equal to a right angle is called a square.

(vi) A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides is called a kite.

2. A quadrilateral is a parallelogram if

(i) its opposite sides are equal, or

(ii) its opposite angles are equal, or

(iii) its diagonals bisect each other, or

(iv) it has one pair of opposite sides equal and parallel.

3. The diagonals of a rhombus bisect each other at right angles.

4. The diagonals of a rectangle are equal.

5. The diagonals of a square are equal and bisect each other at right angles.

6. One angle is more than 180° in concave polygon.

7. One diagonal is in exterior of concave polygon.

8. Both diagonal are in interior of quadrilateral.

9. Sum of interior angles = $(n - 2)\,180°$

(n = number of sides).

10. Each interior angles of regular polygon

$= \dfrac{(n-2)180°}{n}$.

11. Each exterior angle of regular polygon = $\dfrac{360°}{n}$.

12. A line joining any two distinct points in quadrilateral is always in the quadrilateral.

13. A square, rectangle and rhombus are parallelograms.

14. A $\|^{gm}$ is a trapezium but a trapezium is not a $\|^{gm}$.

15. A rectangle or a rhombus is not necessarily a square.

16. A kite is not a $\|^{gm}$.

EXERCISE # 1

Q.1 The sum of all the angles of a quadrilateral is
(A) 180° (B) 270°
(C) 360° (D) 400°

Q.2 The three angles of a quadrilateral are 80°, 70° and 120°. The fourth angle is
(A) 110° (B) 100°
(C) 90° (D) 80°

Q.3 The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. The largest of these angles is
(A) 90° (B) 120°
(C) 150° (D) 102°

Q.4 A quadrilateral having one and only one pair of parallel sides is called
(A) a parallelogram (B) a kite
(C) a rhombus (D) a trapezium

Q.5 A quadrilateral whose opposite sides are parallel is called
(A) a rhombus (B) a kite
(C) a trapezium (D) a parallelogram

Q.6 An isosceles trapezium has
(A) equal parallel sides
(B) equal nonparallel sides
(C) equal opposite sides
(D) none of these

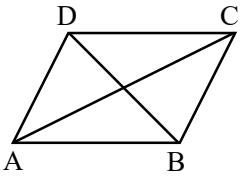
Q.7 If the diagonals of a quadrilateral bisect each other at right angles, then this quadrilateral is
(A) a rectangle
(B) a rhombus
(C) a kite
(D) none of these

Q.8 A square has
(A) all sides equal and diagonals unequal
(B) all sides equal and diagonals equal
(C) all sides unequal and diagonals equal
(D) none of these

Q.9 A quadrilateral having two pairs of equal adjacent sides but unequal opposite sides, is called a
(A) trapezium (B) parallelogram
(C) kite (D) rectangle

Q.10 What do you mean by a regular quadrilateral?
(A) A rectangle (B) A rhombus
(C) A square (D) A trapezium

Q.11 In the adjacent figure, a quadrilateral has been shown.



Name :
(i) its diagonals,
(ii) two pairs of opposite sides,
(iii) two pairs of opposite angles,
(iv) two pairs of adjacent sides,
(v) two pairs of adjacent angles.

Q.12 Two sides of a parallelogram are in the ratio 4 : 3. If its perimeter is 56 cm, find the lengths of its sides.

Q.13 Name each of the following parallelograms.
(i) The diagonals are equal and the adjacent sides are unequal.
(ii) The diagonals are equal and the adjacent sides are equal.
(iii) The diagonals are unequal and the adjacent sides are equal.

Q.14 Which of the following statements are true and which are false ?
(a) The diagonals of a parallelogram are equal.
(b) The diagonals of a rectangle are perpendicular to each other.
(c) The diagonals of a rhombus are equal.

Q.15 Give reasons for the following :

(a) A square can be thought of as a special rectangle.

(b) A square can be thought of as a special rhombus.

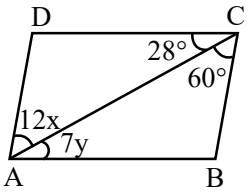
(c) A rectangle can be thought of as a special parallelogram.

