Supporting Information: Categorical Exhaustion and the Relational Foundation

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Section 2 Supporting Information for "Being and Nothingness"

CE.1 Introduction

CE.1.1 Purpose

This Supporting Information provides rigorous foundations for Section 2 of the main text. We establish:

1. **The Uemov Inversion:** Why relations are ontologically prior to relata
2. **The Three Tiers and Role-Substitution:** How relations, relational signatures (properties), and relata populate Uemov's three-tier ontology, and how the same Cl(5) structures appear across tiers by role-substitution
3. **Categorical Necessity:** Why each of the five constraints is independently necessary
4. **Categorical Completeness:** Why no sixth constraint exists
5. **Categorical Independence:** Why no constraint reduces to the others

CE.1.2 Central Claim

The five constraints (β, κ, ρ, λ, τ) are not arbitrary parameters chosen for convenience. They represent the **exhaustive categorical decomposition** of what robust distinguishability requires. This decomposition is:

- **Derived** from the foundational axiom ($\Diamond N \rightarrow \neg N$)
- **Complete** (no sixth category exists)
- **Independent** (no reduction to four is possible)
- **Structured** (four symmetric + one asymmetric)

CE.1.3 Methodological Note

Throughout this document, we maintain strict adherence to relational ontology. We do not speak of: - "Space" as a container in which relations occur - "Time" as a dimension along which things evolve - "Things" as primitives that happen to be related

Instead: - Relations are primitive; relata are derivative - What we call "space" emerges from boundary (β) structure - What we call "time" emerges from ordering (τ) structure at $N \geq 3$ - "Things" are topologically protected patterns in relational structure

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CE.2 The Uemov Inversion: Relations as Primitive

CE.2.1 The Traditional Ontological Order

Standard physics and philosophy adopt what we call the **traditional ontological order**:

Things → Properties → Relations

1. Entities exist as fundamental substances
2. Entities possess properties (mass, charge, position)
3. Entities enter into relations with other entities

Under this view, relations are derivative—they require pre-existing relata to connect.

CE.2.2 The Uemov Insight

A.I. Uemov (1963, *Things, Properties, and Relations*) observed that the ontological priority among things, properties, and relations is not fixed by logic. Any of the three can be taken as fundamental:

Starting Point	Derivative Concepts
Things →	Properties → Relations
Properties →	Things → Relations
Relations →	**Properties → Things**

The third option—**relations as primitive**—is what we adopt. This is not merely a notational preference but a genuine ontological commitment with mathematical consequences.

CE.2.3 Why the Axiom Forces the Inversion

The axiom ($\Diamond N \rightarrow \neg N$) forces the Uemov inversion through the following argument:

Step 1: Nothing cannot exist (from axiom)

Step 2: Therefore, distinction must exist. If there were no distinction, "something" would be indistinguishable from "nothing"—and nothing cannot exist.

Step 3: Distinction is inherently relational. There is no such thing as "A is distinct" (distinct from what?). Only "A is distinct FROM B" is meaningful.

Step 4: Therefore, the primitive is not "things that are distinguished" but "the relation of distinction itself."

Step 5: Relata (the terms of the distinction relation) are defined by their distinguishing relations, not vice versa.

This is not a choice we make for convenience—it is forced by the logical structure of the axiom.

CE.2.4 Relata as Topologically Protected Patterns

If relations are primitive, what are relata?

Relata are topologically protected patterns of maximal distinguishability in the network of distinguishing relations.

Key aspects: - Relata are not pre-existing things that happen to enter into relations - Relata ARE nodes in the network of distinguishing relations - A relatum is defined by its pattern of distinctions with other relata - “Topologically protected” means: cannot be continuously deformed to trivial structure

Analogy: A mountain is not an object placed upon Earth’s surface; it is a feature OF that surface—a pattern of elevation. Similarly, relata are not objects placed within some container; they are patterns of distinguishability—features of the relational structure that the axiom requires.

CE.2.5 Why This Is Not “Just an HMM”

A natural criticism: “The relational structure looks like a Hidden Markov Model—hidden states, emissions, inference. Why invent new language?”

Response: The structural similarity is real but the ontological interpretation differs fundamentally.

Under the traditional ontology: - There are systems (things) with hidden states (properties) - Observers infer from emissions - The HMM is an epistemic tool—a model OF something more fundamental - Things → Properties → Relations (relations are derivative)

Under the Uemov inversion: - The relational structure is fundamental - “Things” (relata) are patterns IN the relational structure - The HMM-like structure is not a model OF something else—it IS how relational structure manifests to embedded relata - Relations → Properties → Things

The “field” language in the main text captures that the relational structure is **ontologically prior**. It’s not that there are things, and they happen to be related. It’s that there IS relational structure, and what we call “things” are features of that structure.

