Physical Constants from Algebraic Structure

A Unified Framework Connecting Geometric Algebra, Indivisible Processes, Causal Fermion Systems, and Thermodynamics

---

Abstract

We present a unified framework in which fundamental physical constants emerge from the algebraic structure of Clifford algebra Cl(5). The fine structure constant $\alpha = 1/137.036$ and the Weinberg angle $\sin^2\theta\_W \approx 0.231$ arise as combinatorial invariants of a constraint structure on bivector magnitudes. This same constraint—which we call "monogamy"—appears identically in Barandes' indivisible stochastic processes (as the failure of Chapman-Kolmogorov factorization), in Finster's Causal Fermion Systems (as the causal action bound), and in thermodynamics (as entropy subadditivity). The dimensional constants c and $\hbar$ are explained as consequences of algebraic isotropy and minimum distinguishable bivector structure respectively. The uncertainty principle $\Delta x \Delta p \geq \hbar/2$ emerges as an indivisibility constraint rather than a measurement limitation. This synthesis suggests that quantum mechanics is fundamentally the thermodynamics of distinguishability, with the electroweak constants serving as thermodynamic parameters of the constraint structure.

---

1. Introduction

1.1 The Problem of Fundamental Constants

Physics contains approximately 26 free parameters that must be measured experimentally. Among these, the dimensionless constants—particularly the fine structure constant $\alpha \approx 1/137$ and the Weinberg angle $\sin^2\theta\_W \approx 0.231$—have long been considered mysterious. Why these particular values?

We propose that these constants are not free parameters but **combinatorial invariants** of an underlying algebraic structure. Specifically, they emerge from the Clifford algebra Cl(5) through constraints on how relational structures (bivectors) can be distributed among features (vectors).

1.2 The Axiom

Our starting point is a single axiom from modal logic:

$$\Diamond N \to \neg N$$

"If nothing is possible, then nothing is not the case"—equivalently, **nothing cannot exist**. This implies that anything which exists must be **distinguishable from nothing**, and distinct things must be **distinguishable from each other**.

1.3 Why Clifford Algebra?

Previous formulations used "constraint space" language that presupposed a container—a geometric arena in which relations occur. This violates the relational ontology we seek, where relations are primary and entities emerge from relational patterns.

Clifford (Geometric) Algebra provides intrinsically relational mathematics:

- The **geometric product** $ab = a \cdot b + a \wedge b$ encodes both correlation (scalar) and oriented relation (bivector)
- **Relations ARE products**, not distances in a pre-existing space
- **Structure emerges from algebraic properties**, not geometric embedding

---

2. The Algebraic Framework

2.1 Clifford Algebra Cl(5)

The algebra Cl(5) is generated by five orthonormal basis vectors e_1, e_2, e_3, e_4, e_5 satisfying:

$$e_i e_j + e_j e_i = 2\delta_{ij}$$

This gives a 32-dimensional algebra with graded structure:

Grade	Dimension	Elements	Physical Interpretation
0	1	Scalars	Magnitudes, correlations
1	5	Vectors	**Features** (distinguishable aspects)
2	10	Bivectors	**Relations** (pairwise structure)
3	10	Trivectors	**N=3 structure** (irreducible triples)
4	5	Quadvectors	Dual to vectors
5	1	Pseudoscalar	Orientation

2.2 Features as Grade-1 Elements

A **feature** is a vector in Cl(5):

$$\mathbf{a} = \sum_{i=1}^{5} a^i e_i \in \mathrm{Cl}(5)_1$$

The axiom requires distinguishability:

- From nothing: $|\mathbf{a}|^2 \geq \varepsilon^2$
- From each other: $|\mathbf{a} - \mathbf{b}|^2 \geq \varepsilon^2$
- From totality: $|\mathbf{a}|^2 \leq 1 - \varepsilon^2$

2.3 Relations as Grade-2 Elements

The **relation** between features **a** and **b** is the bivector:

$$\mathbf{a} \wedge \mathbf{b} \in \mathrm{Cl}(5)_2$$

Key properties:

- Antisymmetric: $\mathbf{a} \wedge \mathbf{b} = -\mathbf{b} \wedge \mathbf{a}$
- Magnitude: $|\mathbf{a} \wedge \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$
- Interpretation: The bivector IS the relation, not a measurement of distance

2.4 The N=3 Structure as Grade-3

Three features form a **trivector**:

$$T = \mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \in \mathrm{Cl}(5)_3$$

This is non-zero if and only if the features are linearly independent. Crucially, the trivector contains information **not reducible** to its constituent bivectors—it represents genuinely three-way structure.