(d) A square is also a parallelogram.

Q.16 A figure is said to be regular if its sides are equal in length and angles are equal in measure. What do you mean by a regular quadrilateral?

Q.17 In a parallelogram PQRS, $\angle S = 115°$, find the measurement of $\angle P$ and $\angle Q$.

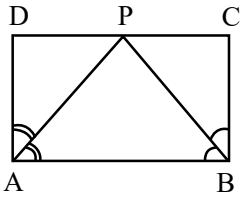
Q.18 In the given figure ABCD is a parallelogram, find x and y.



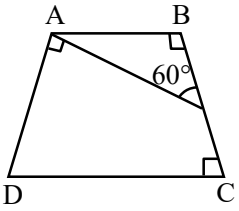
Q.19 If BD is diagonal of a $||^{gm}$ ABCD and $\angle C = 5a°$, $\angle CBD = 3a°$ and $\angle BDC = 2a°$ then find all four angles of $||^{gm}$ ABCD.

Q.20 In $||^{gm}$ ABCD, diagonals AC & BD intersect at O and AC = 5.6 cm, BD = 6.8 cm, find OC & OD.

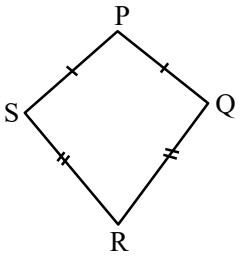
Q.21 In the given figure ABCD is parallelogram and $\angle DAB = 60°$. If the bisectors AP & BP of angles A and B respectively, meet at P on CD, prove that P is mid point of CD



Q.22 In the given figure find $\angle ADC$.



Q.23 PQRS is a kite $\angle P = 70°$, $\angle S = 90.5°$ then find $\angle R$



ANSWER KEY

EXERCISE # 1

1. (C) **2.** (C) **3.** (B) **4.** (D) **5.** (D) **6.** (B) **7.** (B)

8. (B) **9.** (C) **10.** (C)

11. (i) AC, BD (ii) (AB, DC) and (AD, BC) (iii) ($\angle$A, $\angle$C), ($\angle$B, $\angle$D)

(iv) (AB, BC), (AD, DC) (v) ($\angle$A, $\angle$B), ($\angle$B, $\angle$C)

12. 16 cm, 12 cm **13.** (i) Rectangle (ii) Square (iii) Rhombus

14. (a) False (b) False (c) False

15. (a) A rectangle with sides equal becomes a square.

(b) A rhombus with each angle a right angle becomes a square.

(c) A parallelogram with each angle a right angle becomes a rectangle.

(d) The opposite sides of a square are parallel, so it is a parallelogram.

16. A regular quadrilateral is a square.

17. 65°, 115° **18.** $x = 5, y = 4$ **19.** $\angle A = \angle B = \angle C = \angle D = 90°$

20. OC = 2.8 cm, OD = 3.4 cm

22. 60° **23.** 109°

EXERCISE # 2

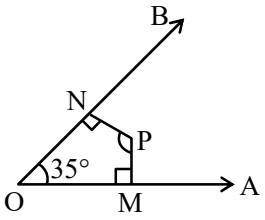
Q.1 One angle of a quadrilateral is 78° and the other angles are equal. Find the measure of each of the equal angles.

Q.2 The angles of a quadrilateral are 100°, 98° and 92°. Find the fourth angle.

Q.3 In a quadrilateral ABCD, the angles A, B, C and D are in ratio 1 : 2 : 3 : 4. Find the measure of each angle of the quadrilateral.

Q.4 The measure of two adjacent angles of a quadrilateral are 125° and 35°, the other two angles are equal. Find the measure of each of the angles.

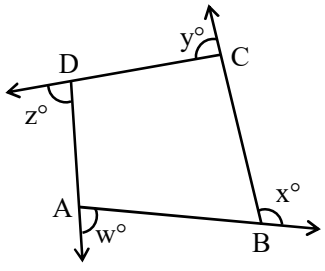
Q.5 In the figure, P is a point in the interior of ∠AOB. PM ⊥ OA and PN ⊥ OB. If ∠AOB = 35°, what is the measure of ∠MPN?



