Section 4: The Emergence of Causality

4.1 The Problem of Time

We have developed a geometric framework—constraint space with its potential Φ, gradient $\nabla\Phi$, metric, and curvature—without assuming time. Yet we experience time: sequences, before and after, cause and effect. Where does this temporal structure come from?

The standard move in physics is to *assume* time as a background dimension. Events are located in spacetime; dynamics is motion through time; causality follows temporal ordering. But this leaves time unexplained—a primitive structure smuggled in at the foundations.

Our framework suggests a different approach: *temporal structure emerges from the geometry of distinguishability when configurations have sufficient complexity*. Time is not a container in which things happen; it is a feature of certain relational structures—specifically, those with $N \geq 3$ distinguishable features.

This section develops this claim. We show that:

1. At $N = 2$, no ordering structure is possible—the configuration is inherently symmetric
2. At $N \geq 3$, ordering structure becomes possible—asymmetry can exist
3. What we call "time" is the ordering parameter for configurations with non-trivial asymmetry
4. What we call "causality" is asymmetric constraint structure between features

The emergence is not temporal (that would be circular) but geometric: certain configurations have structure that others lack.

4.2 The N = 2 Configuration: Symmetric and Orderless

4.2.1 The Minimum Configuration

Section 2 established that the axiom requires $N \geq 2$: at least two distinguishable features in relation. The $N = 2$ configuration is the simplest structure consistent with existence.

Let A and B denote two distinguishable features. Each has a constraint profile:

- $C^A = (C_1^A, C_2^A, C_3^A, C_4^A, C_5^A)$
- $C^B = (C_1^B, C_2^B, C_3^B, C_4^B, C_5^B)$

Their relationship is characterized by the coupling matrix M(A,B) introduced in Section 2.7.

4.2.2 Inherent Symmetry

At N = 2, consider the "ordering" of A and B. We might write "A-B" to denote their relation. But is "A-B" distinguishable from "B-A"?

It is not.

There is no third feature to serve as reference. To distinguish "A first, then B" from "B first, then A" requires a vantage point outside the A-B pair. But at N = 2, no such vantage exists. The labels "A" and "B" are arbitrary; swapping them changes nothing about the relational structure.

More precisely: the coupling matrix M(A,B) is symmetric in the sense that the *structure* it describes is invariant under exchange of A and B. The matrix elements $M(A,B)_{ij}$ encode distinguishability between constraint dimensions, but this encoding doesn't prefer A over B or B over A.

4.2.3 No Ordering Structure

This symmetry has a crucial consequence: **the ordering constraint τ is necessarily zero at N = 2**.

Recall from Section 2 that τ measures asymmetric/ordering structure—the degree to which orderings can be distinguished from their reversals. At N = 2:

- The only "ordering" is A-B
- Its "reversal" is B-A
- These are indistinguishable (no reference frame to distinguish them)
- Therefore $\tau = 0$

This is not an empirical observation but a geometric necessity. The N = 2 configuration lacks the structure required for non-zero τ. Ordering requires something to order *with respect to*, and at N = 2, there is no such thing.

4.2.4 Decomposability Revisited

Section 2.8 noted that N = 2 configurations are decomposable: two symmetric matrices can be simultaneously diagonalized. We can now see why this matters for ordering.

In the diagonal basis, A and B couple through independent modes. Each mode is a simple oscillation—symmetric under reversal. The superposition of symmetric modes is symmetric. There is no way to construct asymmetry from purely symmetric components.

Decomposability ↔ Symmetry ↔ $\tau = 0$

These are not three separate facts but three expressions of the same geometric structure.

4.3 The N = 3 Transition: The Birth of Asymmetry

4.3.1 Three Features

Now consider three distinguishable features: A, B, C. Each has its constraint profile; each pair has its coupling matrix: M(A,B), M(B,C), M(C,A).

Something qualitatively new becomes possible.

4.3.2 Circulation

Consider a "loop" through the three features: $A \to B \to C \to A$. We can ask: is this loop distinguishable from its reversal $A \to C \to B \to A$?

At N = 2, we could not ask this question—there was no loop, only a pair. At N = 3, loops exist, and they can have *chirality*: a handedness that distinguishes clockwise from counterclockwise.