---

3. The Monogamy Constraint

3.1 Finite Relational Capacity

Each feature has **finite capacity** to participate in relations. If feature **b** is involved in two bivectors **a∧b** and **b∧c**, these cannot both be arbitrarily large:

$$|\mathbf{a} \wedge \mathbf{b}| + |\mathbf{b} \wedge \mathbf{c}| \leq \Lambda_{max}$$

This is the **monogamy constraint**—not a geometric condition but an algebraic one about how bivector magnitudes can be distributed.

3.2 The Constraint Structure

For three features at N=3, we have three bivector magnitudes. Normalizing by Λ and denoting x = |**a∧b**|/Λ, y = |**b∧c**|/Λ, z = |**c∧a**|/Λ, the constraints become:

Monogamy constraints:

- $x + y \leq 1$ (capacity of **b**)
- $y + z \leq 1$ (capacity of **c**)
- $z + x \leq 1$ (capacity of **a**)

Non-negativity:

- $x, y, z \geq 0$

3.3 The Combinatorial Invariants

This system of 6 linear constraints on 3 variables has:

- **V = 5** vertices (extreme configurations where 3 constraints are tight)
- **E = 9** edges
- **F = 6** faces
- **χ = V − E + F = 2** (Euler characteristic)

The vertices are:

Vertex	(x, y, z)	Interpretation
V_1	(0, 0, 0)	No bivector structure
V_2	(1, 0, 0)	**a-b** maximal, **c** isolated
V_3	(0, 1, 0)	**b-c** maximal, **a** isolated
V_4	(0, 0, 1)	**c-a** maximal, **b** isolated
V_5	(½, ½, ½)	Democratic—all relations equal

The numbers **$V + \chi = 7$** and **$\dim(Cl(5)_1) \times 3! = 30$** are the key combinatorial invariants.

---

4. Derivation of the Electroweak Constants

4.1 The Fine Structure Constant

The fine structure constant emerges from the full N=3 geometry:

$$\alpha = \frac{\sqrt{3}}{24\pi^2 + \sqrt{7/30}}$$

Origin of each factor:

Factor	Value	Algebraic Origin
$\sqrt{3}$	1.732	Equilateral bivector magnitude: $\lvert \mathbf{a} \wedge \mathbf{b} \rvert$ at $\theta = 60°$
$(2\pi)^2$	39.48	Rotor periodicity: $\exp(B \cdot 2\pi) = 1$, with 2 independent phases
$3! = 6$	6	Antisymmetry of wedge product under permutations
$24\pi^2$	236.87	Combined: $(2\pi)^2 \times 3!$
7	7	$V + \chi$ = constraint complexity
30	30	$\dim_1 \times 3!$ = algebraic context
$\sqrt{(7/30)}$	0.483	Monogamy correction

Numerical result:

$$\alpha = \frac{1.732}{236.87 + 0.483} = \frac{1.732}{237.35} = 0.007297$$

$$\frac{1}{\alpha} = 137.036$$

Agreement with experiment: **1 ppm**

4.2 The Weinberg Angle

The Weinberg angle is simpler—a direct ratio:

$$\sin^2\theta_W = \frac{V + \chi}{\dim(\mathrm{Cl}(5)_1) \times 3!} = \frac{7}{30} = 0.2333$$

Interpretation: The fraction of the algebraic structure subject to monogamy constraints.

Full topological formula (incorporating χ's distinct contribution):

$$\sin^2\theta_W = \frac{7}{30 + 2/7} = \frac{49}{212} = 0.2311$$

Experimental value (at M_Z): 0.23121. **Agreement:** 0.03%

4.3 Why the Difference in Structure?

Constant	Formula	What it measures
α	$\sqrt{3}/(24\pi^2 + \sqrt{7/30})$	Coupling **strength** (involves full geometry)
$\sin^2\theta$_W	7/30	Mixing **ratio** (pure counting)

The fine structure constant involves the **full N=3 structure**—geometry (√3), phases ($(2\pi)^2$), symmetry (3!), plus monogamy correction. The Weinberg angle involves **only** the constraint fraction—no geometry or phases, just combinatorics.

---

5. Connection to Barandes' Indivisible Stochastic Processes

5.1 Barandes' Framework

Barandes shows that quantum mechanics emerges from **indivisible** stochastic processes—processes where:

$$P(A \to C) \neq \sum_{B} P(A \to B) P(B \to C)$$

The Chapman-Kolmogorov equation fails. You cannot decompose the process A→C into sub-processes through intermediate state B without changing the probabilities.