CE.2.6 Consequences for Language

The Uemov inversion requires care with language:

Avoid	Prefer
“Features exist in constraint space”	“Constraint values characterize relational configurations”
“Systems move through the space”	“Gradient relationships exist between configurations”
“Space contains relations”	“What we call ‘space’ emerges from β structure”

Avoid	Prefer
"Things have properties"	"Patterns of relation exhibit characteristic structure"
"Time passes"	"Ordering structure ($\tau > 0$) enables asymmetric relations"

This is not mere pedantry. Container language smuggles in assumptions that contradict the framework's foundations.

CE.2.7 Role-Substitution and the Three Tiers

The Uemov inversion gives an ontological order but leaves open how the three tiers are populated in the framework. We have named relations as primitive (the distinguishability structure itself) and relata as emergent (topologically protected patterns). The middle tier — properties — has been implicit. We now make it explicit and state the framework's commitment to what Uemov's formalism calls **role-substitution**.

Properties as relational signatures. Under the inverted ontology, a property is not an attribute that a relatum possesses. It is the specific way a pattern participates in the distinguishability structure. The fine-structure constant α is not a property of electromagnetism as a substance; it is the signature of how the grade-2 coupling at $N = 5$ participates in the monogamy polytope geometry. The Weinberg angle $\sin^2\theta\_W$ is the signature of how the λ-sector structure participates at the same algebraic locus. A particle's mass is the signature of how its closed-bivector topology participates in the grade-2 edge-sharing structure of K_4. Each is a quantitative tag identifying how a pattern shows up in the larger structure. We adopt **relational signatures** as the framework term for this middle tier. The constants derivations collected in SI_Section7 are, under this reading, derivations of the relational signatures of specific grade structures at specific N.

The role-substitution principle. Uemov's Language of Ternary Description (developed 1999–2003) operates on a principle that carries directly into the framework: the same object can play different ontological roles depending on the description it is part of. The clearest case in the framework is the grade-2 bivector in Cl(5). The bivector is a *relation* when it is the λ-σ object connecting two features (the framework analog of the photon in the minimal-scope regime). It is a *relational signature* when it appears as α or $\sin^2\theta\_W$ — the bivector geometry cast into a numerical tag. It is *relatum-like* when it is closed into a K_4 edge-sharing configuration (the framework analog of the electron). These are not three different objects. They are three role-substitutions of the same underlying Cl(5) structure. Which role the bivector plays is fixed by the description it appears in, not by the bivector itself.

The principle generalises. The grade-3 trivector plays the relation role when it is articulated circulation around a specific triangle at $N = 3$; it plays the signature role when τ_circ is measured as an ensemble average at larger scope; it contributes to the relatum role when it is part of the internal structure of a closed topological knot. The Cl(5) pseudoscalar I_5 plays the relation role when it characterises the monogamy polytope; it plays the signature role when it appears in the Ω-I_5 connection; relata never reduce to pseudoscalars directly but inherit pseudoscalar structure from their grade-5 content. The framework's Cl(5) taxonomy is fixed, but every element in it is available across the three tiers by role-substitution.

Consequences for framework documents. Making role-substitution explicit clarifies several threads that had been treated as independent. The Bridge Photon document's treatment of the λ-σ bivector across the vacuum/medium/statistical/field regimes is role-substitution in the scope axis. The constants derivations are the bivector and polytope structure in signature role. The electron-as-trapped-photon reading is role-substitution between the relation role (propagating bivector) and the relatum role (closed K_4 bivector). Each of these is a case of one Cl(5) structure appearing in multiple tiers under different descriptions. The principle is not new content; it is a unifying name for something the framework has been doing throughout.

CE.2.8 Relational Collapse at the N-Transitions

The Uemov tradition distinguishes **external** relations (between independently existing things that retain their identity regardless of the relation) from **internal** relations (constitutive of what the relata are — severing the relation changes what the relata are, or eliminates them entirely). The phenomenon where a relation shifts from external to internal status is called **relational collapse** (Uemov 1963; developed further in Wawrzyniak 2020, *Philosophia*).

The framework has a structurally identical phenomenon at its key N-transitions, which we now name explicitly:

Collapse at N = 2 → N ≥ 3. At N = 2, a single bivector λ_AB is an external relation between features A and B. The features retain independent identity; each has its own constraint configuration (β, κ, ρ) that does not depend on the relation's existence. At N ≥ 3, closed circulation becomes possible. The τ-content of the cycle is no longer an external relation among features; it is constitutive of what those features are *for ordering purposes*. A feature's position in the cycle A → B → C → A is not something it has independently of the cycle; the cycle makes the positions what they are. This is relational collapse: τ moves from external status (available as a separate coordinate) to internal status (constitutive of the feature-as-cycle-participant).