Geometrically, consider the circulation integral around the loop:

$$\oint \nabla\Phi \cdot dl$$

integrated along the path $A \to B \to C \to A$.

If the three coupling matrices M(A,B), M(B,C), M(C,A) cannot be simultaneously diagonalized, this integral is generically non-zero.

The proof is linear-algebraic: three (or more) symmetric matrices cannot generically be simultaneously diagonalized. When they cannot, the gradient ∇Φ has a component that "circulates"—a curl-like structure that distinguishes the two directions around the loop.

4.3.3 Chirality as Geometric Asymmetry

This non-zero circulation is *chirality*: the loop has a preferred direction. One orientation (say, $A \to B \to C \to A$) is geometrically distinguishable from the other ($A \to C \to B \to A$).

Crucially, this is a geometric fact, not a temporal one. We have not said "A happens before B happens before C." We have said: the gradient structure around the A-B-C triangle has an asymmetry that distinguishes the two orientations.

Visualize a triangle with arrows on its edges:

See diagram at end of document: Chirality at N ≥ 3

If the arrows form a consistent circulation (all clockwise or all counterclockwise), the triangle has chirality. If the arrows conflict, the circulation is zero. At N = 3, consistent circulation becomes possible for the first time.

4.3.4 The Ordering Constraint Becomes Non-Trivial

With chirality possible, τ can be non-zero.

Definition (refined): τ measures the magnitude of circulation asymmetry in the configuration—how strongly orderings are distinguished from their reversals.

At N = 2: τ = 0 necessarily (no loops exist)
At N ≥ 3: τ can be non-zero (circulation can have preferred direction)

Note: τ *can be* non-zero at N ≥ 3, but need not be. A highly symmetric N = 3 configuration might still have τ ≈ 0. The transition at N = 3 enables asymmetry; it does not guarantee it.

4.3.5 Irreducibility and Its Formalization

Section 2.8 established that N ≥ 3 configurations are irreducible: three symmetric matrices cannot generically be simultaneously diagonalized. We now see the full significance:

Irreducibility ↔ Non-zero circulation possible ↔ τ can be non-zero ↔ Ordering structure possible

Irreducibility is not merely a mathematical curiosity. It is the geometric foundation of asymmetry—and asymmetry is the geometric foundation of what we experience as time.

This irreducibility has been formalized categorically by Gorard, who shows that functorial composition of computations fails precisely when intermediate decomposition is impossible—the categorical expression of our N ≥ 3 transition. The failure of simultaneous diagonalization corresponds to the failure of a functor to preserve compositional structure, providing rigorous mathematical grounding for our claim.

4.4 From Asymmetry to Ordering

4.4.1 What Ordering Means

Given a configuration with non-zero τ, what does "ordering" mean?

Consider three features A, B, C with chiral structure (non-zero circulation). The chirality defines a consistent orientation: say, A → B → C → A is the "positive" direction.

This orientation induces an ordering among the features:

- A precedes B (in the positive direction)
- B precedes C

- C precedes A

This is not temporal precedence in the sense of "A exists at an earlier time than B." It is *geometric* precedence: A's position in the oriented structure comes before B's position.

The cyclic structure at small N: Note that at $N = 3$, this ordering is cyclic: A precedes B precedes C precedes A. This is not the linear time we experience. However, as N increases, the structure becomes richer. At large N (many features), the cycles become vanishingly small perturbations on an effectively linear ordering. What we experience as the linear flow of time emerges from countless overlapping cycles whose individual circularity is imperceptible at our scale. The local ordering appears linear because we cannot perceive the vast cycles that close only across cosmological scales.

4.4.2 The Ordering Parameter

We can parameterize positions along the oriented loop by a parameter, call it t. As t increases, we move A → B → C → A → B → ...

This parameter t is *defined by* the ordering structure. It does not exist independently and then get applied to the configuration. The configuration's chirality creates the ordering; t merely labels positions in that ordering.

This is the key insight: What we call "time" is the ordering parameter for configurations with non-trivial τ. Time does not flow through configurations; configurations with sufficient asymmetry *have* ordering structure, and we call that structure "time."