Q.6 Three angles of a quadrilateral are equal. The fourth angle is of measure 120°. What is the measure of each of its equal angles?

Q.7 Two angles of a quadrilateral are 100° and 80°. If one of the other two is double the other, find their measures.

Q.8 The sides of a quadrilateral are produced in order. The exterior angles marked as w°, x°, y° and z°, are in the ratio 5 : 6 : 3 : 4. Find their measures.



Q.9 Write true or false for the following statements.

(a) In a convex quadrilateral, there is at least one angle which is less than 180°.

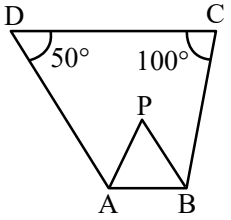
(b) A quadrilateral may have four acute angles.

(c) A quadrilateral may have four obtuse angles.

(d) At least one angle of a concave quadrilateral is more than 180°.

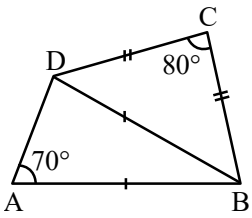
(e) A line segment joining two points in the interior of a quadrilateral lies entirely within the interior of the quadrilateral.

Q.10 In figure, the bisectors of ∠A and ∠B meet at P. If ∠C = 100°, ∠D = 50°, find ∠APB.

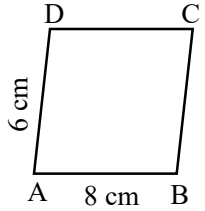


Q.11 If three equal angles of a quadrilateral measure 75°, find the fourth angle.

Q.12 Find the angles of the quadrilateral ABCD.

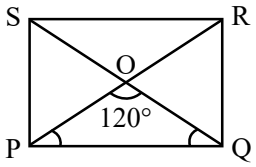


Q.13 Find the perimeter of a parallelogram, if its two adjacent sides measure 8 cm and 6 cm.

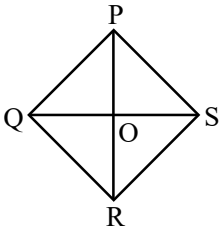


Q.14 Check if the following figure forms a parallelogram, where AB = 5.5 cm, BC = 8 cm, $\angle B = \angle D = 45°$, $\angle C = \angle A = 145°$.

Q.15 PQRS is a rectangle. If $\angle POQ = 120°$, find the angles of ΔPOQ.



Q.16 The diagonals of a rhombus are 6 cm and 8 cm respectively. Find the perimeter of the rhombus.



Q.17 Check if ABCD is a parallelogram given

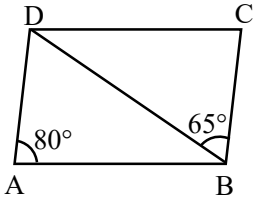
(a) AB = 4.5 cm, BC = 6 cm, AD = 6 cm, DC = 5 cm, $\angle A = 50°$, $\angle B = 130°$

(b) $\angle B = 90°$, $\angle A = 90°$, DC = AB = 7 cm, BC = 7 cm, AD = 7 cm

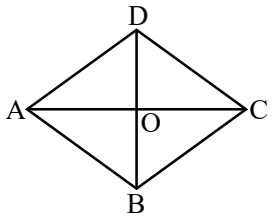
Q.18 Find the perimeter of a rectangle with sides 6 cm and 7.5 cm.

Q.19 The diagonals of a rectangle intersect at O. If $\angle BOC = 75°$, find the angles of the triangle BOC.

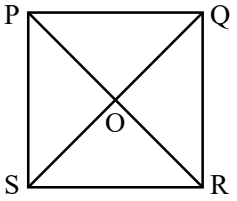
Q.20 If ABCD is a parallelogram and $\angle A = 80°$ and $\angle CBD = 65°$, find the angles of the parallelogram and ΔABD.