Collapse at N = 3 → N = 4. At N = 3, circulation exists but bivectors do not share edges between triangles. At N = 4, K_4 activates: every edge participates in exactly two triangles. The shared bivector is no longer between two independent triangle-structures; it is constitutive of both. Removing the bivector does not separate two independent triangles — it damages both simultaneously because each triangle's existence depended on the shared edge. This is the relational collapse that the grade-2 edge-sharing at K_4 *is*, and it is the structural origin of mass in the framework (per Regimes V2, Section 7.2).

Collapse at N = 4 → N = 5. At N = 5, the full Cl(5) structure closes. The pseudoscalar I_5 = e_β ∧ e_κ ∧ e_ρ ∧ e_λ ∧ e_τ appears as a single object — not a relation among five independent sectors but a structure that is constitutive of the sectors' joint existence. Individual grade content continues to be resolvable, but the pseudoscalar integrates all five grades into a single relation that cannot be decomposed without losing what it is. This is the third and strongest relational collapse the framework invokes.

The N-hierarchy, on this reading, is a hierarchy of progressive relational collapse. At each threshold, relations that had been external (resolvable as separate coordinates) become internal (constitutive of the objects they appear to connect). This is not an additional claim beyond what the framework already commits to; it is a name — borrowed from Uemov — for what the N-transitions *structurally are*. Naming them this way connects the framework to a live philosophical conversation and distinguishes the N-transitions from mere phase transitions of the kind statistical mechanics describes.

CE.2.9 The Modal Triad: Definite, Indefinite, Arbitrary

Uemov's calculus uses three modal operators to characterise the status of objects within a description: **definite** (the object's value is fixed by the description), **indefinite** (the object exists but its value is not fixed by this description), and **arbitrary** (the description is insensitive to the object's value — changes in it produce no change in what the description resolves). We adopt this triad as framework vocabulary for the status of constraints within an inference-relationship.

Definite: A constraint is *definite* for an inference-relationship when the relationship resolves its value cleanly for the configuration in question. The β-value of a spatially well-resolved pattern in a Game of Life analysis is definite for the standard emission set.

Indefinite: A constraint is *indefinite* when the relationship cannot resolve its value in this configuration, though it exists in principle. The knockout experiments in the computational substrates move specific constraints from definite to indefinite status by removing the emissions that would resolve them. The 4-10% survival of inference under knockout is a measure of how robust the five-constraint decomposition is when particular constraints become indefinite.

Arbitrary: A constraint is *arbitrary* when the inference-relationship is structurally insensitive to its value — changes in the constraint produce no change in the inferred structure. The constraint-specific effective N conjecture (Constraint_Relative_Distinguishability_Position_Paper) characterises configurations in which one constraint is definite along one axis while being effectively arbitrary along another. The scale-invariance discussion in Constraints_as_Relationships_Working_Note V3 characterises κ-self-similar regions as configurations where κ is definite while β is effectively arbitrary along the κ-self-similar axis.

The modal triad replaces the language of "constraint manipulation" and "constraint suppression" that had been sliding toward container readings. We do not suppress constraints; we construct inference-relationships in which particular constraints have particular modal status. This is vocabulary, not new physics, but it does structural work: it gives a triadic classification for what kinds of relational configurations are possible, and it connects the framework's inference-relationship language to Uemov's formal modal operators within parametric general systems theory.

CE.2.10 What the Uemov Vocabulary Does and Does Not Add

The four commitments of CE.2.7-CE.2.9 — relational signatures (middle tier), role-substitution, relational collapse, and the modal triad — are formal vocabulary additions. They do not change any framework derivation. Every constants result, every N-transition analysis, every knockout experiment returns the same numerical content under the Uemov-extended vocabulary as under the older working vocabulary.

What they add is articulation discipline. Role-substitution prevents the framework from sliding between senses of Cl(5) objects; relational collapse names the N-transitions as structurally specific phenomena rather than as unexplained shifts in behaviour; the modal triad replaces container-flavoured "manipulation" language with relation-status language.

We do not commit to Uemov's full Language of Ternary Description — the 40 system parameters, the specific inference rules of the calculus, the categorical-systems extensions developed in later literature — as framework machinery. The framework has its own formal apparatus (Cl(5), monogamy polytope, HMM inference, Φ-landscape), and importing Uemov's calculus wholesale would produce duplication without derivational gain. What we commit to is the four vocabulary items above. These are the portable pieces — the parts that do real work in articulating what the framework already does.

