CHAPTER 1 WHOLE NUMBER

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Chapter 1.1 Addition and Subtraction

Summary

- Addition and subtraction are foundational operations used in everyday life and more advanced mathematics.
- Addition involves combining two or more quantities to find a total or sum.
- Subtraction involves finding the difference between two numbers or how much more one is than the other.
- The commutative property applies to addition (e.g., $5 + 3 = 3 + 5$), but not to subtraction.
- The inverse of addition is subtraction and vice versa.
- We use column (vertical) methods and mental strategies for both operations depending on number size.
- Estimation helps check whether the result of an operation is reasonable.
- Addition and subtraction with larger numbers require attention to place value and regrouping (carrying/borrowing).
- These operations apply to real-life contexts like budgeting, time calculations, and measurements.
- Problem-solving often involves identifying keywords like 'total', 'more', 'difference', and 'less' to choose the correct operation.

Examples

- $325 + 148 = 473$ (using vertical algorithm).
- Estimate: $823 + 276 \approx 800 + 300 = 1100$.
- $1{,}003 - 478 = 525$ (borrow from thousands column).
- Using number line: Start at 35 and add 24 → land on 59.
- Word problem: Sara had 45 apples, gave away 18. How many left?
  $45 - 18 = 27$ apples.

Word Problems

Intermediate Questions

1. Add: $327 + 145$.

2. Subtract: $845 - 378$.

3. Estimate: $421 + 608$.

4. Solve: $2{,}100 - 975$.

5. Use the column method to add 654 and 789.

6. Find the difference between 986 and 742.

7. A shop sold 123 pens in the morning and 236 in the afternoon. How many altogether?

8. Subtract: $300 - 128$.

9. What is the missing number: $152 + \_\_ = 275$?

10. Solve: $3{,}000 - 1{,}458$.

Hard Questions

1. Add: $2{,}476 + 3{,}589 + 1{,}204$.

2. Subtract: $8{,}642 - 3{,}917$.

3. Estimate the result of $4{,}928 + 1{,}143$.

4. Explain why $478 - 149 \neq 149 - 478$.

5. Worded problem: Mia had $2,000. She spent $785. How much is left?

6. Solve using vertical method: $6{,}540 + 3{,}768$.

7. If $7{,}254 - x = 3{,}128$, what is x?

8. Round and estimate: $9{,}672 - 4{,}398$.

9. A bus travels $1{,}230$ km in total. It has covered 874 km. How much further?

10. Add: $5{,}429 + 2{,}781 + 3{,}290$.

Practice Questions

1. Mentally find the answers to these sums. Hint: Use the partitioning strategy.

a. $23 + 41$

b. $71 + 26$

c. $138 + 441$

d. $246 + 502$

e. $937 + 11$

f. $1304 + 4293$

g. 140 273 + 238 410

h. 390 447 + 201 132

i. 100 001 + 101 010

2. Mentally find the answers to these differences. Hint: Use the partitioning strategy.

a. $29 - 18$

b. $57 - 21$

c. $249 - 137$

d. $1045 - 1041$

e. $4396 - 1285$

f. $10101 - 100$

3. Mentally find the answers to these sums. Hint: Use the compensating strategy.

a. $15 + 9$

b. $64 + 11$

c. $19 + 76$

d. $18 + 115$

e. $31 + 136$

f. $245 + 52$

Chapter 1.2 Addition and Subtraction Algorithms

Summary

- Addition and subtraction algorithms are step-by-step written methods to solve calculations with larger numbers.
- The vertical or column method is the most common algorithm used for addition and subtraction.
- In column addition, digits are lined up according to their place value (ones under ones, tens under tens, etc.).
- Start adding from the rightmost digit (units), carry over any values greater than 9 to the next column.
- In column subtraction, begin from the right and borrow from the next left digit if the top digit is smaller than the bottom.
- Accuracy in aligning digits by place value is essential to prevent errors.
- Carrying and borrowing (regrouping) must be clearly indicated to maintain clarity.
- Algorithms are particularly helpful for multi-digit numbers and when mental math is not practical.
- These methods are also applied in real-life contexts like finance, data analysis, and quantity tracking.
- Checking your answer by estimating or reversing the operation is a good verification strategy.

Examples

- Addition: $368 + 457$

  Step 1: Line up digits:

  $$\begin{array}{r} 368 \\ \underline{+\,457} \end{array}$$

  Step 2: Add units $(8 + 7 = 15)$, write 5, carry 1.

  Step 3: Add tens $(6 + 5 + 1 = 12)$, write 2, carry 1.

  Step 4: Add hundreds $(3 + 4 + 1 = 8)$, final answer: 825.

- Subtraction: $602 - 45$
- Borrow from hundreds since $2 < 8$ → regroup and subtract → final answer: 144.

Word Problems

Intermediate Questions

1. Use column method to add 437 and 268.

2. Add: $534 + 789$ using written algorithm.

3. Subtract: $900 - 587$ using borrowing.

4. Use column subtraction to calculate $752 - 439$.

5. Find $1{,}320 + 875$ using the algorithm.

6. What is $846 - 297$ using the vertical method?

7. Write and solve: $3{,}450 + 2{,}671$.

8. Line up and solve: $4{,}208 - 1{,}359$.

9. Estimate and then check $1{,}201 - 499$ using algorithm.

10. Add $643 + 298$ using a structured algorithm.

Hard Questions

1. Add: $4{,}532 + 2{,}689 + 1{,}471$ using column method.

2. Subtract: $6{,}205 - 3{,}819$ using vertical algorithm.

3. Explain how carrying affects multi-digit addition.

4. Solve: $8{,}321 - 4{,}756$ and verify by adding result to smaller number.

5. Calculate: $7{,}832 + 9{,}421 + 6{,}789$ using written algorithm.

6. Find and explain steps for $10{,}000 - 6{,}328$.

7. Solve: $6{,}207 - 3{,}586$ using column subtraction with regrouping.

8. Why is place value alignment critical in written algorithms?

9. Calculate and verify: $5{,}005 + 3{,}901$.

10. Subtract: $4{,}004 - 2{,}125$ using a written algorithm.

Practice Questions

1. Give the answer to each of these sums. Check your answer with a calculator.

a. $\begin{array}{r} 36 \\ \underline{+\ 51} \end{array}$

b. $\begin{array}{r} 74 \\ \underline{+\ 25} \end{array}$

c. $\begin{array}{r} 17 \\ \underline{+\ 24} \end{array}$

d. $\begin{array}{r} 47 \\ \underline{+\ 39} \end{array}$

e. $\begin{array}{r} 129 \\ \underline{+\ \ \ 97} \end{array}$

f. $\begin{array}{r} 458 \\ \underline{+\ 287} \end{array}$

g. $\begin{array}{r} 1041 \\ \underline{+\ \ \ 882} \end{array}$

h. $\begin{array}{r} 3092 \\ \underline{+\ 1988} \end{array}$

2. Show your working to find the answer to each of these differences.

a. $32 - 16$

b. $124 - 77$

c. $613 - 128$

d. $1004 - 838$

3. Find the missing numbers in these sums.

a. $$\begin{array}{r} 3\,\square \\ +\ 5\ 3 \\ \hline \square\ 1 \end{array}$$

b. $$\begin{array}{r} 1\,\square\,4 \\ +\ \ 7\,\square \\ \hline \square\ 9\ 1 \end{array}$$

c. $$\begin{array}{r} \square\,\square \\ +\ \square\ 4\ 7 \\ \hline 9\ 1\ 4 \end{array}$$

4. Find the missing numbers in these differences.

a. $$\begin{array}{r} 6\,\square \\ -\ \ 2\ 8 \\ \hline \square\ 4 \end{array}$$

b. $$\begin{array}{r} 2\,\square\,5 \\ -\ \square\ 8\,\square \\ \hline 8\ 1 \end{array}$$

c. $$\begin{array}{r} 3\ \square\,\square\ 2 \\ -\ \ 9\ 2\,\square \\ \hline \square\ 1\ 6\ 5 \end{array}$$

Chapter 1.3 Multiplication

Summary

- Multiplication is a mathematical operation used to find the total number of items in equal-sized groups.
- It is repeated addition: e.g., 4×3 means 4 groups of 3, or $3 + 3 + 3 + 3 = 12$.
- Multiplication is commutative: the order of numbers does not affect the result (e.g., $4 \times 5 = 5 \times 4$).
- It is also associative: changing grouping does not change the result (e.g., $(2 \times 3) \times 4 = 2 \times (3 \times 4)$).
- Multiplication has an identity element: multiplying by 1 does not change a number.
- Multiplication by 0 always gives 0.
- We use mental strategies, vertical algorithm, and area models to multiply.
- Knowing times tables helps to multiply larger numbers quickly and accurately.
- Multiplication is used in many real-life contexts: shopping, calculating area, scaling recipes, budgeting, etc.
- In vertical multiplication, we multiply each digit separately and add partial products.

Examples

- $4 \times 3 = 12$ (4 groups of 3).
- $7 \times 0 = 0$ (multiplying by 0 gives 0).
- $6 \times 1 = 6$ (identity property).
- Vertical method: $23 \times 4 \rightarrow (20 \times 4) + (3 \times 4) = 80 + 12 = 92$.
- Mental strategy: $6 \times 15 = 6 \times (10 + 5) = 60 + 30 = 90$.

Word Problems

Intermediate Questions

1. What is 6×4?

2. Multiply 13×3 using mental maths.

3. Use the vertical method to multiply 24×5.

4. Explain why $7 \times 0 = 0$.

5. Solve: 8×6.

6. What is 9×1?

7. Multiply: 12×2.

8. What is the total of 5 groups of 7?

9. Use repeated addition to show 4×3.

10. Calculate: 10×15.

Hard Questions

1. Multiply 43×6 using vertical algorithm.

2. Explain the identity and zero properties of multiplication.

3. Solve: 35×12 using partial products.

4. Estimate 78×5 by rounding.

5. Use area model: $23 \times 4 = (20 + 3) \times 4 = ?$

6. If a pen costs $\$7$, what is the cost of 28 pens?

7. Solve and check: 65×8.

8. Multiply 109×3 using vertical method.

9. Explain why multiplication is commutative with an example.

10. A box has 24 items. How many items in 36 boxes?

Practice Questions

1. Using your knowledge of multiplication tables, give the answer to these products.

a. 8×7

b. 6×9

c. 12×4

d. 11×11

e. 6×12

f. 7×5

g. 12×9

h. 13×3

2. Find the results to these products mentally. Hint: Use the distributive law strategy – subtraction for **a** to **d** and addition for **e** to **h**.

a. 3×19

b. 6×29

c. 4×28

d. 38×7

e. 5×21

f. 4×31

g. 6×42

h. 53×3

3. Give the result of each of these products, using the multiplication algorithm.

a. $\begin{array}{r} 33 \\ \times \quad 2 \\ \hline \end{array}$

b. $\begin{array}{r} 43 \\ \times \quad 3 \\ \hline \end{array}$

c. $\begin{array}{r} 72 \\ \times \quad 6 \\ \hline \end{array}$

d. $\begin{array}{r} 55 \\ \times \quad 3 \\ \hline \end{array}$

e. $\begin{array}{r} 129 \\ \times \quad 2 \\ \hline \end{array}$

f. $\begin{array}{r} 407 \\ \times \quad 5 \\ \hline \end{array}$

g. $\begin{array}{r} 526 \\ \times \quad 5 \\ \hline \end{array}$

h. $\begin{array}{r} 3509 \\ \times \quad 9 \\ \hline \end{array}$

4. Find the missing digits in these products.

a.
$$\begin{array}{r} 3\ 9 \\ \times \quad 7 \\ \hline 2\square 3 \end{array}$$

b.
$$\begin{array}{r} 2\ 5 \\ \times \quad \square \\ \hline 1\ 2\ 5 \end{array}$$

c.
$$\begin{array}{r} 7\ 9 \\ \times \quad \square \\ \hline \square 3\,7 \end{array}$$

d.
$$\begin{array}{r} 1\ 3\ 2 \\ \times \quad \square \\ \hline 10\square 6 \end{array}$$

e.
$$\begin{array}{r} 2\square \\ \times \quad 7 \\ \hline \square 8\,9 \end{array}$$

f.
$$\begin{array}{r} \square\square \\ \times \quad 9 \\ \hline 3\ 5\ 1 \end{array}$$

g.
$$\begin{array}{r} 2\ 3\square \\ \times \quad 5 \\ \hline 1\square 6\ 0 \end{array}$$

h.
$$\begin{array}{r} \square\square\ 4 \\ \times \quad \square \\ \hline \square 1\ 9\ 8 \end{array}$$

Chapter 1.4 Multiplying Larger Numbers

Summary

- Multiplying larger numbers requires careful use of place value and multi-step algorithms.
- We often use the vertical algorithm (also called long multiplication) for multiplying 2-digit and 3-digit numbers.
- Each digit of the second number is multiplied by each digit of the first, considering place value.
- Partial products are calculated separately and then added together.
- Estimation helps us check if our final answer is reasonable.
- Use rounding and compatible numbers to quickly estimate large multiplications.
- Real-world uses include bulk buying, area calculations, scaling, and budgeting.
- Zeroes must be placed carefully in partial products based on the digit's position (tens, hundreds, etc.).
- Mistakes often happen when digits are not aligned properly, so layout is very important.
- Multiplication can be verified using reverse operations like division or estimation.