Q.21 ABCD is a rhombus. If AB = 10 cm, BD = 16 cm, find AC.

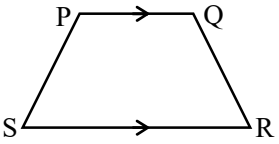


Q.22 PQRS is a square. If PO = 4.5 cm, find PR and QS.

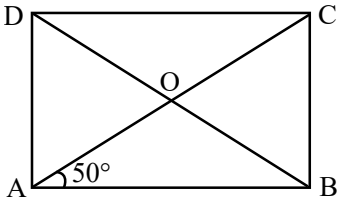


Q.23 ABCD is a rectangle. Diagonal AC divides $\angle C$ in the ratio 2 : 3. Find $\angle ACD$ and $\angle ACB$.

Q.24 PQRS is a trapezium in which PQ || SR. If $\angle P = 100°$ and $\angle Q = 110°$, find $\angle S$ and $\angle R$.



Q.25 ABCD is a rectangle. If $\angle OAB = 50°$, find $\angle AOB$.



Q.26 Find the perimeter of a rhombus whose diagonal are 5 cm and 12 cm.

Q.27 The perimeter of a rhombus is 40 cm. If one diagonal is 16 cm, find the other diagonal.

Q.28 ABCD is a quadrilateral with some special properties. So identify ABCD according to the following conditions (each part is separate question).

(a) AB = CD and BC = AD.

(b) AB || CD.

(c) Diagonals AC and BD bisect each other.

(d) AB = CD, AD = BC, $\angle A = 90°$.

(e) AB = BC = CD = AD.

(f) $\angle A = \angle C$ and $\angle B = \angle D$.

(g) AB = BC = CD = AD, $\angle A = 90°$.

(h) Diagonals AC and BD bisect at 90°.

(i) Diagonals AC and BD bisect and are equal.

(j) Diagonals AC and BD bisect at 90° and are equal.

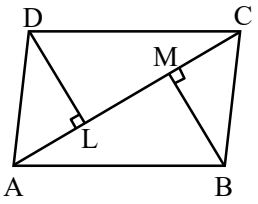
Q.29 ABCD is a square with one diagonal of length 12 cm. What is the length of other diagonal ?

Q.30 ABCD is a rhombus with diagonals 16 and 12 cm. Find the perimeter of this rhombus.

Q.31 Given a parallelogram ABCD, with DL $\perp$ AC and BM $\perp$ AC. Prove that

(a) AD = BC (b) $\angle DAC = \angle ACB$

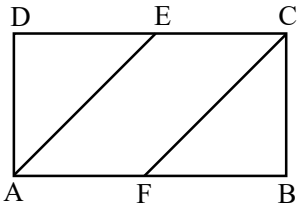
(c) $\Delta ADL \cong \Delta BCM$ (d) DL = BM



Q.32 ABCD is a rectangle with AE || CF prove with reasons, that

(a) $\angle DEA = \angle CFB$ (b) $\Delta ADE \cong \Delta BFC$

(c) AE = CF



ANSWER KEY

EXERCISE # 2

1. 94° **2.** 70° **3.** 36°, 72°, 108°, 144° **4.** 100°

5. 145° **6.** 80° **7.** 60°, 120° **8.** 100°, 120°, 60°, 80°

9. (a) F (b) F (c) F (d) T (e) T **10.** 75° **11.** 135°

12. 120°, 90° **13.** 28 cm **14.** BC $\nparallel$ AD **15.** 30°

16. 20 cm **17.** (a) No (b) Yes **18.** 27 cm

19. 52.5° **20.** $\angle ADB = 65°$, $\angle ABD = 35°$, $\angle C = 80°$ **21.** 12 cm

22. 9 cm, 9 cm **23.** 36°, 54° **24.** 80°, 70°

25. 80° **26.** 26 cm **27.** 12 cm

28. (a) Parallelogram (b) trapezium (c) parallelogram (d) rectangle (e) rhombus
(f) parallelogram (g) square (h) kite (i) rectangle (j) square

29. 12 cm **30.** 40 cm