CE.3 The Five Categories: Derivation from Distinguishability

CE.3.1 The Analytical Method

We ask: What aspects must be specified for distinction to be robust?

"Robust" means: - Not fragile (survives perturbation) - Not instantaneous (maintains identity across configurations) - Not isolated (participates in larger relational structure)

For each aspect, we identify an irreducible requirement—something that cannot be derived from the other aspects.

CE.3.2 Category 1: Demarcation (β)

The Question: Where is the boundary between A and B?

The Requirement: For A to be distinguished from B, there must be demarcation—gradient structure marking where the region associated with A transitions to that associated with B.

Why Irreducible: Without boundary, A and B blur into undifferentiated continuum. No amount of pattern, resource, integration, or ordering can substitute for demarcation. You cannot distinguish what has no edge.

Formal characterization: β measures the magnitude of gradients in the relational structure. High β means sharp demarcation; low β means diffuse boundaries.

Failure mode: β → 0 implies no gradient, no demarcation, no distinction. The structure approaches the undifferentiated nothingness that cannot exist.

CE.3.3 Category 2: Differentiation (κ)

The Question: How do A and B differ in structure?

The Requirement: For A to be distinguished from B, there must be structural difference. If A and B have identical patterns, they are indistinguishable—and by Leibniz's principle, not two things but one.

Why Irreducible: Boundary alone is insufficient. Two regions can be sharply demarcated yet structurally identical—like two drops of pure water. Pattern provides the qualitative difference that makes distinction meaningful.

Formal characterization: κ measures structural complexity/compressibility. High κ means rich, incompressible pattern; low κ means simple, compressible pattern.

Failure mode: κ → 0 implies no structural difference, no basis for distinction. A and B collapse into identity.

CE.3.4 Category 3: Instantiation (ρ)

The Question: What realizes the distinction?

The Requirement: Distinction requires capacity—something to BE configured differently. Pattern without medium is abstract, not actual. For distinguishability to *obtain*, there must be resource for configurations to be realized and sustained.

Why Irreducible: Boundary and pattern specify *what* distinction would look like. Resource specifies that it *actually obtains*. Abstract possibility is not the same as actuality—and the axiom concerns what actually exists, not what merely could exist.

Formal characterization: ρ measures the density of configurational capacity. High ρ means rich capacity for distinction; low ρ means impoverished capacity.

Failure mode: ρ → 0 implies no capacity for distinction to be realized. Existence becomes merely formal—and formal-but-not-actual is another form of the nothingness that cannot exist.

CE.3.5 Category 4: Unification (λ)

The Question: What makes A one thing rather than many fragments?

The Requirement: For A to be distinguished from B, A must be unified—its aspects must cohere. Without integration, "A" is not a relatum but a scattering of unconnected fragments, and the distinction "A versus B" dissolves into many smaller distinctions with no overall structure.

Why Irreducible: Boundary, pattern, and resource can all be present, yet the structure may lack coherence. A cloud of dust has boundary, pattern, and resource—but it is not unified in the way a crystal is. Integration captures the correlation structure that binds aspects into wholes.

Formal characterization: λ measures correlation/coherence across the relational structure. High λ means strong integration; low λ means fragmentation.

Failure mode: $\lambda \to 0$ implies no coherence, no unity, no stable relatum. The structure fragments below the threshold of robust distinguishability.

CE.3.6 Category 5: Asymmetry (τ)

The Question: Can orderings be distinguished from their reversals?

The Requirement: For complex relational structure to exist, there must be the capacity for asymmetry—for distinguishing "A relates to B" from "B relates to A" in contexts where this matters.

Why Irreducible: The first four categories (β, κ, ρ, λ) are all **symmetric—**they do not distinguish directions. Boundary between A and B equals boundary between B and A. Pattern of A-B relation equals pattern of B-A relation. And so on.

τ is the **unique asymmetric category**. It enables directional structure that no combination of symmetric categories can provide.

Critical clarification: τ measures geometric asymmetry (chirality), NOT temporal persistence. We have not introduced time. What we call "time" emerges from configurations with $\tau > 0$ at $N \geq 3$, as developed in Section 4 of the main text.

Formal characterization: τ measures the degree to which orderings can be distinguished from their reversals. High τ means strong chirality; $\tau = 0$ means perfect symmetry.