4.4.3 The Direction of Ordering

The chirality picks out a direction: the "positive" circulation versus the "negative" circulation. But which is which? What determines that A → B → C is "forward" rather than "backward"?

Two possibilities:

Arbitrary convention: The labels "forward" and "backward" are conventional. The chirality distinguishes the two directions but doesn't privilege one over the other. We choose to call one direction "positive" and stick with that choice.

Gradient alignment: The direction of increasing Φ provides a non-arbitrary distinction. Define "forward" as the direction in which Φ tends to increase (or decrease least). This aligns ordering with the efficiency of distinguishability.

The second option connects ordering to the potential structure developed in Section 3. The "arrow" of the ordering parameter points in the direction of the gradient—toward configurations of higher Ω/K.

4.4.4 The Thermodynamic Arrow

At large N, Section 3.2.7 established that Φ connects to thermodynamic quantities and that $d\Phi/d\lambda \geq 0$

characterizes viable configurations. The direction of increasing Φ corresponds to the direction of increasing entropy.

Thus: **the direction of ordering aligns with the thermodynamic arrow.**

This is not imposed but derived. Ordering emerges from chirality; the direction of ordering is fixed by gradient alignment; gradient alignment corresponds to entropy increase. The "arrow of time" is the geometric asymmetry of configurations with non-zero τ, oriented by the potential landscape.

4.5 Causality as Asymmetric Constraint

4.5.1 What Is Causality?

In ordinary language, "A causes B" means something like: A's occurrence brings about B's occurrence; B depends on A; changing A would change B.

In our framework, there is no "occurrence" in the temporal sense—we have not assumed events happening in time. We need a geometric notion of causality.

Definition: A *constrains* B (written A → B) if the configuration of A restricts the possible configurations of B more than B restricts A.

This is asymmetric constraint. The coupling matrix M(A,B) encodes how A and B constrain each other. If this constraint is asymmetric—if A constrains B more than B constrains A—we have a causal relation.

4.5.2 Symmetry of Constraint at N = 2

At N = 2, constraint is symmetric. M(A,B) encodes the distinguishability relationship between A and B, but this relationship is mutual: A is distinguishable from B exactly as much as B is distinguishable from A.

There is no "A causes B" at N = 2 because there is no asymmetry. A and B co-constrain each other symmetrically.

4.5.3 Asymmetric Constraint at N ≥ 3

At N ≥ 3, asymmetry becomes possible. Consider three features A, B, C. The constraint structure is:

- M(A,B): how A and B constrain each other
- M(B,C): how B and C constrain each other
- M(C,A): how C and A constrain each other

These matrices need not be "symmetric" in the sense of creating balanced constraint. A might constrain B strongly while B constrains A weakly. This asymmetry is possible because C provides an external reference.

Example: Suppose A's configuration largely determines B's configuration (given C), but B's configuration only weakly constrains A's. Then A → B in the causal sense: A constrains B more than B constrains A.

4.5.4 Causality and Ordering

There is a deep connection between causal asymmetry and temporal ordering:

Claim: In configurations with non-zero τ, the direction of asymmetric constraint aligns with the direction of ordering.

That is: if A precedes B in the ordering (A comes before B in the oriented loop), then A constrains B more than B constrains A.

This follows from the shared geometric origin of both structures. The gradient structure that creates circulation (non-zero τ) is the same structure that creates asymmetric constraint. Both arise from the irreducibility of the $N \geq 3$ configuration—the non-simultaneous-diagonalizability of the coupling matrices. When M(A,B), M(B,C), and M(C,A) cannot be simultaneously diagonalized, the resulting asymmetry manifests both as circulation (ordering) and as imbalanced constraint (causality). These are two aspects of a single geometric fact, not independent phenomena that happen to correlate.

4.5.5 Causal Chains

At $N = 3$, we have a simple causal structure: A → B → C → A (if that's the direction of ordering). At higher N, more complex causal structures emerge:

- Chains: A → B → C → D → ...
- Branches: A → B and A → C
- Convergences: B → D and C → D
- Cycles: possible but constrained by overall circulation

The network of asymmetric constraints forms a *causal structure*. This structure is not imposed on a pre-existing spacetime; it emerges from the geometry of constraint relationships at $N \geq 3$.