Examples

- 23×46:

  Step 1: $23 \times 6 = 138$

  Step 2: $23 \times 40 = 920$

  Final: $138 + 920 = 1{,}058$.
- Estimate 78×42 → Round to $80 \times 40 = 3{,}200$.
- Multiplying 3-digit numbers: 124×32

  → Multiply each digit of 32 with 124 and add.
- Use the vertical method for 215×37 → total = 7,955.
- Partial product method: $(200 \times 30) + (200 \times 7) + (10 \times 30) + (10 \times 7) + (5 \times 30) + (5 \times 7)$.

Word Problems

Intermediate Questions

1. Multiply: 34×21 using long multiplication.

2. Estimate: 49×82 by rounding.

3. Use vertical algorithm: 63×25.

4. Multiply: 312×4.

5. What is 203×17?

6. Solve: 126×39 using standard algorithm.

7. Multiply: 45×38 and show all steps.

8. Explain why estimation is useful in multiplication.

9. Multiply: 128×23 using vertical method.

10. What is 73×50?

Hard Questions

1. Calculate: 237×46 using full working.

2. Estimate and solve: 189×67.

3. Use long multiplication: 405×82.

4. Multiply: 524×31 and explain the steps.

5. What is the result of $1{,}005 \times 76$?

6. Find 219×88 using vertical layout.

7. Explain how to multiply a 3-digit by 2-digit number.

8. Multiply and verify: 413×29.

9. Use estimation to check your result: 316×41.

10. Solve: 628×47 and round your answer to the nearest hundred.

Practice Questions

1. Use the strategy to find these products.

a. 17×20

b. 36×40

c. 92×70

d. 45×500

e. 138×300

f. 92×5000

g. 317×200

h. 1043×9000

2. If both numbers in a multiplication problem have at least three digits, then the algorithm needs to be expanded. Use the algorithm to find these products.

a. $\begin{array}{r} 294 \\ \times\ 136 \\ \hline \end{array}$

b. $\begin{array}{r} 1013 \\ \times\ \ 916 \\ \hline \end{array}$

c. $\begin{array}{r} 3947 \\ \times\ 1204 \\ \hline \end{array}$

d. $\begin{array}{r} 47126 \\ \times\ \ 3107 \\ \hline \end{array}$

Chapter 1.5 Division

Summary

- Division is a mathematical operation used to split a number into equal parts or groups.
- It is the inverse of multiplication (e.g., $20 \div 4 = 5$ because $5 \times 4 = 20$).
- Division involves a dividend (the number being divided), a divisor (the number of groups), and a quotient (the result).
- There are two main written methods: short division (bus stop method) and long division.
- Short division is used for smaller numbers or single-digit divisors.
- Long division is used for dividing larger numbers and helps break down division step-by-step.
- Sometimes, division results in a remainder or a decimal.
- Estimating the quotient before dividing can make calculations easier.
- Division is used in real-life for sharing, calculating averages, converting units, etc.
- Checking division can be done by multiplying the quotient by the divisor and adding any remainder.

Examples

- Short division: $96 \div 4 = 24$.
- Long division: $125 \div 5 \rightarrow$ 5 into 12 is 2 (10), remainder 2 $\rightarrow$ bring down 5 $\rightarrow$ 5 into 25 is 5 $\rightarrow$ answer: 25.
- Divide $1{,}234 \div 2$ → estimate $1{,}200 \div 2 = 600$.
- Check: $26 \div 4 = 6$ remainder 2 $\rightarrow 6 \times 4 + 2 = 26$.
- In context: 120 apples shared among 8 students → $120 \div 8 = 15$ apples each.

Word Problems

Intermediate Questions

1. What is $84 \div 7$?

2. Use short division to calculate $96 \div 4$.

3. Divide: $128 \div 2$ using the bus stop method.

4. If 5 books are shared between 15 students, how many does each student get?

5. Use long division to solve $247 \div 7$.

6. Estimate: $400 \div 8$.

7. Divide: $36 \div 5$ and write the answer with remainder.

8. What is the result of $1{,}000 \div 10$?

9. Solve: $324 \div 6$.

10. Explain why $60 \div 1 = 60$.

Hard Questions

1. Divide: $842 \div 4$ using long division.

2. Estimate: $725 \div 9$.

3. Use long division to calculate $1{,}236 \div 3$.

4. Explain the difference between short and long division.

5. Divide: $2{,}508 \div 6$ using the written algorithm.

6. What is $978 \div 9$ with remainder?

7. Solve and verify: $636 \div 4$.

8. Use estimation to check: $880 \div 5$.

9. Calculate: $1{,}043 \div 7$ using long division.

10. How would you divide $1{,}525$ jellybeans between 25 children?

Practice Questions

1. Find the answer to these using a mental strategy. Hint: Use the distributive law strategy.

a. $63 \div 3$

b. $76 \div 4$

c. $57 \div 3$

d. $205 \div 5$

e. $203 \div 7$

f. $189 \div 9$

g. $906 \div 3$

h. $490 \div 5$

2. Use the short division algorithm to find the quotient and remainder.

a. $3\overline{)71}$

b. $7\overline{)92}$

c. $2\overline{)139}$

d. $6\overline{)247}$

e. $4\overline{)2173}$

f. $3\overline{)61\,001}$

g. $5\overline{)4093}$

h. $9\overline{)90\,009}$

3. Use the short division algorithm to find the quotient and remainder.

a. $526 \div 4$

b. $1691 \div 7$

c. $2345 \div 6$

d. $92\,337 \div 8$

Chapter 1.6 Estimating and Rounding

Summary

- Estimation involves finding an approximate answer that is close enough to the exact value for practical purposes.
- Rounding simplifies a number while keeping it close to the original, making mental calculations quicker and easier.
- Common rounding places include nearest $10, 100, 1{,}000$, or decimal places.
- Rounding rules: If the digit to the right is 5 or more, round up. If it is less than 5, round down.
- Estimation is helpful for checking calculations, budgeting, shopping, and when precision is not necessary.
- Compatible numbers are rounded values that make calculations easier (e.g., 49 rounded to 50).
- Front-end estimation focuses on the most significant digits, ignoring the rest.
- Rounding decimals involves identifying the place value (e.g., tenths, hundredths) and applying the same rule.
- Estimation and rounding are both essential for making quick decisions in everyday life and problem solving.
- It’s important to choose an appropriate level of rounding depending on the context.

Examples

- Round 58 to the nearest $10 \rightarrow 60$.
- Round $2{,}943$ to the nearest $1{,}000 \rightarrow 3{,}000$.
- Estimate $149 + 267$ by rounding $\rightarrow 150 + 270 = 420$.
- Round 8.763 to 2 decimal places $\rightarrow 8.76$.
- Estimate 74×29 by rounding $\rightarrow 70 \times 30 = 2{,}100$.

Word Problems

Intermediate Questions

1. Round 47 to the nearest 10.

2. Estimate: $196 + 407$ by rounding.

3. What is 983 rounded to the nearest 100?

4. Round $6{,}739$ to the nearest $1{,}000$.

5. Estimate the total of $118 + 263$.

6. Round 8.64 to 1 decimal place.

7. What is 5.827 rounded to 2 decimal places?

8. Estimate: 39×81 using compatible numbers.

9. Round $2{,}345$ to the nearest 10.

10. Why is rounding useful in everyday life?

Hard Questions

1. Estimate: $487 + 914 + 239$.

2. Round $6{,}482$ to the nearest 100 and $1{,}000$.

3. Estimate 627×48 by rounding both numbers.

4. Explain the front-end estimation method with an example.

5. Estimate $1{,}392 - 758$ using rounding.

6. Round 0.7639 to 3 decimal places.

7. Estimate the cost of 7 items priced at $\$12.89$ each.

8. Use compatible numbers to estimate: 45×39.

9. Estimate the perimeter of a rectangle with sides 49.5 m and 18.2 m.

10. Round 78.349 to the nearest whole number.

Practice Questions

1. Round these numbers as indicated.

a. 59 (nearest 10)

b. 32 (nearest 10)

c. 124 (nearest 10)

d. 185 (nearest 10)

e. 231 (nearest 100)

f. 894 (nearest 100)

g. 96 (nearest 10)

h. 584 (nearest 100)

i. 1512 (nearest 1000)

2. Estimate the answers to these problems by first rounding both numbers as indicated.

a. $72 + 59$ (nearest 10)

b. $138 - 61$ (nearest 10)

c. $275 - 134$ (nearest 10)

d. $841 + 99$ (nearest 10)

e. $203 - 104$ (nearest 100)

f. $815 + 183$ (nearest 100)

g. $990 + 125$ (nearest 100)

h. $96 + 2473$ (nearest 100)

i. $1555 - 555$ (nearest 1000)

Chapter 1.7 Order of Operations Investigation

Summary

- Order of operations ensures that mathematical expressions are solved in a consistent and correct way.
- The standard rule is BIDMAS/BODMAS: Brackets, Indices/Orders, Division/Multiplication (left to right), Addition/Subtraction (left to right).
- Brackets must be simplified first before applying other operations.
- Indices (or powers/orders) must be calculated after brackets and before division or multiplication.
- Division and multiplication are performed from left to right, depending on which appears first.
- Addition and subtraction are also handled from left to right.
- Misapplying the order of operations can lead to incorrect answers.
- Using a step-by-step layout helps track progress in complex expressions.
- Real-life uses include spreadsheets, coding, finance, and construction calculations.
- Visualising number operations in stages helps strengthen problem-solving skills and accuracy.

Examples

- $6 + 4 \times 2 = 6 + 8 = 14$ (multiplication before addition)
- $(6 + 4) \times 2 = 10 \times 2 = 20$ (brackets first)
- $18 \div 3 + 2 = 6 + 2 = 8$ (left to right for $\div$ and $+$)
- $8 + 2 \times (3^2) = 8 + 2 \times 9 = 8 + 18 = 26$
- $(12 - 4) \div 2 + 3 = 8 \div 2 + 3 = 4 + 3 = 7$

Word Problems

Intermediate Questions

1. Evaluate: $5 + 3 \times 2$

2. Solve: $(4 + 6) \div 2$

3. What is $10 - 3 + 2$?

4. Simplify: $3 + 2 \times 5$

5. Evaluate: $(8 - 3) \times 4$

6. What is the order of operations used in maths?

7. Solve: $6 \times 2 - 3$

8. Evaluate: $4 + 6 \times 2 - 1$

9. Solve: $7 + (2 \times 3)$

10. Evaluate: $(5 + 5) \div 5$

Hard Questions

1. Simplify: $(10 - 2) \times (6 \div 3)$

2. Evaluate: $4 + 2 \times (3 + 5)$

3. What is the value of: $8 + (12 \div 4) \times 2$?

4. Solve: $(6 + 2 \times 3) - 4$

5. Explain why $3 + 4 \times 2 \neq (3 + 4) \times 2$

6. Evaluate: $24 \div (6 \div 2)$

7. What is the result of $5^2 - 3 \times 2$?

8. Simplify: $3 \times (4 + 5) - 6 \div 2$

9. Evaluate: $(10 + 2) \times (6 - 4)$

10. If $x = 3$, evaluate: $2x + 5 \times (x - 1)$

Practice Questions

1. Use order of operations to find the answers to the following.

a. $2 + 3 \times 7$

b. $5 + 8 \times 2$

c. $10 - 20 \div 2$

d. $22 - 16 \div 4$

e. $6 \times 3 + 2 \times 7$

f. $1 \times 8 - 2 \times 3$

g. $18 \div 9 + 60 \div 3$

h. $2 + 3 \times 7 - 1$

i. $40 - 25 \div 5 + 3$

j. $63 \div 3 \times 7 + 2 \times 3$

k. $78 - 14 \times 4 + 6$

l. $300 - 100 \times 4 \div 4$

2. These computations involve brackets within brackets. Ensure you work with the inner brackets first.

a. $2 \times [(2 + 3) \times 5 - 1]$

b. $[10 \div (2 + 3) + 1] \times 6$

c. $26 \div [10 - (17 - 9)]$

d. $[6 - (5 - 3)] \times 7$

e. $2 + [103 - (21 + 52)] - (9 + 11) \times 6 \div 12$

CHAPTER 10 EQUATIONS

Table of Contents

Chapter 10.1 Introduction to Equations

Summary

- An equation is a mathematical statement that shows two expressions are equal, using the '=' sign.
- Equations often contain a variable (e.g. x) that we can solve for.
- Solving equations means finding the value of the variable that makes the equation true.
- Equations are useful in representing real-life problems mathematically.
- We can use balance and inverse operations to maintain equality while solving equations.