5.2 The Correspondence

Our Framework	Barandes
N=2 (no monogamy)	**Divisible** processes
N=3 (monogamy active)	**Indivisible** processes
Monogamy: $\lambda\_AB + \lambda\_BC \leq \Lambda$	Chapman-Kolmogorov failure
Trivector irreducibility	Process irreducibility

The monogamy constraint IS indivisibility.

When **b**'s relational capacity is bounded, it cannot fully mediate both the **a**↔**b** and **b**↔**c** correlations. The intermediate state **b** is "shared" in a way that prevents factorization.

5.3 GA Formulation

In Cl(5), a trivector **a∧b∧c** is **irreducible**—it cannot be written as a product of lower-grade elements. This algebraic irreducibility corresponds exactly to Barandes' process indivisibility.

The transition from divisible to indivisible (classical to quantum) occurs at **N=3**, where:

- Trivector structure becomes non-trivial
- Monogamy constraints activate
- Chapman-Kolmogorov fails

Quantum behavior is N≥3 relational structure.

6. Connection to Causal Fermion Systems

6.1 Finster's Framework

In Causal Fermion Systems, spacetime emerges from a measure space of operators. The key structure is the **causal action principle**:

$$S[\rho] = \iint \mathcal{L}(x,y)\, d\rho(x) d\rho(y)$$

where L(x,y) is built from the spectrum of the operator product xy.

The Lagrangian penalizes configurations where points are:

- Too similar (indistinguishable)
- Too different (causally disconnected)

The optimum is where points are **just distinguishable enough**.

6.2 The Correspondence

Our Framework	CFS
Feature **a**	Spacetime point x
Geometric product ab	Operator product xy
Scalar a·b	Eigenvalue magnitude
Bivector a∧b	Causal structure
Distinguishability bounds	Causal action constraints
Monogamy constraint	Causal bound: L(a,b) + L(b,c) bounded

The causal action principle IS the distinguishability optimization.

6.3 Emergence of Lorentzian Signature

In CFS, Lorentzian signature (−,+,+,+) emerges from the causal action—it's not assumed.

In our framework, the Lorentzian signature reflects the **ontological difference** between:

- Directions meaningful at N=2 (spatial: e_1, e_2, e_3)
- Directions meaningful only at N≥3 (ordering: e_5)

The e_4 direction emerges when circulation becomes possible (N≥3). The "minus sign" in $ds^2 = -c^2dt^2 + dx^2$ encodes this categorical difference, not a metric magnitude difference.

Both frameworks derive Lorentzian structure rather than assuming it.

---

7. Connection to Thermodynamics

7.1 Entropy as Grade Mixing

A general element of Cl(5) is a **mixed multivector**:

$$M = s + \mathbf{v} + B + T + Q + P$$

spanning all grades. Define grade probabilities:

$$p_k = \frac{|M|_k^2}{\sum_j |M|_j^2}$$

where |M|_k is the magnitude of the grade-k component. Then:

$$S = -\sum_k p_k \ln p_k$$

- **Low entropy:** M concentrated in one grade (pure state)
- **High entropy:** M spread across grades (mixed state)

7.2 The Efficiency Potential

The efficiency potential $\Phi = \ln(\Omega/K)$ measures hidden structure:

- Ω: Total degrees of freedom (dim(Cl(5)) = 32)
- K: Observer-accessible degrees of freedom

Observer Resolution	K	Φ = ln(32/K)
Full (all grades)	32	0
Grades 1,2,3	25	0.25
Grades 1,2	15	0.76
Grade 1 only	5	1.86

7.3 Monogamy as Entropy Subadditivity

The thermodynamic analog of monogamy is **entropy subadditivity**:

$$S(A,B) + S(B,C) \leq S(A,C) + S(B)$$

Correlations through B are limited by B's entropy capacity—exactly the monogamy constraint in thermodynamic language.

7.4 Thermodynamic Interpretation of Constants

Constant	Thermodynamic Meaning
α	Entropy production efficiency
$\sin^2\theta_W$	Entropy partition fraction (constrained/total)
7 = V + χ	Effective entropy of constrained sector
30 = 5 × 3!	Maximum entropy of unconstrained sector

The electroweak constants are thermodynamic parameters of the constraint structure.

---

8. The Dimensional Constants: c, *ℏ*, and Uncertainty

8.1 The Speed of Light (c = 1)

Algebraic origin: Cl(5) is **isotropic**—all basis vectors satisfy $e_i^2 = +1$ with no algebraic preference for any direction.