Transitivity at large N: At small N, causal structure may have cycles (A → B → C → A). At large N, the dominant causal chains become effectively transitive: if A → B and B → C, then A → C. This transitivity is not exact but statistical—a consequence of the law of large numbers applied to overlapping causal influences. The apparent strict transitivity of macroscopic causation emerges from the averaging of many microscopic causal relationships.

4.6 Time Without Time

4.6.1 The Apparent Paradox

We have described the "emergence of time" without using temporal language (we hope). But there seems to be a paradox: how can time emerge if emergence is itself a temporal notion?

The resolution: **emergence here is not temporal but structural.**

We are not claiming "first there was an $N = 2$ configuration, then it became $N = 3$, and then time emerged." That would presuppose the time we're trying to derive.

Instead, we are claiming: configurations with $N \geq 3$ features *have* ordering structure; configurations with $N = 2$ do not. This is a structural fact about what kinds of configurations exist, not a narrative about what happened.

4.6.2 The Tenseless Description

The full description is tenseless:

- There exist configurations with $N = 2$ features. These configurations lack ordering structure. $\tau = 0$ for them.
- There exist configurations with $N \geq 3$ features. These configurations can have ordering structure. τ can be non-zero for them.
- Among $N \geq 3$ configurations, some have significant τ (strong ordering) and some have $\tau \approx 0$ (weak ordering).
- Configurations with significant τ have what we call "temporal structure."
- We, as complex configurations ourselves, have significant τ, which is why we experience time.

No temporal sequence is implied. The statement "$N = 2$ configurations lack time" does not mean "they exist before time begins"—it means "they are the kind of configuration that lacks ordering structure."

4.6.3 Why We Experience Time

We are configurations with $N >> 3$. Our structure has robust chirality, strong asymmetric constraint, significant τ. From our position in this structure, the ordering parameter is inescapable—we cannot help but experience sequence, before/after, cause/effect.

This is not an illusion. Time is real for configurations with ordering structure. But it is not fundamental; it is emergent. Configurations without sufficient N do not have time—not because time hasn't reached them, but because they lack the structural complexity that constitutes time.

4.6.4 The Block Universe Reconsidered

The "block universe" view in physics holds that all times exist equally—past, present, and future are equally real, and the flow of time is an illusion.

Our framework offers a nuanced version:

- The constraint field Φ exists tenselessly. All viable configurations exist as the structure of this field.
- Among these configurations, some have ordering structure ($N \geq 3$ with $\tau > 0$).
- For those configurations, "past" and "future" are meaningful—they label positions in the ordering structure.
- But the ordering structure itself exists tenselessly. It is not that past events *were* and future events *will be*; all events in the ordering structure *are*.

The flow of time is not exactly an illusion—it reflects real asymmetry (chirality, gradient direction). But it is not a fundamental flow through a temporal dimension; it is the structure of configurations with sufficient complexity.

4.7 The Constraint Independence Question

4.7.1 Are the Five Constraints Independent?

Section 2 presented the five constraints as conceptually independent. Section 2.4 noted that their mathematical independence varies with N. We can now be more precise.

4.7.2 Constraint Status at N = 2

At $N = 2$:

β (Boundary): Defined. The gradient structure creating two distinguishable features IS the boundary constraint. $\beta > 0$ is what makes $N = 2$ possible rather than $N = 1$.

κ (Pattern): Maximal relative to what's possible. With only two features, there are no intermediate states through which correlations could factor. The structure is inherently non-factorizable—any correlation between A and B is direct, unmediated. This non-factorizability is precisely the character of quantum coherence and entanglement: states that cannot be decomposed into independent components. The $N = 2$ regime is thus inherently "quantum" in this structural sense; quantum coherence is not a mysterious addition to classical physics but the natural character of minimal ($N = 2$) configurations. Section 5 develops this connection further.

ρ (Resource): Defined. Some capacity must sustain the distinction between A and B.

λ (Integration): Not independent of β. At $N = 2$, the correlation between A and B (λ) and the boundary distinguishing them (β) are aspects of the same structure. There is no third feature to create independent correlation patterns.