Examples

- $x + 5 = 8$ → Solve: $x = 3$
- $2x = 10$ → Solve: $x = 5$
- $x - 4 = 7$ → Solve: $x = 11$
- $\frac{x}{3} = 6$ → Solve: $x = 18$

Word Problems

Intermediate Questions

1. What is an equation?

2. What symbol is used to show equality?

3. What does it mean to solve an equation?

4. Solve: $x + 4 = 10$

5. Solve: $3x = 12$

6. What is a variable in an equation?

7. Find the missing value: $x - 2 = 5$

8. What is x in the equation $x \div 2 = 7$?

9. Solve: $x + 6 = 11$

10. Write an equation for: 'A number increased by 3 equals 9'

Hard Questions

1. Solve: $x - 7 = 2 \times 3$

2. Explain the steps to solve $x + 5 = 12$

3. Write and solve an equation: 'Double a number is 18'

4. Explain the role of inverse operations in solving equations

5. Find x: $x \div 5 + 3 = 7$

6. Solve: $2x - 3 = 11$

7. If $3x = 2x + 6$, find x

8. Translate: 'Triple a number minus 1 equals 8' into an equation

9. Write a real-life situation that can be represented as an equation

10. Solve and check: $x - 4 = 9$

Practice Questions

1. If $b = 4$, state whether each of the following equations is true or false.

a. $5b + 2 = 22$

b. $10 \times (b - 3) = b + b + 2$

c. $12 - 3b = 5 - b$

d. $b \times (b + 1) = 20$

2. If $a = 10$ and $b = 7$, state whether each of these equations is true or false.

a. $a + b = 17$

b. $a \times b = 3$

c. $a \times (a - b) = 30$

d. $a \times (a - b) = 30$

e. $3a = 5b - 5$

h.f. $b \times (a - b) = 20$

g. $10 - a = 7 - b$

h. $1 + a - b = 2b - a$

3. Write equations for each of the following.

a. The sum of 3 and x is equal to 10.

b. When L is multiplied by 5, the result is 1005.

c. The sum of a and b is 22.

d. When d is doubled, the result is 78.

e. The product of 8 and x is 56.

f. When p is tripled, the result is 21.

g. One-quarter of t is 12.

h. The sum of R and p is equal to the product of R and p.

Chapter 10.2 Solving Equations by Inspection

Summary

- Solving equations by inspection means finding the solution by thinking logically rather than using algebraic steps.
- This method is useful for simple equations that can be solved mentally.
- We test values that make both sides of the equation equal.
- It's often the first method taught before learning formal algebraic techniques.

Examples

- $x + 2 = 5$ → What number added to 2 gives 5? Answer: 3
- $2x = 10$ → What number multiplied by 2 gives 10? Answer: 5
- $x - 4 = 6$ → What number minus 4 gives 6? Answer: 10
- $x \div 3 = 4$ → What number divided by 3 gives 4? Answer: 12

Word Problems

Intermediate Questions

1. Solve by inspection: $x + 3 = 7$

2. What is x if $5x = 15$?

3. Find the value of x: $x - 6 = 4$

4. $x \div 2 = 5$. What is x?

5. What number added to 9 gives 14?

6. Find x if $x - 5 = 0$

7. Solve by inspection: $2x = 8$

8. Find x: $x + 6 = 10$

9. $x - 3 = 1$. What is x?

10. What number divided by 5 gives 3?

Hard Questions

1. Find the value of x in $x + 4 = 12$ using inspection.

2. Explain how inspection helps solve $x - 2 = 9$.

3. Guess and check x: $x \div 4 = 6$

4. What is x if $3x = 27$?

5. Solve $x - 7 = 2$ by trial and error.

6. Use mental maths to solve: $x + 10 = 25$

7. What number doubled equals 22?

8. Solve by inspection: $x \div 3 = 7$

9. Find the number that, when subtracted by 8, equals 12.

10. Why is solving by inspection important before learning algebraic methods?

Practice Questions

1. Solve the following equations by inspection.

a. $2p - 1 = 5$

b. $3p + 2 = 14$

c. $4R - 4 = 8$

d. $4W + 4 = 24$

e. $2b - 1 = 1$

f. $5V + 1 = 21$

g. $5g + 5 = 20$

h. $4(e - 2) = 4$

i. $45 = 5(d + 5)$

j. $3d - 5 = 13$

k. $8 = 3N - 4$

l. $8 = 3o - 1$

2. Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

a. $4 \times (x + 1) - 5 = 11$

b. $7 + x = 2 \times x$

c. $(3x + 1) \; 2 = 8$

d. $10 - x = x + 2$

e. $2 \times (x + 3) + 4 = 12$

f. $15 - 2x = x$

Chapter 10.3 Equivalent Equations

Summary

- Equivalent equations are equations that have the same solution.
- We create equivalent equations by performing the same operation on both sides (e.g. add, subtract, multiply, divide).
- These operations keep the equation balanced and maintain equality.
- This concept is key in solving more complex algebraic equations.

Examples

- $x + 5 = 8$ → Subtract 5 from both sides → $x = 3$
- $2x = 10$ → Divide both sides by 2 → $x = 5$
- $x - 4 = 6$ → Add 4 to both sides → $x = 10$
- $x \div 3 = 7$ → Multiply both sides by 3 → $x = 21$

Word Problems

Intermediate Questions

1. What are equivalent equations?

2. Why do we perform the same operation on both sides?

3. Make an equivalent equation to $x + 2 = 7$ by subtracting 2.

4. Create an equivalent equation to $2x = 8$ by dividing both sides.

5. What happens if you add the same number to both sides of an equation?

6. Solve: $x + 6 = 10$ using equivalent equations.

7. What does it mean to 'balance' an equation?

8. Find x if $x - 5 = 9$ using an equivalent equation.

9. Make $x = 3$ the solution to a new equation.

10. Create two different equivalent equations with the solution $x = 4$.

Hard Questions

1. Explain how multiplying both sides keeps an equation equivalent.

2. Solve: $3x + 2 = 11$ using equivalent steps.

3. How do you reverse an operation in an equation?

4. Find x in $5x = 30$ using equivalent equations.

5. Describe the steps to solve $x - 7 = 4$ by forming equivalent equations.

6. Give an example of two different-looking equations that have the same solution.

7. Solve: $2x - 4 = 10$ using equivalent equations.

8. Make $x = 2$ the solution of three different equations.

9. Create an equation that is equivalent to $x \div 2 = 6$.

10. What makes two equations equivalent? Give examples.

Chapter 10.4 Solving Equations Algebraically

Summary

- Solving equations algebraically involves using mathematical operations to isolate the variable.
- The goal is to perform inverse operations to 'undo' what's being done to the variable.
- Each step must maintain balance by performing the same operation on both sides.
- Algebraic solving is essential for handling more complex equations in later years.

Examples

- $x + 7 = 12$ → Subtract 7 from both sides → $x = 5$
- $2x = 14$ → Divide both sides by 2 → $x = 7$
- $x - 4 = 9$ → Add 4 to both sides → $x = 13$
- $3x - 2 = 10$ → Add 2 $\rightarrow 3x = 12$ → Divide by 3 $\rightarrow x = 4$

Word Problems

Intermediate Questions

1. Solve: $x + 5 = 13$

2. What is x in the equation $4x = 20$?

3. Solve: $x - 6 = 2$

4. Use inverse operations to solve $x \div 3 = 5$

5. What does 'algebraically' mean in solving equations?

6. Solve: $2x + 1 = 9$

7. Find x: $\frac{x}{2} - 4 = 0$

8. What is the first step to solve $x - 3 = 8$?

9. Solve: $x - 9 = -3$

10. Find x in $5x + 2 = 17$

Hard Questions

1. Solve: $3x - 4 = 11$ step-by-step

2. Explain how inverse operations work in algebraic solving

3. If $2x + 5 = 15$, find x

4. What is the solution to $4x - 3 = 5$?

5. Solve and check: $7x = 42$

6. Solve: $(x \div 2) + 6 = 10$

7. If $5x - 2 = 3x + 6$, find x

8. Explain how to isolate x in multi-step equations

9. Solve: $2(x - 1) = 10$

10. Why is algebraic solving more reliable for complex equations?

Practice Questions

1. Solve the following equations algebraically.

a. $6m = 54$

b. $g - 9 = 2$

c. $s \times 9 = 81$

d. $i - 9 = 1$

e. $7 + t = 9$

f. $8 + R = 11$

g. $4y = 48$

h. $7 + s = 19$

i. $7 + s = 19$

j. $12 = l + 8$

k. $1 = v \div 2$

l. $19 = 7 + y$

m. $k \div 5 = 1$

n. $2 = y - 7$

o. $8z = 56$

p. $13 = 3 + t$

q. $b \times 10 = 120$

r. $p - 2 = 9$

s. $5 + a = 13$

t. $n - 2 = 1$

2. For each of the following equations:
   i. Solve the equation algebraically, showing your steps.
   ii. Check your solution by substituting the value into the LHS and RHS.

a. $6f - 2 = 64$

b. $\frac{k}{4} + 9 = 10$

c. $5x - 4 = 41$

d. $3(a - 8) = 3$

e. $5k - 9 = 31$

f. $\frac{a}{3} + 6 = 8$

g. $2n - 8 = 14$

h. $\frac{n}{4} + 6 = 8$

i. $1 = 2g - 7$

j. $30 = 3R - 3$

k. $3z - 4 = 26$

l. $17 = 9 + 8p$

m. $10d + 7 = 47$

n. $38 = 6t - 10$

o. $9u + 2 = 47$

p. $7 = 10c - 3$

q. $10 + 8q = 98$

r. $80 = 4(y + 8)$

s. $4(R + 8) = 40$

t. $7 + 6u = 67$

Chapter 10.5 Equations with Fractions and Brackets

Summary

- Equations with fractions require eliminating the fractions to simplify solving.
- Multiply both sides of the equation by the denominator to remove fractions.
- Brackets in equations must be expanded before solving.
- Apply the distributive law: $a(b + c) = ab + ac$ to remove brackets.
- Always use inverse operations after simplifying the equation.