Failure mode: $\tau = 0$ implies perfect symmetry—no distinguishable orderings, no directional structure. Section 4 shows this has profound consequences: at $N = 2$, $\tau = 0$ necessarily; ordering structure requires $N \geq 3$.

CE.3.7 Summary: The Five Questions

Category	Question	Constraint	Character
Demarcation	Where is the boundary?	β	Symmetric
Differentiation	How do they differ?	κ	Symmetric
Instantiation	What realizes it?	ρ	Symmetric
Unification	What makes each one?	λ	Symmetric
Asymmetry	Which way does it go?	τ	**Asymmetric**

The 4+1 structure (four symmetric, one asymmetric) is not accidental but reflects the logical structure of distinguishability.

CE.4 Completeness: Why No Sixth Category

CE.4.1 The Completeness Claim

Claim: The five categories exhaust what robust distinguishability requires. Any proposed sixth category either: 1. Reduces to one of the five 2. Reduces to a combination of the five 3. Fails to address a genuine requirement of distinguishability

CE.4.2 Three Exhaustion Arguments

We provide three independent arguments for completeness, each approaching from a different direction.

Argument 1: Interrogative Exhaustion

For any distinction A vs B, what questions can we ask?

Question	Category	Constraint
Where is the boundary?	Demarcation	β
How do they differ?	Differentiation	κ
What sustains them?	Instantiation	ρ
What unifies each?	Unification	λ
Which way does it go?	Asymmetry	τ

What other question could there be? Every question about distinction reduces to one of these five.

Argument 2: Ontological Exhaustion

What aspects of being must be specified for something to exist as something?

Aspect of Being	Question	Constraint
Location/Extension	Where?	β
Quality/Property	What kind?	κ
Substance/Actuality	How real?	ρ
Unity/Identity	How one?	λ
Direction/Process	Which way?	τ

These correspond to classical ontological categories: - β ↔ Extension (Descartes), Topos (Aristotle) - κ ↔ Quality (Aristotle), Form (Plato) - ρ ↔ Substance (Aristotle), Matter (Plato) - λ ↔ Unity (Parmenides), Synthesis (Kant) - τ ↔ Process (Whitehead), Temporality (Heidegger)

The convergence with classical ontology suggests these categories are not arbitrary but reflect the structure of existence itself.

Argument 3: Information-Theoretic Exhaustion

What must be specified to transmit a distinction?

1. **Where** the distinction is (boundary information) → β
2. **What** the distinction is (structural information) → κ
3. **How much** supports it (magnitude information) → ρ
4. **How coherent** it is (correlation information) → λ
5. **Which direction** it goes (directional information) → τ

This is a complete specification. Additional information would be redundant with or derived from these five.

CE.4.3 Refutation of Candidate Sixth Categories

We consider proposed sixth categories and show each reduces:

Candidate: Scale/Size - "How big is A relative to B?" - **Reduces to:** β (boundary extent) and ρ (resource magnitude) - Not independent

Candidate: Duration/Persistence - "How long does A last?" - **Reduces to:** τ (ordering structure) integrated along a path - Not independent - Note: "Duration" presupposes temporal structure, which emerges from τ at $N \geq 3$

Candidate: Complexity - "How complex is A?" - **Reduces to:** κ (pattern structure) - Complexity IS pattern structure - Not independent

Candidate: Stability - "How stable is A against perturbation?" - **Reduces to:** function of all five constraints - High stability = robust β, coherent κ, sufficient ρ, strong λ, appropriate τ - Not independent (it's a function of the five, not a sixth dimension)

Candidate: Multiplicity - "How many features are there?" - This is N, the count of features - N is a parameter describing how many configurations exist, not a property of each configuration - The constraints describe *each* feature; N describes *how many* - Not a constraint dimension but a separate parameter

Candidate: Relation/Connection - "How is A related to B?" - This is what the coupling matrix M(A,B) describes - The coupling operates IN the five constraint dimensions - Relation is not a sixth dimension but structure *in* the five dimensions - Not independent

Candidate: Causation - "Does A cause B?" - **Reduces to:** asymmetric coupling ($\tau > 0$) plus coupling structure - Causation = asymmetric coupling in the presence of ordering - Not independent

CE.4.4 The Completeness Theorem

Theorem CE.1 (Categorical Completeness). The five categories (demarcation, differentiation, instantiation, unification, asymmetry) are exhaustive: any aspect of robust distinguishability can be expressed as a function of these five.

Proof sketch (by exhaustion). We have shown: 1. Each category addresses an irreducible question about distinction 2. The categories correspond to classical ontological analysis 3.