Multi-framework interpretation:

Framework	Why c = 1
GA	Algebraic isotropy of Cl(5)
Barandes	All directions in state space equivalent
CFS	Causal action doesn't prefer any direction
Thermodynamics	Entropy is isotropic

c = 1 is about isotropy, not "speed."

The Lorentzian signature (−,+,+,+) doesn't come from different "stiffnesses" in different directions. It comes from the ontological distinction between N=2 structure (spatial) and N=3 structure (temporal/ordering).

8.2 Planck's Constant ($\hbar$ = 1)

Algebraic origin: The minimum distinguishable bivector combined with rotor periodicity.

In GA, rotors $R = \exp(B\theta/2)$ generate evolution. The periodicity $\exp(B\cdot 2\pi) = 1$ is **algebraic**, not geometric. Combined with the minimum distinguishable bivector from N=3 structure, this sets the scale.

Multi-framework interpretation:

Framework	What $\hbar$ represents
GA	Minimum bivector × rotor period
Barandes	Minimum "indivisibility unit"
CFS	Minimum "causal cell"
Thermodynamics	Minimum distinguishable phase space cell

$\hbar$ is the scale of distinguishability/indivisibility.

8.3 The Uncertainty Principle

Standard formulation: $\Delta x \Delta p \geq \hbar/2$

GA formulation: Conjugate variables span a bivector plane. The uncertainty relation states that the **bivector magnitude** must exceed the minimum:

$$|\Delta x \wedge \Delta p| \geq \hbar/2$$

Multi-framework interpretation:

Framework	What uncertainty means
GA	Minimum bivector magnitude in phase space
Barandes	Phase space is **indivisible** below this scale
CFS	Causal uncertainty—minimum causal cell
Thermodynamics	Minimum entropy cell

Uncertainty is about indivisibility, not measurement disturbance.

The uncertainty principle doesn't say "measurement disturbs the system." It says "the relational structure of phase space cannot be divided below the minimum bivector scale." This is a geometric/algebraic fact, not an epistemological limitation.

---

9. The Unified Picture

9.1 One Constraint, Four Languages

The monogamy constraint on bivector magnitudes:

$$|\mathbf{a} \wedge \mathbf{b}| + |\mathbf{b} \wedge \mathbf{c}| \leq \Lambda$$

appears in four equivalent forms:

Framework	Expression	Physical Meaning
GA/Relational	Bivector bound	Finite relational capacity
Barandes	C-K failure	Process indivisibility
CFS	Causal bound	Causal action constraint
Thermodynamics	Subadditivity	Entropy bound

9.2 The Grade-Framework Correspondence

GA Grade	Barandes	CFS	Thermodynamics
0 (scalar)	Transition probability	Causal strength	Energy
1 (vector)	State	Spacetime point	Microstate
2 (bivector)	Indivisibility measure	Causal structure	Pairwise entropy
3 (trivector)	Quantum interference	Spacetime volume	3-body correlation
Rotor exp(Bθ/2)	Unitary evolution	Causal propagator	Time evolution

9.3 The Central Thesis

Quantum mechanics is the thermodynamics of distinguishability.

Specifically:

- States are grade-1 elements (vectors)
- Correlations are grade-2 elements (bivectors)
- N=3 structure is grade-3 (trivectors)—where quantum behavior emerges
- Evolution is rotors
- Temperature is rotor frequency
- Entropy is grade mixing
- Monogamy is entropy subadditivity

The "mystery" of quantum mechanics dissolves: it's the natural structure of constrained distinguishability at $N \geq 3$.

Terminological note: We use "quantum" to connect with established physics, but the underlying structure is purely geometric—distinguishability constraints in Cl(5). The framework does not invoke wavefunctions, Hilbert spaces, or measurement collapse. What physics calls "quantum behavior" is here identified as indivisible stochastic structure (Barandes) emerging at $N \geq 3$, where monogamy constraints activate and trivector irreducibility appears.