Examples

- $\frac{x}{2} = 4$ → Multiply both sides by 2 → $x = 8$
- $\frac{1}{3x} = 5$ → Multiply both sides by 3 → $x = 15$
- $2(x + 3) = 12$ → Expand: $2x + 6 = 12$ → Solve: $x = 3$
- $3(x - 1) = 15$ → Expand: $3x - 3 = 15$ → $x = 6$

Word Problems

Intermediate Questions

1. Solve: $\frac{x}{4} = 3$

2. What is x in $2(x + 2) = 10$?

3. Expand: $3(x - 4)$

4. How do you eliminate fractions in an equation?

5. Solve: $\frac{1}{5x} = 6$

6. What is the distributive law?

7. Simplify and solve: $2(x + 1) = 12$

8. What is x if $\frac{x}{3} - 2 = 4$?

9. Expand and simplify: $4(x - 2) + 1 = 17$

10. What should be done first when brackets are in an equation?

Hard Questions

1. Solve: $3(x - 2) = 2x + 4$

2. Find x: $\left(\frac{x}{2}\right) + 3 = 7$

3. Explain how multiplying removes fractions in equations.

4. Solve: $\frac{2}{3x} = 10$

5. Expand and solve: $5(x + 2) = 35$

6. If $\frac{1}{4x} - 1 = 3$, what is x?

7. Explain the steps in solving equations with brackets and fractions.

8. Solve and check: $2(x - 3) = x + 4$

9. What's the value of x in $4x - 1 = 3(x + 2)$?

10. Why is it important to simplify before solving equations?

Practice Questions

1. Solve the following equations algebraically.

a. $\frac{m}{6} = 2$

b. $\frac{c}{9} = 2$

c. $\frac{s}{8} = 2$

d. $\frac{r}{5} = 2$

e. $\frac{3u}{5} = 12$

f. $\frac{2y}{9} = 4$

g. $\frac{5x}{2} = 10$

h. $\frac{3a}{8} = 6$

i. $\frac{4h}{5} = 8$

j. $\frac{3j}{5} = 9$

k. $\frac{5v}{9} = 5$

l. $\frac{3q}{4} = 6$

2. Solve the following equations algebraically. Check your solutions using substitution.

a. $\frac{y+5}{11} = 1$

b. $14 + \frac{4t}{9} = 18$

c. $\frac{s+2}{5} = 1$

d. $\frac{f-2}{7} = 1$

e. $\frac{7m+7}{4} = 21$

f. $\frac{b-2}{2} = 1$

g. $1 = \frac{4r-13}{3}$

h. $12 = \frac{2z}{7} + 10$

i. $3 = \frac{7+4d}{9}$

j. $\frac{4a-6}{5} = 6$

3. Expand each of the following.

a. $2(x + 1)$

b. $5(2b + 3)$

c. $2(3a - 4)$

d. $5(7a + 1)$

e. $4(3x + 4)$

f. $3(8 - 3y)$

g. $12(4a + 3)$

h. $2(V - 4)$

4. Solve these equations by expanding the brackets first.

a. $6(3 + 2d) = 54$

b. $8(7x - 7) = 56$

c. $3(2x - 4) = 18$

d. $27 = 3(3 + 6e)$

e. $44 = 4(3a + 8)$

f. $30 = 6(5r - 10)$

g. $10 = 5(9V - 7)$

h. $3(2R - 9) = 39$

5. Solve the following equations by first expanding the brackets. You will need to simplify the expanded expressions by collecting like terms.

a. $5i + 5(2 + 2i) = 25$

b. $3(4c - 5) + c = 50$

c. $5(4k + 2) + k = 31$

d. $4R + 6(4R - 4) = 60$

e. $44 = 4f + 4(2f + 2)$

f. $40 = 5t + 6(4t - 3)$

Chapter 10.6 Formulas and Applications

Summary

- A formula is a mathematical rule or relationship written using symbols and variables.
- Formulas are used to solve problems in real-life contexts such as geometry, finance, and science.
- To use a formula, substitute known values into the variables and solve for the unknown.
- It is important to rearrange formulas when solving for different variables.
- Always follow the order of operations when substituting values into a formula.

Examples

- Area of a rectangle: $A = l \times w \rightarrow If\ l = 8, w = 3 \rightarrow A = 24$
- Perimeter of a square: $P = 4s$ → If $s = 5 \rightarrow P = 20$
- Speed formula: $\text{Speed} = \text{Distance} \div \text{Time}$ → If $D = 60\text{ km}, T = 2\,h \rightarrow \text{Speed} = 30\text{ km/h}$
- Volume of a cube: $V = s^3$ → If $s = 2 \rightarrow V = 8$

Word Problems

Intermediate Questions

1. What is a formula?

2. Substitute $l = 5$ and $w = 2$ into $A = l \times w$

3. Find the perimeter using $P = 2(l + w)$, if $l = 6$ and $w = 3$

4. Use $V = s^3$ to find the volume if $s = 4$

5. Calculate area: $A = l \times w$, with $l = 7$ and $w = 2$

6. Solve: $D = s \times t$, with $s = 10$ and $t = 3$

7. Use $P = 4s$ to find the perimeter when $s = 9$

8. What does it mean to substitute in a formula?

9. Evaluate $A = \pi r^2$, with $r = 2$ (use $\pi \approx 3.14$)

10. Use a formula to calculate the total cost: $T = p \times q$, if $p = \$5$ and $q = 8$

Hard Questions

1. Rearrange the formula $D = s \times t$ to find t

2. Solve: $A = \frac{1}{2} \times b \times h$, with $b = 10$ and $h = 5$

3. Explain how formulas are used in everyday life

4. If $V = l \times w \times h$, find V with $l = 3, w = 4, h = 2$

5. Use $I = PRT$ (simple interest), with $P = \$1000, R = 0.05, T = 2$

6. Rearrange the speed formula to find Distance

7. Why is it important to substitute correctly in formulas?

8. Solve: $C = 2\pi r$, with $r = 3$ (use $\pi \approx 3.14$)

9. Create your own formula for calculating total price with tax

10. Explain how to solve a formula when two variables are unknown

Practice Questions

1. Launz buys a car and a trailer for a combined cost of 40 000. The trailer costs 2000.

   a. Define a pronumeral for the car's cost.

   b. Write an equation to describe the problem.

   c. Solve the equation algebraically.

   d. Hence, state the cost of the car.

2. Jonas is paid 17 per hour and gets paid a bonus of 65 each week. One particular week he earned 643.

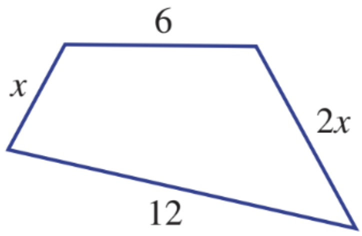
   a. Define a pronumeral for the number of hours Jonas worked.

   b. Write an equation to describe the problem.

   c. Solve the equation algebraically.

   d. How many hours did Jonas work in that week?

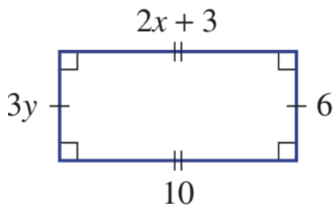
3. The perimeter of the shape shown is 30. Find the value of x.



4. Alexa watches some television on Monday, then twice as many hours on Tuesday, then twice as many hours again on Wednesday. If she watches a total of $10\frac{1}{2}$ hours from Monday to Wednesday, how much television did Alexa watch on Monday?

5. A rectangle has width w and height I. The perimeter and area of the rectangle are equal. Write an equation and solve it by inspection to find some possible values for w and I. (Note: There are many solutions to this equation. Try to find a few.)

6. Find the values of x and y in the rectangle shown.



CHAPTER 11 MEASUREMENT

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Chapter 11.1 Measurement Systems

Summary

- Measurement systems are used to quantify length, mass, capacity, time, and temperature.
- The two most commonly used systems are the metric system and the imperial system.
- The metric system is based on powers of ten and is widely used around the world.
- Metric units of length include millimetres (mm), centimetres (cm), metres (m), and kilometres (km).
- Consistency in units is crucial when calculating or comparing measurements.

Examples

- $1 \text{ kilometre } = 1000 \text{ metres}$
- $1 \text{ metre } = 100 \text{ centimetres } = 1000 \text{ millimetres}$
- To convert 3.5 km to metres: $3.5 \times 1000 = 3500 \text{ m}$
- To convert 750 mm to metres: $750 \div 1000 = 0.75 \text{ m}$

Word Problems

Intermediate Questions

1. What are the two main systems of measurement?

2. List three metric units of length.

3. Convert 1.2 kilometres to metres.

4. How many centimetres are in 3.4 metres?

5. What is the base unit of length in the metric system?

6. Convert 5600 mm to metres.

7. Convert 85 cm to millimetres.

8. Which is longer: 2500 mm or 2.3 m?

9. What are metric units based on?

10. Convert 0.07 km to metres.

Hard Questions

1. Convert 6.25 m to both cm and mm.

2. Explain how the metric system is structured.

3. What is the advantage of using the metric system over imperial units?

4. How many kilometres is 3400 m?

5. Convert 0.5 m to mm and cm.

6. If a race is 4.2 km long, how many metres is that?

7. Explain why unit conversion is important in real-world measurement tasks.

8. Compare 3.8 m and 3890 mm.

9. A piece of rope is 0.9 km long. How many cm is that?

10. How many millimetres are in 7.25 m?

Chapter 11.2 Using and Converting Metric Lengths

Summary

- Metric length units include millimetres (mm), centimetres (cm), metres (m), and kilometres (km).
- To convert between these units, multiply or divide by powers of 10:
- - 10 mm = 1 cm, 100 cm = 1 m, 1000 m = 1 km.
- Use multiplication to convert to a smaller unit and division to convert to a larger unit.
- Metric conversions are commonly used in measuring distances, heights, and lengths in everyday situations.

Examples

- Convert 3.5 m to cm: 3.5 × 100 = 350 cm
- Convert 400 cm to metres: 400 ÷ 100 = 4 m
- Convert 2.75 km to metres: 2.75 × 1000 = 2750 m
- Convert 1500 mm to m: 1500 ÷ 1000 = 1.5 m

Word Problems

Intermediate Questions

1. Convert 2.3 m to centimetres.
2. How many millimetres are in 8.5 cm?
3. Convert 1200 mm to metres.
4. What is 0.6 km in metres?
5. Convert 450 cm to m.
6. How many centimetres are in 3.2 m?
7. Convert 0.9 m to millimetres.
8. What is 7000 m in km?
9. Convert 62 cm to mm.
10. How many metres are there in 3200 mm?

Hard Questions

1. Explain how to convert 0.045 km to mm.
2. A field is 2.4 km long. How many mm is that?
3. Convert 1870 mm to km.
4. Why is the metric system easier to convert than the imperial system?
5. What is 0.008 km in cm?
6. A marathon is 42.195 km. Express it in metres.
7. Compare 0.07 km and 68 m. Which is greater?
8. If a car travels 300 m in 20 seconds, how far is that in km?
9. Write a real-life situation requiring metric conversions.
10. Convert 9870 mm to m and km.

Practice Questions

1. Convert these measurements to the units shown in brackets.

a. 5 cm (mm)

b. 2 m (cm)

c. 3.5 km (m)

d. 26.1 m (cm)

e. 40 mm (cm)

f. 500 cm (m)

g. 4200 m (km)

h. 472 mm (cm)

i. 6.84 m (cm)

j. 0.02 km (m)

k. 9261 mm (cm)

l. 9261 mm (cm)

2. Convert to the units shown in the brackets.

a. 3 m (mm)

b. 6 km (cm)

c. 2.4 m (mm)

d. 0.04 km (cm)

e. 47 000 cm (km)

f. 913 000 mm (m)

g. 216 000 mm (km)

h. 0.5 mm (m)

Chapter 11.3 Perimeter

Summary

- Perimeter is the total distance around the edge of a 2D shape.
- To find the perimeter, add the lengths of all sides.
- Common units include $\mathrm{mm}, \mathrm{cm}, \mathrm{m}$, and km, depending on the size of the shape.
- Formulas exist for regular shapes:
  - Square: $P = 4 \times side$
  - Rectangle: $P = 2 \times (length + width)$
  - Triangle: $P = a + b + c$

Examples

- Perimeter of square with side $6 \mathrm{~cm}$: $P = 4 \times 6 = 24 \mathrm{~cm}$
- Rectangle with length $5 \mathrm{~m}$, width $3 \mathrm{~m}$: $P = 2 \times (5 + 3) = 16 \mathrm{~m}$
- Triangle with sides $4 \mathrm{~cm}, 5 \mathrm{~cm}, 6 \mathrm{~cm}$: $P = 4 + 5 + 6 = 15 \mathrm{~cm}$
- Perimeter of a hexagon with each side $2 \mathrm{~cm}$: $P = 6 \times 2 = 12 \mathrm{~cm}$

Word Problems

Intermediate Questions

1. Define perimeter.

2. Find the perimeter of a square with side 9 cm.

3. Calculate the perimeter of a rectangle 8 m long and 3 m wide.

4. Find the perimeter of a triangle with sides $5 \text{ cm}, 7 \text{ cm}, 6 \text{ cm}$.

5. What is the formula for the perimeter of a square?

6. How many sides does a regular hexagon have?

7. A rectangle has $l = 12\text{ m}, w = 5\text{ m}$. Find its perimeter.

8. What are common units used for perimeter?

9. Calculate the perimeter of a pentagon with side 4 cm.

10. What is the perimeter of a $10\text{ m} \times 4\text{ m}$ rectangle?

Hard Questions

1. Explain how to calculate the perimeter of an irregular shape.

2. A park has a rectangular fence $150\ \mathrm{m}$ long and $90\ \mathrm{m}$ wide. Find the perimeter.

3. Design a garden $7\ \mathrm{m} \times 5\ \mathrm{m}$ and calculate fencing needed.

4. Convert and find: perimeter of square with side $0.4\ \mathrm{m}$ in mm.

5. Compare perimeter of square (side $8\ \mathrm{cm}$) and rectangle ($4\ \mathrm{cm} \times 12\ \mathrm{cm}$).

6. Find the missing side if perimeter is 20 cm and two sides are 5 cm and 7 cm.

7. Perimeter of a triangle with base 6 m and equal sides: 4 m. Find total.

8. If a shape has perimeter 30 cm and 5 equal sides, what is the length of one side?

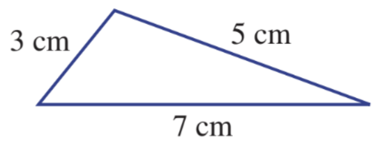
9. Explain why perimeter is important in building and design.