The categories provide complete information-theoretic specification 4. All candidate sixth categories reduce to the five

Therefore, no sixth category exists. ■

CE.5 Independence: Why No Reduction to Four

CE.5.1 The Independence Claim

Claim: No constraint can be derived from the other four. Each addresses an irreducible requirement.

CE.5.2 Pairwise Independence

For each pair of constraints, we exhibit configurations where one varies while the other is held fixed:

Pair	Configuration where independent
β, κ	Sharp boundary (high β) can enclose simple or complex pattern (variable κ)
β, ρ	Sharp boundary (high β) can enclose high or low resource density (variable ρ)
β, λ	Sharp boundary (high β) can enclose integrated or fragmented structure (variable λ)
β, τ	Sharp boundary (high β) can enclose symmetric or chiral structure (variable τ)
κ, ρ	Complex pattern (high κ) can exist with much or little resource (variable ρ)
κ, λ	Complex pattern (high κ) can be integrated or fragmented (variable λ)
κ, τ	Complex pattern (high κ) can be symmetric or chiral (variable τ)
ρ, λ	High resource (high ρ) can be integrated or fragmented (variable λ)
ρ, τ	High resource (high ρ) can be symmetric or chiral (variable τ)
λ, τ	High integration (high λ) can be symmetric or chiral (variable τ)

CE.5.3 Independence from Combinations

Could any constraint be derived from a combination of the other four?

β from (κ, ρ, λ, τ): No. Pattern, resource, integration, and ordering can all be specified without determining where boundaries lie. Boundary is topological information not contained in the others.

κ from (β, ρ, λ, τ): No. Boundary, resource, integration, and ordering can all be specified without determining the structural pattern. Pattern is informational content not contained in the others.

ρ from (β, κ, λ, τ): No. Boundary, pattern, integration, and ordering can all be specified without determining the resource capacity. Resource is actuality not contained in the others.

λ from (β, κ, ρ, τ): No. Boundary, pattern, resource, and ordering can all be specified without determining how parts cohere. Integration is correlation structure not contained in the others.

τ from (β, κ, ρ, λ): No. This is the most important case.

CE.5.4 The Irreducibility of τ

Theorem CE.2. No combination of symmetric constraints (β, κ, ρ, λ) can produce the asymmetric constraint (τ).

Proof.

Consider any function f(β, κ, ρ, λ). Under the exchange operation that swaps "A relates to B" with "B relates to A":

- β is invariant (boundary between A-B equals boundary between B-A)
- κ is invariant (pattern of A-B relation equals pattern of B-A relation)
- ρ is invariant (resource in A-B equals resource in B-A)
- λ is invariant (integration of A-B equals integration of B-A)

Therefore f(β, κ, ρ, λ) is invariant under the exchange.

But τ is NOT invariant—it measures precisely the degree to which orderings differ from their reversals.

No invariant function can produce a non-invariant quantity. ■

This theorem establishes that τ is categorically distinct from the other four. The 4+1 structure is not arbitrary but forced by the symmetric/asymmetric distinction.

CE.5.5 Empirical Validation

The independence claim has empirical support:

Principal Component Analysis: Analysis across 256 cellular automata rules and 47 Game of Life patterns shows five principal components account for >95% of behavioral variance. If any constraint were reducible to others, fewer components would suffice.

Knockout Experiments: Removing any single constraint dimension from predictive models reduces accuracy by >20%. If any constraint were derivable from others, its removal would not degrade performance.

Variance Inflation Factors: All VIF values < 2, indicating no problematic multicollinearity among the five constraints.

CE.6 The Structure of the Categories

CE.6.1 The 4+1 Partition

The five categories partition into two classes:

Symmetric (4): β, κ, ρ, λ - Invariant under exchange of relata - Describe "what exists" - Present at all $N \geq 2$

Asymmetric (1): τ - Not invariant under exchange - Describes "directional structure" - Non-trivial only at $N \geq 3$

This partition is basis-independent—it does not depend on how we parameterize the constraints.

CE.6.2 Logical Hierarchy (Not Temporal)

The categories have a logical (not temporal) hierarchy:

Level 1: Demarcation (β) - Most basic requirement - Without boundary, no distinction exists

Level 2: Differentiation (κ) and Instantiation (ρ) - Given boundary, need content (κ) and realization (ρ) - These can vary independently of each other

Level 3: Unification (λ) - Given content and realization, need coherence - Binds aspects into unified relata

Level 4: Asymmetry (τ) - Given unified relata, enables directional structure - Requires $N \geq 3$ to be non-trivial

This hierarchy is logical, not temporal. We are not claiming β "comes first in time"—time has not been introduced. We are saying β is more basic in the sense that later categories presuppose earlier ones.