τ (Ordering): Zero necessarily. As established above, ordering requires $N \geq 3$.

4.7.3 Constraint Status at $N \geq 3$

At $N \geq 3$, all five constraints become independently variable:

β: Multiple boundaries exist (A-B, B-C, C-A) and can vary independently.

κ: Pattern structure can vary—some correlations may factor through intermediate features, others not.

ρ: Resource can be distributed differently across features.

λ: Correlations can span different feature pairs with different strengths—independent of boundary structure.

τ: Can be non-zero. The degree of ordering structure varies across configurations.

4.7.4 Independence as Emergent

The full independence of the five constraints is itself emergent—it requires $N \geq 3$. At $N = 2$, the constraints are not all separable; some collapse into each other.

This is not a defect of the framework but a feature. The $N = 2$ configuration is minimal—it has the simplest structure consistent with existence. The full richness of five independent constraints requires more complex configurations.

4.8 Connections to Physical Formalism

Without developing the physical interpretation in detail (that is Section 5's task), we note parallels to existing frameworks that derive causality algebraically.

4.8.1 Finster's Causal Fermion Systems

Felix Finster's Causal Fermion Systems framework defines causal structure through eigenvalues of operator products, without assuming spacetime:

- Spacetime points are operators $x \in F$ on a Hilbert space
- For two points x, y, the product xy has eigenvalues $\lambda_1, ..., \lambda_{2n}$
- Causal relations (timelike, spacelike, lightlike) are defined by eigenvalue patterns
- Time direction comes from an antisymmetric functional $C(x,y)$

The parallel to our framework:

- Our features are like Finster's operators
- Our coupling matrices are like his operator products

- Our τ (ordering structure) is like his C(x,y)
- Both derive causality from algebraic structure, not assumed spacetime

4.8.2 Barandes' Indivisible Stochastic Processes

Jacob Barandes interprets quantum mechanics through "indivisible" stochastic processes—transitions that cannot be factored through intermediate states:

- Divisible processes: $\Gamma(t \leftarrow t_0) = \Gamma(t \leftarrow t') \cdot \Gamma(t' \leftarrow t_0)$
- Indivisible processes: this factorization fails

The parallel:

- Our $N = 2$ configurations are inherently indivisible (no intermediate features)
- At $N \geq 3$, divisibility becomes possible (correlations can factor through intermediates)
- The transition from indivisible to divisible parallels the emergence of classical from quantum behavior

Barandes' framework is particularly relevant because it derives quantum behavior from stochastic structure without assuming wavefunctions as fundamental—aligning with our relational ontology where wavefunctions are descriptions of correlation patterns, not primitive objects.

4.8.3 The Common Theme

Both Finster and Barandes derive temporal/causal structure from more primitive algebraic or stochastic structure. Our framework does something analogous: deriving ordering (τ) and causality from the geometry of distinguishability.

The common theme: **time and causality are not fundamental but emergent from relational structure of sufficient complexity.**

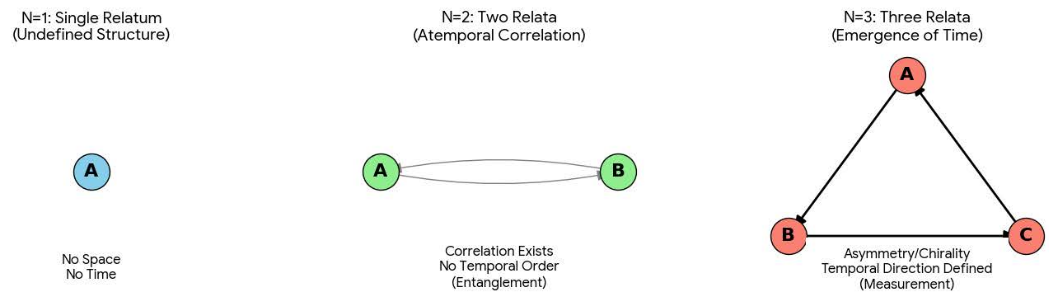
4.9 Summary

From the geometry of constraint space developed in Section 3, we have derived:

1. **$N = 2$ is symmetric:** Two features lack the structure for ordering. The configuration A-B is indistinguishable from B-A. Ordering constraint $\tau = 0$ necessarily.
2. **$N \geq 3$ enables asymmetry:** Three or more features can have circulation—a preferred orientation around loops. This is chirality: geometric asymmetry.
3. **Ordering emerges from chirality:** Configurations with non-zero circulation have ordering structure. The ordering parameter (what we call "time") labels positions in this structure.