10. What is the perimeter of a square field with area 49 m^2?

Practice Questions

1. Find the perimeter of these shapes. (Diagrams are not drawn to scale.)

a.



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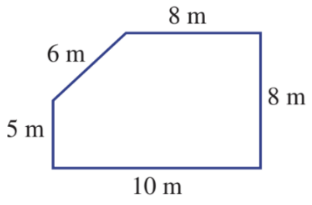

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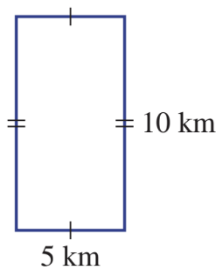

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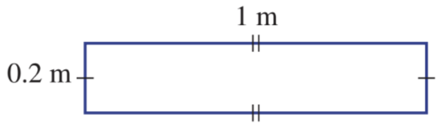

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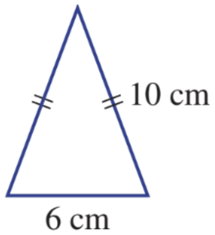

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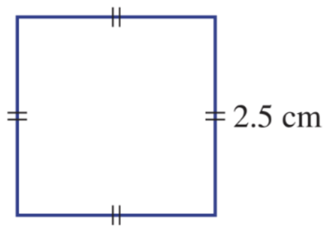
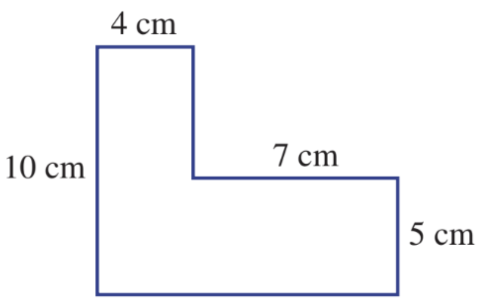


2. Find the perimeter of these shapes. All corner angles are 90°.

a.



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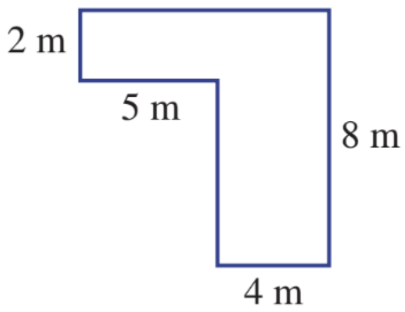
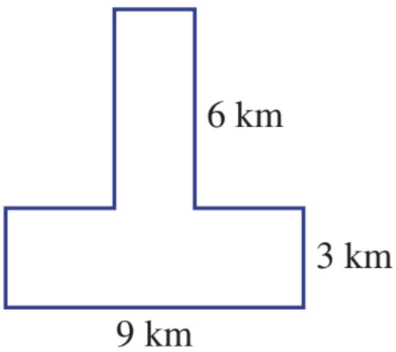
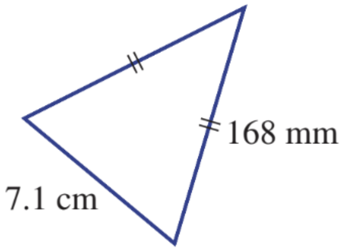


c.

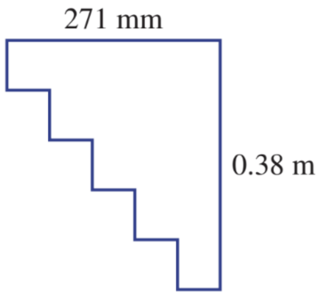


3. Find the perimeter of each of these shapes. Give your answers in centimetres.

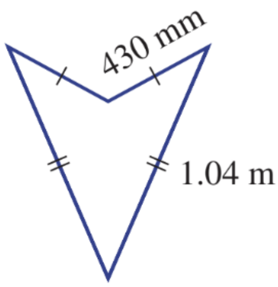
a.



b.

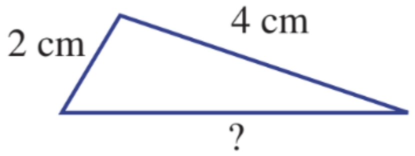


c.



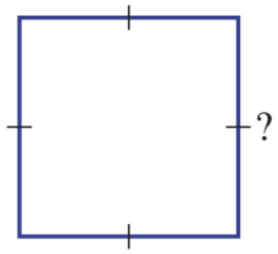
4. The perimeter of each shape is given. Find the missing length of each.

a.



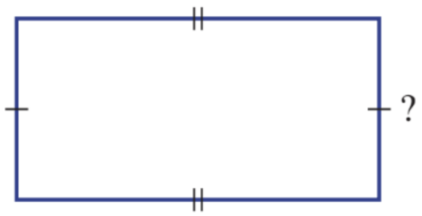
$P = 11$ cm

b.



$P = 20$ m

c.



$P = 38$ km

Chapter 11.4 Areas and Rectangles

Summary

- Area is the amount of space inside a 2D shape, measured in square units (e.g., $\mathrm{cm}^2, \mathrm{m}^2$).
- The formula for the area of a rectangle is: $A = length \times width$ ($A = l \times w$).
- Area is useful for measuring surfaces like floors, walls, or gardens.
- Ensure that length and width are in the same unit before calculating.

Examples

- Area of rectangle with $l = 6\ \mathrm{cm}, w = 4\ \mathrm{cm}$: $A = 6 \times 4 = 24\ \mathrm{cm}^2$
- $A = l \times w$ where $l = 8\ \mathrm{m}, w = 2\ \mathrm{m} \rightarrow A = 16\ \mathrm{m}^2$
- Convert $300\ \mathrm{cm} \times 2\ \mathrm{m}$ to metres before multiplying → $3\ \mathrm{m} \times 2\ \mathrm{m} = 6\ \mathrm{m}^2$
- $A = 5.5\ \mathrm{m} \times 3\ \mathrm{m} = 16.5\ \mathrm{m}^2$

Word Problems

Intermediate Questions

1. What is the formula for the area of a rectangle?

2. Find the area of a $5\ \mathrm{m}$ by $4\ \mathrm{m}$ rectangle.

3. Calculate the area: $l = 12\ \mathrm{cm}, w = 7\ \mathrm{cm}$.

4. What units are used to measure area?

5. If $l = 3\ \mathrm{m}$ and $w = 2.5\ \mathrm{m}$, find the area.

6. Find the area of a rectangle: $l = 10\ \text{cm}, w = 3\ \text{cm}$.

7. What is the area of a $2\ \text{m} \times 1.5\ \text{m}$ rectangle?

8. If width is $8\ \text{cm}$ and area is $64\ \text{cm}^2$, find the length.

9. Is the area of a square found the same way as a rectangle?

10. How do you ensure the units are consistent before finding area?

Hard Questions

1. Find the missing dimension: $A = 36 \text{ cm}^2, w = 6 \text{ cm}$.

2. A rug is 2.2 m long and 1.5 m wide. Find the area.

3. Explain why consistent units are important when calculating area.

4. Convert and calculate: $l = 120 \text{ cm}, w = 0.8 \text{ m}$.

5. Design a rectangular garden and calculate its area.

6. Compare area of 2 rectangles: $(6\ \mathrm{m}\ \times\ 3\ \mathrm{m})$ vs. $(4\ \mathrm{m}\ \times\ 5\ \mathrm{m})$.

7. How many $1\ \mathrm{m}^2$ tiles are needed to cover a 5 m × 6 m floor?

8. A wall is $4.5\ \mathrm{m}$ wide and $2.4\ \mathrm{m}$ tall. What is its area?

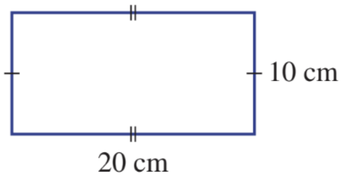
9. Find the area: $l\ =\ 3.2\ \mathrm{m}, w\ =\ 2.25\ \mathrm{m}$.

10. If area = $72\ \mathrm{cm}^2$ and length = $9\ \mathrm{cm}$, what is the width?

Practice Questions

1. Find the area of these rectangles and squares. Diagrams are not drawn to scale.

a.



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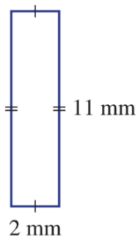

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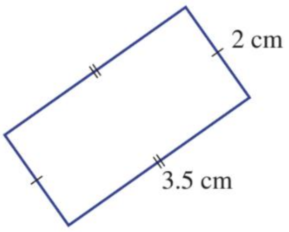

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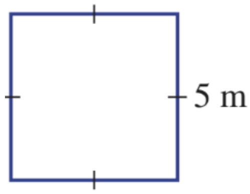

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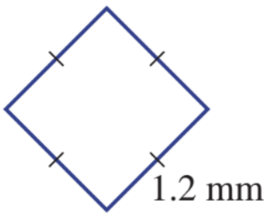

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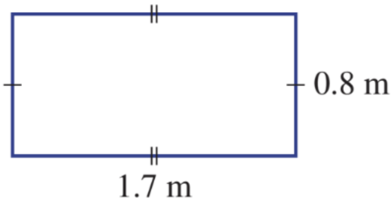

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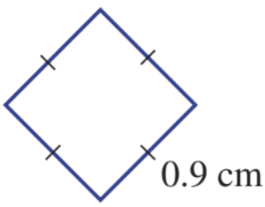

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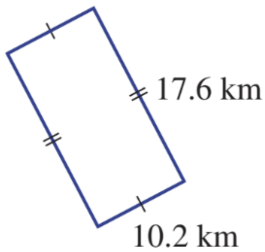


Chapter 11.5 Area of a Triangle

Summary

- The area of a triangle is the amount of space inside the triangle's boundaries.
- The formula is: $A = \frac{1}{2} \times base \times height$.
- The base is any one side of the triangle; the height is the perpendicular distance from that base to the opposite vertex.
- Units for area are square units (e.g., $\mathrm{cm}^2, \mathrm{m}^2$).
- Ensure base and height are in the same unit before calculating.