CE.6.3 Connection to Geometric Algebra

The 4+1 structure has a natural expression in Clifford algebra Cl(5):

Grade	Structure	Constraint Connection
0	Scalar	Normalization
1	Vector	Features (relata)
2	Bivector	Pairwise relations (β, κ, ρ, λ)
3	Trivector	Three-way structure (τ emerges)
4	Quadvector	Higher correlations
5	Pseudoscalar	Total orientation

The emergence of τ at $N \geq 3$ corresponds to the appearance of trivector (grade-3) structure. At $N = 2$, only bivector (grade-2) structure exists, which is inherently symmetric.

This is developed fully in SI: Geometric Algebra Foundations.

CE.7 Philosophical Foundations

CE.7.1 Relation to Classical Ontology

The five categories align with classical ontological analysis:

Aristotle's Categories: - Substance ↔ ρ (what makes something actual) - Quality ↔ κ (what makes it this kind) - Quantity ↔ β, ρ (extension and magnitude) - Relation ↔ λ (connection to others)

Kant's Categories: - Quantity ↔ β (unity, plurality, totality) - Quality ↔ κ (reality, negation, limitation) - Relation ↔ λ (substance, causality, community) - Modality ↔ ρ, τ (possibility, existence, necessity)

Whitehead's Categories: - Extension ↔ β - Pattern ↔ κ - Actuality ↔ ρ - Prehension ↔ λ - Process ↔ τ

The convergence suggests our categories are not invented but discovered—they reflect the structure of existence that classical thinkers also identified.

CE.7.2 Structural Realism

The framework aligns with structural realism (Ladyman & Ross, French): - Relations are primary - "Things" are nodes in relational structure - Physical content is structural content

Our contribution: deriving this from the axiom rather than assuming it.

CE.7.3 Status: Ontological and Epistemological

Are the constraints features of reality or features of our knowledge of reality?

Answer: Both—but not in a way that makes them arbitrary.

The constraints are: - **Ontologically grounded:** Derived from what existence requires (the axiom) - **Epistemologically necessary:** Any inference about configurations requires these five aspects

This dual status follows from the Uemov inversion. If relations are primitive, then the structure of relations IS the structure of reality. There is no gap between "how we describe" and "what exists" at the fundamental level.

CE.8 Objections and Responses

CE.8.1 Objection: The Categories Are Anthropocentric

Objection: The five categories reflect human cognition, not the structure of reality.

Response: The categories are derived from the axiom, not from observation of human cognition. They are what distinguishability requires, regardless of who (or what) is doing the distinguishing.

If the categories happen to align with human cognition, this is because human cognition has evolved to track features of existence—not because we have projected our cognition onto existence.

Moreover, the empirical validation (PCA, knockout experiments) was performed on non-biological systems (cellular automata) with no connection to human cognition.

CE.8.2 Objection: The Completeness Proof Is Not Rigorous

Objection: Showing that candidate sixth categories reduce is not the same as proving no sixth category exists.

Response: We acknowledge this limitation. A fully rigorous completeness proof would require a formal characterization of "category" and a proof that the five exhaust all instances.

What we have provided is: 1. Multiple independent exhaustion arguments (interrogative, ontological, information-theoretic) 2. Refutation of all proposed sixth categories 3. Convergence with classical ontological analysis 4. Convergence with Clifford algebra structure Cl(5) 5. Empirical validation (five dimensions capture >95% variance)

Together, these provide strong evidence for completeness.

CE.8.3 Objection: The Independence Proof Is Circular

Objection: Showing that constraints can vary independently assumes they are independent.

Response: The independence arguments (CE.5) show that for any constraint X, there exist configurations where X varies while the other four remain fixed. This is constructive, not circular.

The PCA analysis provides independent statistical confirmation: five principal components account for the variance, not four.

CE.8.4 Objection: Different Decompositions Might Be Possible

Objection: Even if five constraints are necessary, perhaps different sets of five could work equally well.

Response: Different basis choices in a 5-dimensional configuration characterization are mathematically equivalent. However:

1. The categorical analysis provides natural kinds, not arbitrary basis vectors
2. The 4+1 symmetric/asymmetric split is basis-independent

3. The correspondence to classical ontology suggests the categories are natural, not arbitrary

Alternative decompositions would be related to ours by transformation—mathematically equivalent but perhaps less conceptually transparent.

CE.9 Summary

CE.9.1 Main Results

1. **The Uemov Inversion (CE.2):** Relations are ontologically prior to relata. This is forced by the axiom, not chosen for convenience.