---

10. Summary of Results

10.1 What We Derive

Quantity	Formula	Value	Accuracy
α	$\sqrt{3}/(24\pi^2 + \sqrt{7/30})$	1/137.036	1 ppm
$\sin^2\theta_W$ (vertex-only)	7/30	0.2333	—
$\sin^2\theta_W$ (full topological)	49/212	0.2311	0.03%

10.2 What We Explain

Quantity	Explanation
$c = 1$	Algebraic isotropy of Cl(5)
$\hbar = 1$	Minimum bivector + rotor periodicity
$\Delta x \Delta p \geq \hbar/2$	Minimum bivector bound (indivisibility)
Lorentzian signature	N=2/N=3 ontological boundary

10.3 What We Unify

Connection	Identification
Monogamy ↔ Indivisibility	Same constraint on bivectors
Monogamy ↔ Causal action	Same bound structure
Monogamy ↔ Subadditivity	Same entropy constraint
N=3 emergence ↔ Quantum behavior	Same threshold

---

11. Discussion

11.1 The Relational Ontology

Throughout this framework, **relations are primary**. The Clifford algebra Cl(5) is not a "space containing things"—it IS the relational structure. Features (vectors) and relations (bivectors) are elements of the algebra, not objects in a container.

This resolves the tension in earlier formulations that spoke of "constraint space" while claiming relations are ontologically prior.

11.2 Why Cl(5)?

The dimension 5 appears to be the minimum for:

1. N=3 independent features (requires ≥3)
2. Non-trivial monogamy (requires "excess" bivector space)
3. Separate ordering direction (requires 3 + 1 + 1 = 5)

A rigorous derivation of "why 5" from the axiom alone remains for future work.

11.3 Dimensional vs Dimensionless Constants

The framework treats these differently:

Dimensionless (α, $\sin^2\theta_W$): Pure numbers derivable as combinatorial invariants. Their values are predictions.

Dimensional (c, $\hbar$): Depend on unit conventions. We explain WHY they can be set to 1, but cannot "derive" their SI values (those are human conventions).

11.4 Predictions and Tests

The framework makes specific predictions:

- $\alpha = \sqrt{3}/(24\pi^2 + \sqrt{7/30})$ exactly
- $\sin^2\theta_W = 7/30$ (vertex-only) or 49/212 (full topological structure)
- N-dependence of constants (α larger at small N, corresponding to high energy)

These can be tested against precision measurements.

---

12. Conclusion

We have presented a unified framework in which:

1. **The Clifford algebra Cl(5)** provides the fundamental mathematical structure
2. **The monogamy constraint** on bivector magnitudes creates a polytope with V=5, χ=2

3. **The fine structure constant** $\alpha = \sqrt{3}/(24\pi^2 + \sqrt{(7/30)})$ emerges from the full N=3 geometry
4. **The Weinberg angle** $\sin^2\theta\_W = 49/212$ emerges as the topological structure of the constraint polytope
5. **The same constraint** appears in Barandes (indivisibility), CFS (causal bound), and thermodynamics (subadditivity)
6. **The dimensional constants** c, $\hbar$ and the uncertainty principle are aspects of algebraic isotropy and minimum distinguishable structure

The central insight is that **quantum mechanics is the thermodynamics of distinguishability**. The "mysterious" features of quantum theory—superposition, entanglement, uncertainty—are natural consequences of constrained relational structure at $N \geq 3$.

The electroweak constants are not arbitrary parameters but **thermodynamic ratios** determined by the combinatorial structure of Cl(5). Physics, at its deepest level, is the working out of what must be true given that **nothing cannot exist**.

Appendix A: Notation Summary

Symbol	Meaning
Cl(5)	Clifford algebra with 5 generators
e_i	Basis vectors (i = 1,...,5)
a, **b**, **c**	Feature vectors (grade-1)
a∧b	Bivector (grade-2, oriented relation)
a∧b∧c	Trivector (grade-3, N=3 structure)
a·b	Scalar product (grade-0)
ab = **a·b** + **a∧b**	Geometric product
$R = \exp(B\theta/2)$	Rotor (rotation generator)
V	Number of vertices (= 5)
χ	Euler characteristic (= 2)
α	Fine structure constant

Symbol	Meaning
θ_W	Weinberg angle

Appendix B: Key Formulas

Fine structure constant:

$$\alpha = \frac{\sqrt{3}}{24\pi^2 + \sqrt{7/30}} = \frac{1}{137.036}$$

Weinberg angle:

$$\sin^2\theta_W = \frac{7}{30} = 0.2333 \quad \text{(vertex-only)}$$

$$\sin^2\theta_W = \frac{49}{212} = 0.2311 \quad \text{(full topological)}$$

Monogamy constraint:

$$|\mathbf{a} \wedge \mathbf{b}| + |\mathbf{b} \wedge \mathbf{c}| \leq \Lambda$$

Bivector magnitude:

$$|\mathbf{a} \wedge \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$$

Rotor periodicity:

$$e^{B \cdot 2\pi} = 1 \quad \text{(algebraic, not geometric)}$$

Version 2 — Updated for terminology consistency with SI documents