Examples

- base = $8\ \mathrm{cm}$, height = $5\ \mathrm{cm}$ $\rightarrow A = 1/2 \times 8 \times 5 = 20\ \mathrm{cm}^2$
- $b = 10\ \mathrm{m}, h = 3\ \mathrm{m} \rightarrow A = 1/2 \times 10 \times 3 = 15\ \mathrm{m}^2$
- $b = 4.2\ \mathrm{m}, h = 2\ \mathrm{m} \rightarrow A = 1/2 \times 4.2 \times 2 = 4.2\ \mathrm{m}^2$
- Triangle with $b = 6\ \mathrm{cm}$ and $h = 7\ \mathrm{cm}$ $\rightarrow A = 1/2 \times 6 \times 7 = 21\ \mathrm{cm}^2$

Word Problems

Intermediate Questions

1. What is the formula for the area of a triangle?

2. Find the area: base $= 6 \text{ cm}$, height $= 5 \text{ cm}$.

3. If $b = 10 \text{ m}$ and $h = 2 \text{ m}$, what is the area?

4. What is the unit of area?

5. Why is the area of a triangle half the base × height?

6. Convert base = $0.4\ \mathrm{m}$, height = $60\ \mathrm{cm}$, then find area.

7. Find the area: base = $9.5\ \mathrm{cm}$, height = $4\ \mathrm{cm}$.

8. If area = $24\ \mathrm{m}^2$ and base = $8\ \mathrm{m}$, find height.

9. What is the area of a triangle with $b = 7\ \mathrm{cm}, h = 3\ \mathrm{cm}$?

10. How do you determine the height in a triangle?

Hard Questions

1. A triangle has $b = 12\ \mathrm{m}$ and $h = 4.5\ \mathrm{m}$. Find area.

2. Explain why the triangle area formula works using a rectangle.

3. Compare area of two triangles: ($b = 10\ \mathrm{cm}, h = 6\ \mathrm{cm}$) and ($b = 12\ \mathrm{cm}, h = 5\ \mathrm{cm}$).

4. If $A = 30\ \mathrm{cm}^2$ and $b = 10\ \mathrm{cm}$, find h.

5. Find the base: $A = 42\ \mathrm{m}^2, h = 6\ \mathrm{m}$.

6. Convert all to cm: $b = 2.4 \text{ m}, h = 80 \text{ cm}$, then find A.

7. Design a triangle with area = 18 m^2 and $b = 6 \text{ m}$.

8. Draw a triangle and label base and height.

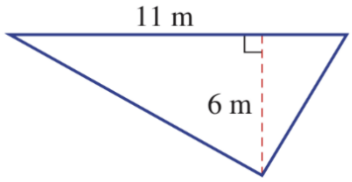
9. Explain what happens if base and height aren't perpendicular.

10. Find A: triangle with $b = 7.2 \text{ m}$ and $h = 2.5 \text{ m}$.

Practice Questions

1. For each of these triangles, what length would be used as the height?

a.



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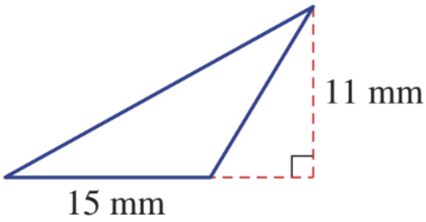

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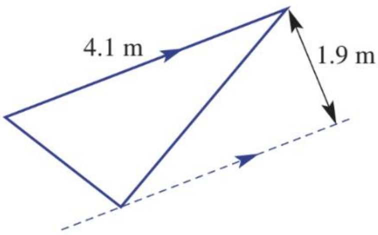

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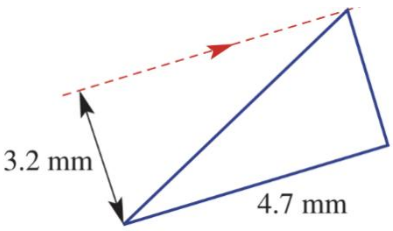
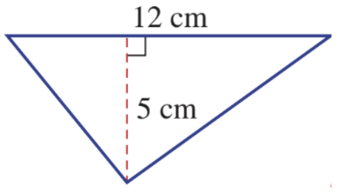


2. Find the area of each triangle given.

a.



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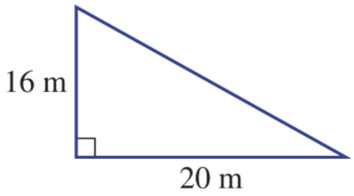

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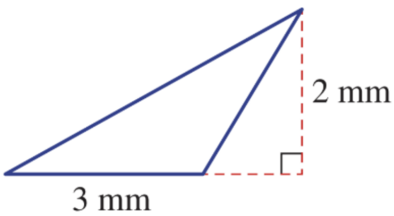

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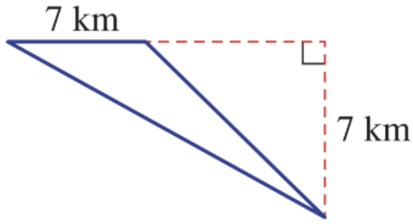

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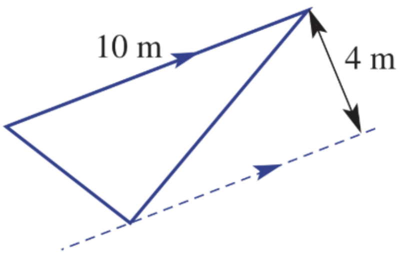

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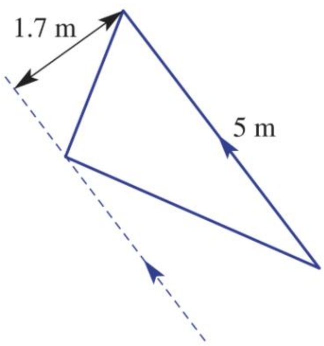


Chapter 11.6 Area of a Parallelogram

Summary

- A parallelogram is a 2D shape with opposite sides that are equal and parallel.
- The formula for the area of a parallelogram is: $A = base \times height$.
- The base is any side, and the height is the perpendicular distance to the opposite side.
- Area is measured in square units like cm^2 or m^2.
- Unlike a rectangle, the sides can be slanted, but the height is always vertical.

Examples

- base $= 8$ cm, height $= 5$ cm $\rightarrow$ A $= 8 \times 5 = 40\ \mathrm{cm}^2$
- $b = 10$ m, $h = 4$ m $\rightarrow A = 10 \times 4 = 40\ \mathrm{m}^2$
- $b = 3.5$ m, $h = 2.5$ m $\rightarrow A = 8.75\ \mathrm{m}^2$
- $b = 9$ cm, $h = 6$ cm $\rightarrow A = 9 \times 6 = 54\ \mathrm{cm}^2$

Word Problems

Intermediate Questions

1. What is the formula for the area of a parallelogram?

2. Find the area: base $= 6$ cm, height $= 4$ cm.

3. What's the area if base $= 11$ m and height $= 3$ m?

4. Explain how a parallelogram differs from a rectangle.

5. Calculate area: $b = 10$ cm, $h = 5$ cm.

6. If base $= 2.4 \text{ m}$ and height $= 1.5 \text{ m}$, what is the area?

7. What units are used to measure area?

8. Find area: base $= 7.5 \text{ m}$, height $= 2 \text{ m}$.

9. How do you identify the base and height in a parallelogram?

10. Convert base $= 120 \text{ cm}$, height $= 0.8 \text{ m}$ and find area.

Hard Questions

1. Compare triangle and parallelogram area formulas.

2. Explain why slanted sides don't affect the area.

3. If $A = 72\ \mathrm{cm}^2$ and base $= 9\ \mathrm{cm}$, find height.

4. A garden is shaped like a parallelogram: $b = 5.2\ \mathrm{m}, h = 3\ \mathrm{m}$. Find the area.

5. Convert: $b = 2\ \mathrm{m}, h = 150\ \mathrm{cm}$ → Find area in m^2.

6. If base is unknown, but $A = 24\ \mathrm{m}^2$ and $h = 6\ \mathrm{m}$, find base.

7. Design a parallelogram with area $= 60\ \mathrm{cm}^2$ using any b and h.

8. Explain how the formula relates to a rectangle.

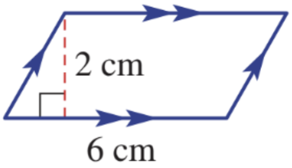
9. Calculate area when $b = 9.4\ \mathrm{m}, h = 3.1\ \mathrm{m}$.

10. Why is perpendicular height important when finding area?

Practice Questions

1. For each of these parallelograms, state the side length of the base and the height that might be used to find the area.

a.



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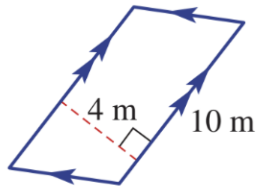

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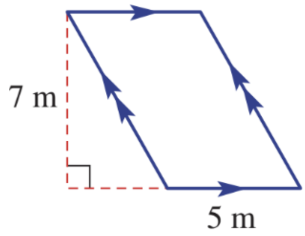

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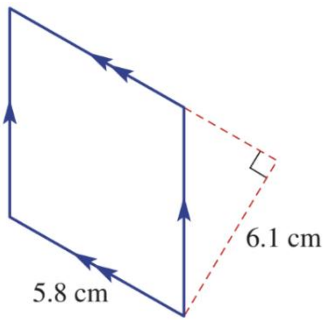

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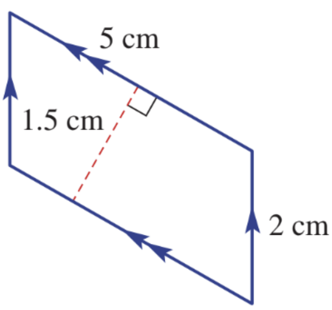

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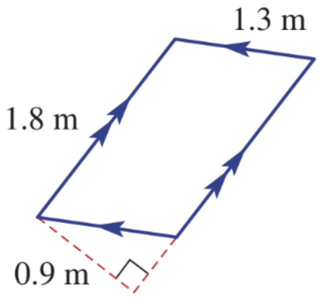
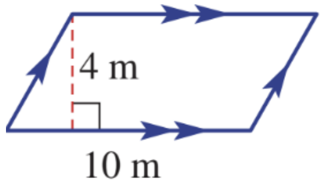


2. Find the area of these parallelograms.

a.



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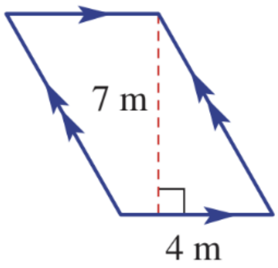

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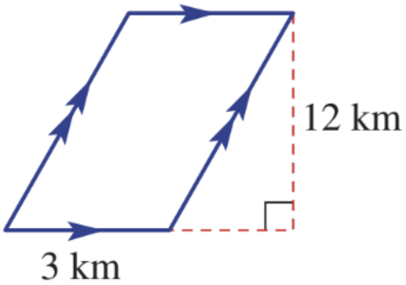

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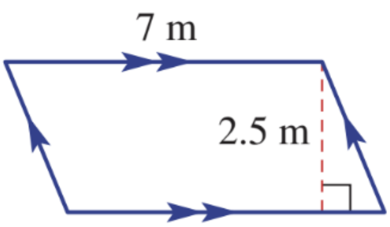

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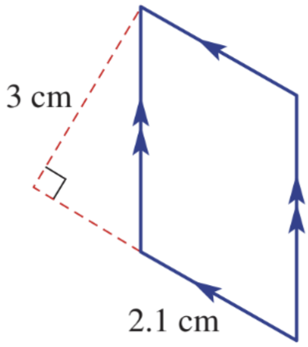

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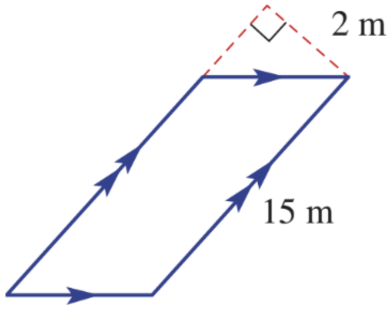

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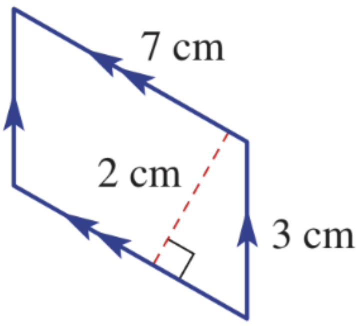

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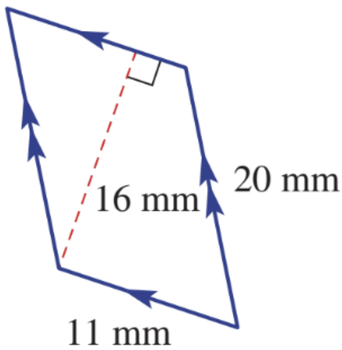


Chapter 11.7 Area of Composite Shapes

Summary

- Composite shapes are made by combining basic shapes like rectangles, triangles, and circles.
- To find the area of a composite shape, break it into simpler shapes and calculate each area separately.
- Then, add or subtract the areas depending on whether parts are joined or removed.
- This method is useful in real-life applications like designing floor plans or parks.
- Always ensure all measurements are in the same units before performing calculations.