2. **Categorical Necessity (CE.3):** Each of the five constraints addresses an irreducible aspect of robust distinguishability:

    - Demarcation (β): where is the boundary?
    - Differentiation (κ): how do they differ?
    - Instantiation (ρ): what realizes the distinction?
    - Unification (λ): what makes each a unity?
    - Asymmetry (τ): what enables ordering?

3. **Categorical Completeness (CE.4):** No sixth category exists. All aspects of distinguishability reduce to the five.

4. **Categorical Independence (CE.5):** No constraint reduces to the others. All five are necessary. In particular, no combination of symmetric constraints can produce the asymmetric constraint τ.

5. **The 4+1 Structure (CE.6):** Four symmetric + one asymmetric. The asymmetric category (τ) enables ordering and requires $N \geq 3$.

6. **Philosophical Grounding (CE.7):** The categories align with classical ontology and structural realism. They are derived from the axiom, not assumed.

CE.9.2 Connection to Other Supporting Information

Document	Establishes
This document (CE)	Why five categories (philosophical/logical)
SI: Circulation Proof	How ordering emerges at $N \geq 3$ (mathematical)
SI: Geometric Algebra	Grade structure and the 4+1 partition (algebraic)
SI: Efficiency Potential	The $\Phi = \ln(\Omega/K)$ structure (field-theoretic)

The approaches converge: - **Categorically:** Five exhaust the requirements of distinguishability - **Mathematically:** Five is forced by the circulation/non-diagonalizability structure - **Algebraically:** Five corresponds to Cl(5) with natural grade interpretation - **Empirically:** Five constraints govern all observed systems

CE.9.3 What This Document Does Not Establish

- The specific mathematical form of each constraint (see Section 2 of main text)
- The N-dependence of constraint independence (see Section 4 of main text)
- The physical interpretation of constraints (see Section 5 of main text)
- The emergence of temporal structure from τ at $N \geq 3$ (see SI: Circulation Proof)

This document establishes only the categorical structure: why five, why these five, and why no more or fewer.

CE.10 Conclusion

The five constraints are not arbitrary parameters chosen for convenience. They represent the exhaustive categorical decomposition of what robust distinguishability requires.

This decomposition is: - **Derived** from the foundational axiom ($\Diamond N \rightarrow \neg N$) - **Complete** (no sixth category exists) - **Independent** (no reduction to four is possible) - **Structured** (4 symmetric + 1 asymmetric) - **Grounded** in classical ontology and structural realism

The number five is thus not accidental but necessary—forced by the logical structure of distinguishability itself.

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Appendix: Glossary of Terms

Axiom: $\Diamond N \rightarrow \neg N$ — The possibility of absolute nothingness entails its negation.

Category: An irreducible aspect of what robust distinguishability requires.

Configuration: A point in the space of constraint values; what the relational structure is doing at a particular locus.

Constraint: A measurable aspect of relational structure; one of β, κ, ρ, λ, τ.

Distinguishability: The relational primitive; A is distinguishable FROM B.

External relation: A relation between relata that retain independent identity regardless of the relation's existence. Contrast with internal relation.

Feature: A topologically protected pattern in relational structure; a relatum.

Internal relation: A relation constitutive of what the relata are; severing the relation changes what the relata are or eliminates them entirely. Contrast with external relation.

Modal triad: The threefold classification of a constraint's status within an inference-relationship as *definite* (resolved), *indefinite* (exists but not resolved), or *arbitrary* (the relationship is insensitive to its value). Adapted from Uemov's Language of Ternary Description.

Relational collapse: The phenomenon where a relation shifts from external to internal status. In the framework, occurs at the $N = 2 \rightarrow 3$, $N = 3 \rightarrow 4$, and $N = 4 \rightarrow 5$ transitions. Term borrowed from Uemov.

Relational signature: A quantitative tag identifying how a specific pattern participates in the distinguishability structure. Framework term for the middle tier of the Uemov ontology (the properties tier). Examples: α, $\sin^2\theta\_W$, particle masses, γ_τ/γ_pair scaling ratio.

Relatum (pl. relata): A node in the network of distinguishing relations; defined by its pattern of distinctions with other relata.

Robust: Stable against perturbation, maintaining identity across configurations, participating in larger relational structure.

Role-substitution: The principle that a single object can play different ontological roles (relation, relational signature, relatum) depending on the description it is part of. Adapted from Uemov's Language of Ternary Description. The grade-2 bivector in Cl(5) is the paradigm framework case.

Topologically protected: Cannot be continuously deformed to trivial structure; requires discrete transition to dissolve.

Uemov inversion: The ontological order Relations → Properties → Things, rather than the traditional Things → Properties → Relations.