4. **Direction comes from gradient:** The direction of ordering aligns with the direction of increasing Φ—toward greater efficiency of distinguishability. This is the thermodynamic arrow.
5. **Causality is asymmetric constraint:** A causes B if A constrains B more than B constrains A. This asymmetry requires $N \geq 3$ and aligns with ordering direction.
6. **Time is structural, not fundamental:** Configurations with ordering structure have time; configurations without it do not. This is a structural fact, not a temporal narrative.
7. **Constraint independence is emergent:** At $N = 2$, constraints are not fully separable. At $N \geq 3$, all five become independently variable.
8. **Physical parallels exist:** Finster's Causal Fermion Systems and Barandes' indivisible processes derive causality algebraically, paralleling our derivation from distinguishability geometry.

The framework is now complete through the emergence of causality and time. Section 5 will examine how this abstract structure connects to physical formalism—how the five constraints map to physical quantities, and how the geometry of constraint space relates to the geometry of spacetime.



Chirality at N ≥ 3

The emergence of circulation and ordering structure

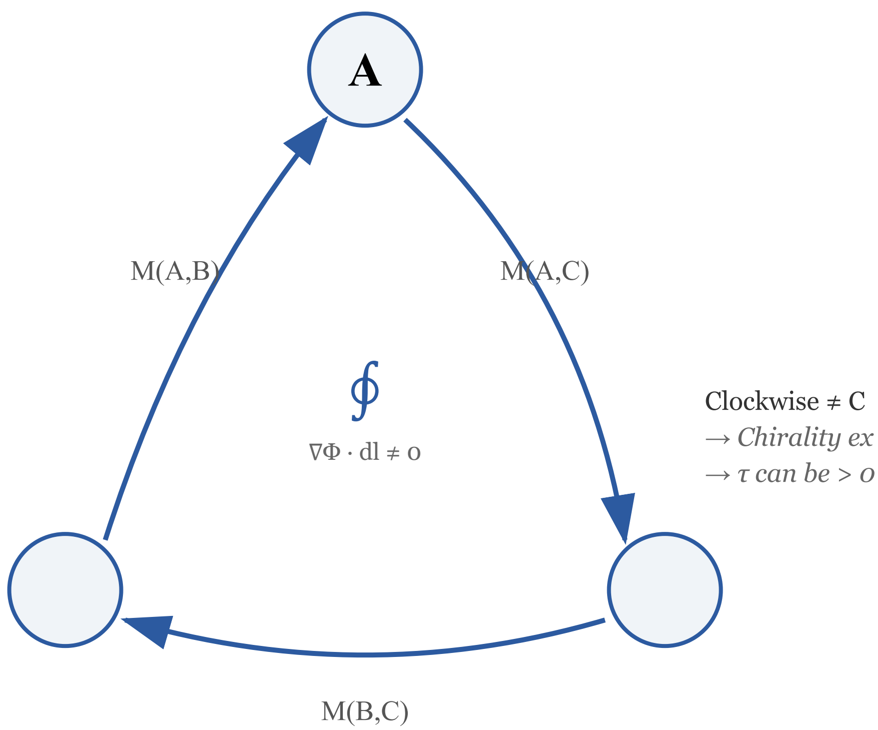


Figure: At N = 3, three features (A, B, C) form a loop with three coupling matrices M(A,B), M(B,C), M(C,A). Because three symmetric matrices cannot generically be simultaneously diagonalized, the circulation integral $\oint \nabla\Phi \cdot dl$ around the loop is generically non-zero. This creates *chirality*—a preferred orientation that distinguishes clockwise from counterclockwise traversal. This geometric asymmetry is the origin of ordering structure ($\tau>0$) and, ultimately, what we experience as temporal direction.