Examples

- A shape consists of a rectangle and a triangle. Area = Area of rectangle + Area of triangle.
- Shape: rectangle ($l = 8\ \mathrm{m}, w = 3\ \mathrm{m}$) and triangle ($b = 4\ m, h = 2\ m$) $\rightarrow A = 24 + 4 = 28\ \mathrm{m}^2$.
- L-shaped figure: divide into 2 rectangles, find area of each, then add them.
- If a circular segment is cut from a square, subtract the circle's area from the square's area.

Word Problems

Intermediate Questions

1. What is a composite shape?

2. How do you find the area of a composite shape?

3. Break and solve: A shape made of two rectangles ($6\text{ m} \times 4\text{ m}$ and $3\text{ m} \times 2\text{ m}$).

4. Find the area: rectangle ($5\text{ m} \times 3\text{ m}$) + triangle ($b = 4\text{ m}, h = 2\text{ m}$).

5. Why do we break shapes into simpler parts?

6. Find area: two rectangles, $4 \text{ cm} \times 6 \text{ cm}$ and $3 \text{ cm} \times 2 \text{ cm}$.

7. If a triangle sits atop a rectangle, how is area calculated?

8. What units are used for area in composite shapes?

9. Explain how subtraction can be used in composite shapes.

10. Draw and label a composite shape using 2 rectangles.

Hard Questions

1. Design a composite shape using a rectangle and triangle. Find the total area.

2. Find the area of an L-shape: $10\text{ m} \times 4\text{ m}$ and $4\text{ m} \times 3\text{ m}$.

3. Explain why correct unit conversion is crucial before finding area.

4. Composite shape: square ($5\text{ m} \times 5\text{ m}$) with semicircle removed ($r = 2\text{ m}$). Find area.

5. Break and solve: rectangle ($6\text{ m} \times 5\text{ m}$) with triangle ($b = 4\text{ m}, h = 3\text{ m}$) attached.

6. Find area of a playground shaped like two joined rectangles.

7. If a triangle is removed from a rectangle, how do you find total area?

8. Calculate area: L-shape divided into 2 rectangles of $9\ \mathrm{m}^2$ and $15\ \mathrm{m}^2$.

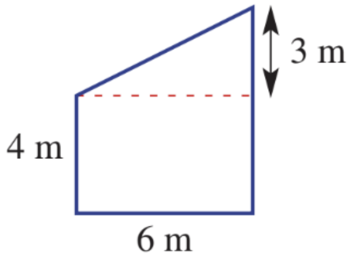
9. Draw a figure composed of three shapes and explain area calculation.

10. How does decomposition help in real-world problems like architecture?

Practice Questions

1. Find the area of these composite shapes by adding together the area of simpler shapes.

a.



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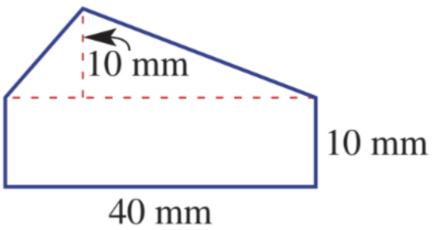

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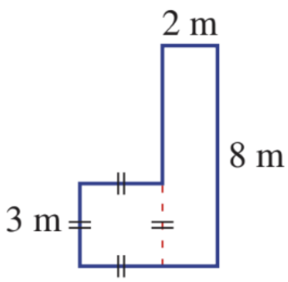

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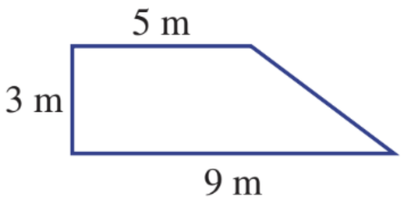

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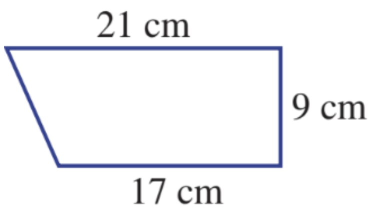

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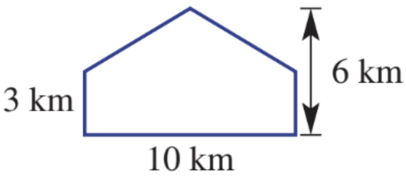
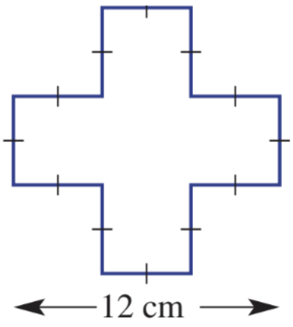


2. By finding the missing lengths first, calculate the area of these composite shapes.

a.



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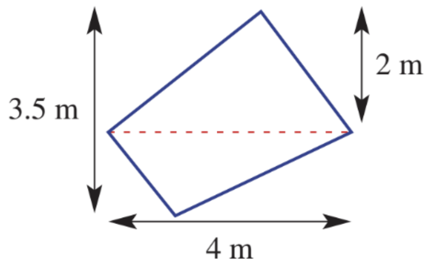

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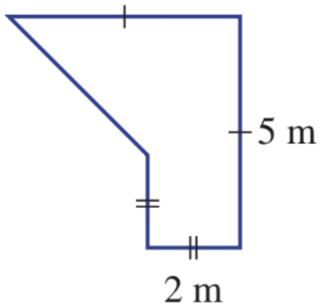

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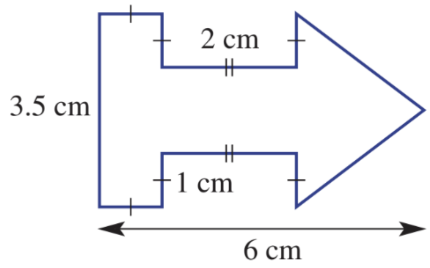


Chapter 11.8 Volume of Rectangular Prisms

Summary

- Volume is the amount of space an object occupies, measured in cubic units (e.g., cm^3, m^3).
- A rectangular prism is a 3D shape with 6 rectangular faces.
- The formula for volume is: $V = length \times width \times height$.
- All dimensions must be in the same unit before performing calculations.
- Volume is useful in real-life scenarios like packing, storage, and construction.

Examples

- $V = l \times w \times h \rightarrow l = 4\ \mathrm{m}, w = 3\ \mathrm{m}, h = 2\ \mathrm{m} \rightarrow V = 4 \times 3 \times 2 = 24\ \mathrm{m}^3$
- Box: $10\ \mathrm{cm} \times 5\ \mathrm{cm} \times 2\ \mathrm{cm} \rightarrow V = 100\ \mathrm{cm}^3$
- Fish tank: $0.6\ m \times 0.4\ m \times 0.5\ m \rightarrow \mathrm{V} = 0.12\ m^3$
- Shipping container: $2.5\ \mathrm{m} \times 1.5\ \mathrm{m} \times 2\ \mathrm{m} \rightarrow V = 7.5\ \mathrm{m}^3$

Word Problems

Intermediate Questions

1. What is the formula for the volume of a rectangular prism?

2. Find the volume: $l = 5 \text{ m}, w = 2 \text{ m}, h = 3 \text{ m}$.

3. Calculate volume: $l = 8 \text{ cm}, w = 4 \text{ cm}, h = 2 \text{ cm}$.

4. What unit is used for volume?

5. Convert and find volume: $l = 1.2 \text{ m}, w = 80 \text{ cm}, h = 0.5 \text{ m}$.

6. Explain why all measurements must be in the same unit.

7. What is the volume of a box: 10 cm × 10 cm × 10 cm?

8. If $V = 96 \text{ m}^3$, and $l = 4 \text{ m}, w = 3 \text{ m}$, find h.

9. Volume of a box: $l = 6.5 \text{ m}, w = 2 \text{ m}, h = 1.2 \text{ m}$.

10. Why is volume important in packaging?

Hard Questions

1. Design a box with volume = $120\ \mathrm{m}^3$. Give possible dimensions.

2. Explain how volume differs from area.

3. Convert dimensions: $2\ \mathrm{m}\ \times\ 120\ \mathrm{cm}\ \times\ 0.8\ \mathrm{m}$ → find *V* in m^3.

4. A pool is $10\ \mathrm{m}\ \times\ 4\ \mathrm{m}\ \times\ 1.5\ \mathrm{m}$. Find volume.

5. Compare two prisms: $3\ \mathrm{m}\ \times\ 4\ \mathrm{m}\ \times\ 2\ \mathrm{m}$ and $2\ \mathrm{m}\ \times\ 5\ \mathrm{m}\ \times\ 2\ \mathrm{m}$.

6. If $V = 180 \text{ cm}^3$ and $h = 5 \text{ cm}, w = 3 \text{ cm}$, find l.

7. Create a real-world problem involving volume.

8. Convert 25000 cm^3 to m^3.

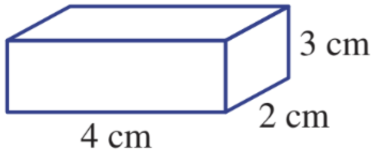
9. A tank is half-filled. Full $V = 600\ L$. How much water is inside?

10. Explain how to estimate volume when measuring isn't possible.

Practice Questions

1. Copy and complete the working shown for each of these solids.

a.



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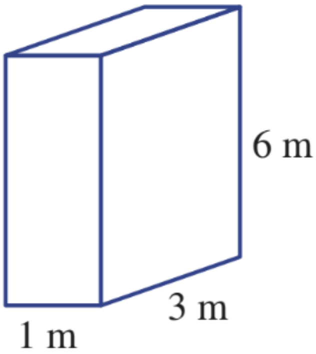

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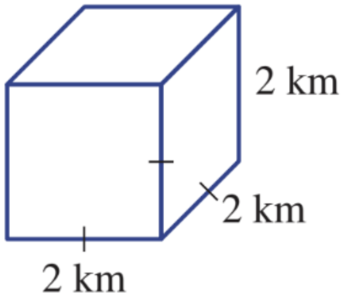
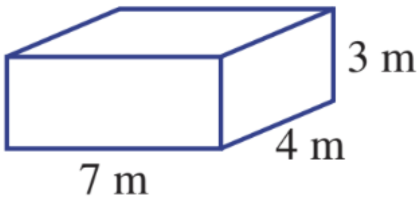


2. Find the volume of these rectangular prisms.

a.



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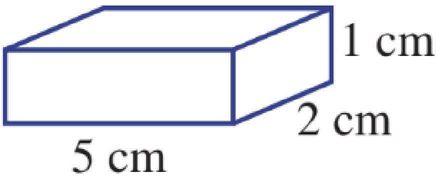

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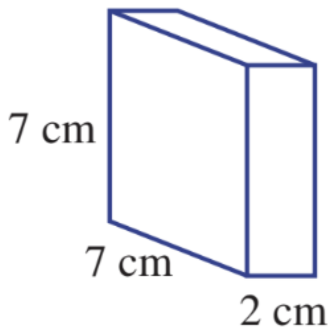

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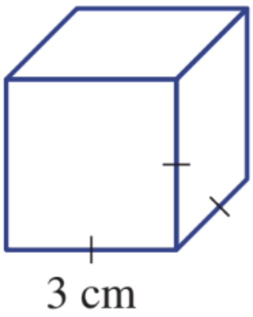

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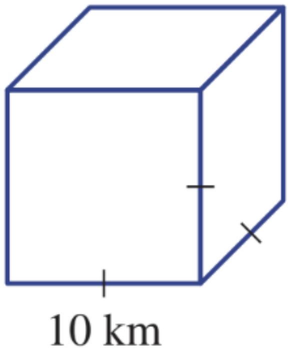

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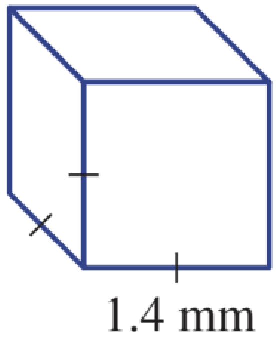

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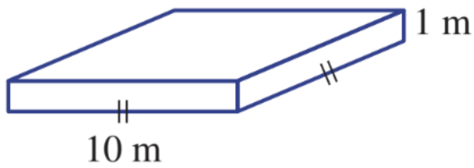

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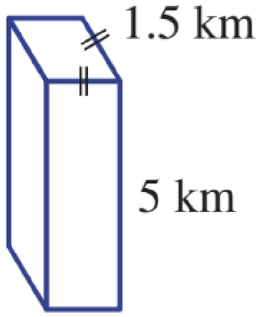


Chapter 11.9 Capacity

Summary

- Capacity refers to the amount a container can hold, typically measured in litres (L) or millilitres (mL).
- $1\text{ L} = 1000\text{ mL};\ 1000\text{ L} = 1\text{ kilolitre (kL)}$.
- Capacity is often used for measuring liquids (e.g. water, juice, petrol).
- To convert between units, use powers of 10, similar to metric length and volume.
- Volume (in cm^3 or m^3) is closely related to capacity: $1\text{ cm}^3 = 1\text{ mL}$ and $1000\text{ cm}^3 = 1\text{ L}$.

Examples

- Convert 2.5 L to mL $\rightarrow 2.5 \times 1000 = 2500\text{ mL}$
- $750\text{ mL} = 0.75\text{ L}$
- Volume of a jug: $3000\text{ cm}^3 = 3000\text{ mL} = 3\text{ L}$
- How much water in 1.5 kL tank? $\rightarrow 1.5 \times 1000 = 1500\text{ L}$

Word Problems

Intermediate Questions

1. Define capacity.

2. Convert 3.5 L to mL.

3. How many mL in 2 L?

4. What is 1500 mL in litres?

5. 1 kL equals how many litres?

6. A bottle holds $600\ \mathrm{mL}$. How many litres is that?

7. How are cm^3 and mL related?

8. If a cup holds $250\ \mathrm{mL}$, how many cups fill a $1\ \mathrm{L}$ jug?

9. Convert $4.8\ \mathrm{L}$ to mL.

10. How many litres in $2500\ \mathrm{mL}$?

Hard Questions

1. Explain how capacity and volume are related.

2. Convert: $2.3 \text{ kL } = ? \text{ mL}$

3. If $1 \text{ m}^3 = 1000 \text{ L}$, how many litres is 0.8 m^3?

4. A container holds 3.75 L. How many mL is that?

5. Convert 5000 cm^3 to litres.

6. How much water in 2 tanks of 2.4 kL each?

7. Design a container with capacity of exactly 1 L using dimensions.

8. Convert 0.09 kL to L and mL.

9. Explain why conversions are important in cooking or science.

10. A bath holds 220 L. How many mL is that?

Practice Questions

1. Convert to the units shown in brackets.

a. 2 L (mL)

b. 0.1 L (mL)

c. 6 ML (kL)

d. 24 kL (L)

e. 24 kL (L)

f. 3500 mL (L)

g. 70 000 mL (L)

h. 2500 kL (ML)

i. 0.257 L (mL)

j. 9320 mL (L)

k. 3.847 ML (kL)

l. 47 000 L (kL)

m. 0.5 kL (L)

n. 0.5 kL (L)

o. 0.42 L (mL)

p. 170 L (kL)

2. Convert to the units shown in brackets.

a. 6 ML (L)

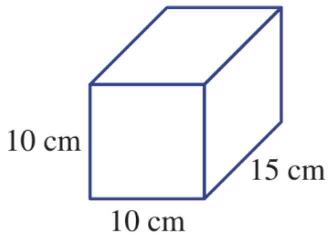
b. 320 000 L (ML)

c. 0.004 kL (mL)

d. 992 700 mL (kL)

3. Find the capacity of each of these containers, in litres.

a.



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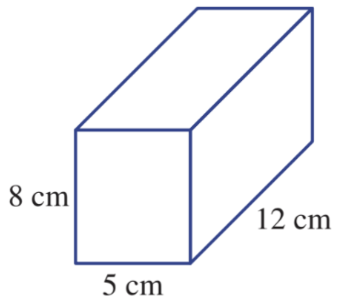

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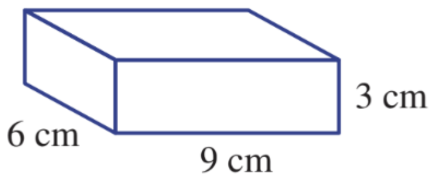

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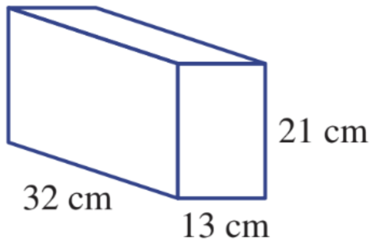

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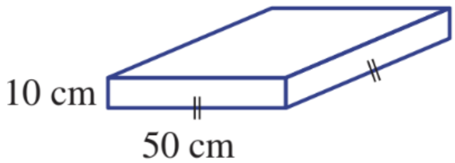

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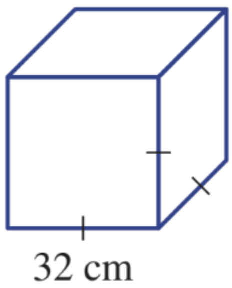


Chapter 11.10 Mass and Temperature

Summary

- Mass is the amount of matter in an object and is measured in grams (g), kilograms (kg), or tonnes (t).
- Temperature is a measure of how hot or cold something is, measured in degrees Celsius ($°C$).
- $1 \text{ kg} = 1000 \text{ g}; \ 1 \text{ tonne} = 1000 \text{ kg}.$
- Room temperature is about $20°C$; water freezes at $0°C$ and boils at $100°C$.
- Mass is measured with scales, and temperature with thermometers.

Examples

- Convert 2.5 kg to grams → $2.5 \times 1000 = 2500 \text{ g}$
- $500 \text{ g} = 0.5 \text{ kg}$
- Water at $37°C$ is body temperature.
- A car weighs 2.4 tonnes = 2400 kg
- Room is cooled from $28°C$ to $22°C$ → change = $-6°C$

Word Problems

Intermediate Questions

1. What is mass?

2. What is temperature measured in?

3. Convert 3.5 kg to grams.

4. If a bag weighs 800 g, how many kg is that?

5. Convert 2500 g to kg.

6. What is the boiling point of water in $°C$?

7. Water freezes at what temperature?

8. How much does 1.2 tonnes weigh in kg?

9. What is room temperature approximately?

10. A temperature changes from $20°C$ to $15°C$. What is the difference?

Hard Questions

1. Explain how mass and weight differ.

2. Convert 3.75 t to kg and grams.

3. If a cake is baked at $180°C$ and cools to $25°C$, find temperature change.

4. What is 4600 g in kg and tonnes?

5. A person weighs 72.5 kg. Convert to grams.

6. Explain why temperature scales matter in science.

7. Compare freezing and boiling point of water.

8. Design a recipe with ingredients needing mass conversion.

9. If a fridge is set at $5°C$ and outside is $30°C$, find temperature gap.

10. Convert 0.045 t to kg and g.

Practice Questions

1. Convert to the units shown in brackets.

a. 2 t (kg)

b. 70 kg (g)

c. 2.4 g (mg)

d. 2300 mg (g)

e. 4620 mg (g)

f. 21 600 kg (t)

g. 0.47 t (kg)

h. 312 g (kg)

i. 27 mg (g)

j. $\frac{3}{4}$ t(kg)

k. $\frac{1}{8}$ kg(g)

l. 10.5 g (kg)

m. 210 000 kg (t)

n. 0.47 t (kg)

o. 592 000 mg (g)

p. 0.08 kg (g)

2. Add all the mass measurements and give the result in kg.

   a. 3 kg, 4000 g, 0.001 t

   b. 2.7 kg, 430 g, 930 000 mg, 0.0041 t

CHAPTER 2 GEOMETRY

Table of Contents

Chapter 2.1 Points, Lines and Angles

Summary

- Points, lines, and angles are fundamental concepts in geometry.
- A point has no size, only location. It is typically represented by a dot.
- A line is a straight one-dimensional figure that extends infinitely in both directions.
- A line segment is part of a line with two endpoints.
- A ray is part of a line that has one endpoint and extends infinitely in one direction.
- Two lines that never meet are called parallel lines.
- The intersection of two lines forms an angle.
- Angles are formed by two rays that share a common endpoint, called the vertex.
- Types of angles include acute (less than $90°$), right ($90°$), obtuse (more than $90°$ but less than $180°$), and reflex (more than $180°$).
- Angle measurement is in degrees (°), and a full circle is $360°$.
- A point: A dot on a piece of paper.
- A line: A line that goes on forever in both directions.
- A ray: A line with one endpoint and extending to the right (→).
- Acute angle: $45°$ (less than $90°$).
- Right angle: $90°$ (square corner).
- Obtuse angle: $120°$ (greater than $90°$).
- Reflex angle: $270°$ (greater than $180°$ but less than $360°$).

Word Problems

Intermediate Questions

1. What is the difference between a line and a line segment?

2. Name the types of angles based on their degree measurement.

3. What is a ray?

4. How do you measure an angle?

5. Draw an acute angle.

6. What does it mean when two lines are parallel?

7. What is a right angle?

8. How many degrees are in a full circle?

9. What is the common point where two lines meet?

10. Which type of angle is greater than 90° but less than 180°?

Hard Questions

1. Explain why lines that are parallel never intersect.

2. What is the angle formed by two intersecting lines?

3. How do you calculate the size of an angle formed by two intersecting rays?

4. Find the reflex angle if the acute angle is $45°$.

5. Explain the difference between a line and a ray.

6. If two lines are perpendicular, what is the angle between them?

7. How do you classify angles based on their measurement?

8. How many degrees are in the sum of adjacent angles on a straight line?

9. Draw a diagram showing parallel and perpendicular lines.

10. What is the total angle measurement of angles around a point?

Chapter 2.2 Measuring Angles

Summary

- Angles are measured in degrees (°) using a protractor.
- A protractor is an instrument used to measure the size of an angle.
- A full circle has $360°$, and angles are measured starting from $0°$ and increasing clockwise or counterclockwise.
- To measure an angle, place the midpoint of the protractor on the angle's vertex and align one side of the angle with the baseline.
- Read the degree measurement where the second side of the angle crosses the protractor scale.
- A right angle is exactly $90°$.
- An acute angle is less than $90°$.
- An obtuse angle is greater than $90°$ but less than $180°$.
- A straight angle is $180°$.
- Using a protractor is essential for accurately measuring angles in geometric constructions.
- Measuring a right angle: Place protractor on vertex, align one side with the $0°$ mark, and read $90°$.
- Measuring an acute angle: Place protractor, align, and read $45°$.
- Measuring an obtuse angle: Place protractor, align, and read $120°$.
- Straight angle: A straight line measured as $180°$.
- Using a protractor to measure a $70°$ angle.

Word Problems

Intermediate Questions

1. What is the purpose of a protractor?

2. How do you measure a $90°$ angle?

3. What type of angle is $30°$?

4. How many degrees are in a full circle?

5. What is the size of a straight angle?

6. Measure: $45°$ angle.

7. What is the angle between the hour hand and minute hand at 3:00?

8. How do you measure an obtuse angle?

9. What type of angle is greater than $90°$ but less than $180°$?

10. How do you identify a right angle using a protractor?

Hard Questions

1. Measure the angle between two lines and explain the steps.

2. If an angle measures $125°$, is it acute, obtuse, or reflex?

3. How would you measure a reflex angle greater than $180°$?

4. Measure and label a $60°$ angle using a protractor.

5. If two angles add up to $180°$, what type of angle are they?

6. Explain how to measure a reflex angle of $270°$.

7. What is the size of an angle formed by the hands of a clock at 10:10?

8. Measure an angle of $110°$ using a protractor and show steps.

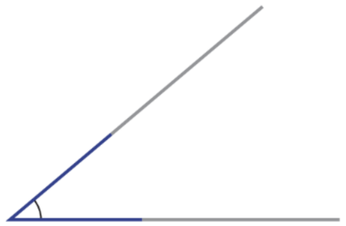
9. What are the differences in measurement between an acute angle and an obtuse angle?

10. How can you use the protractor to measure angles in a triangle?

Practice Questions

1. Give the answer to each of these sums. Check your answer with a calculator.

a.



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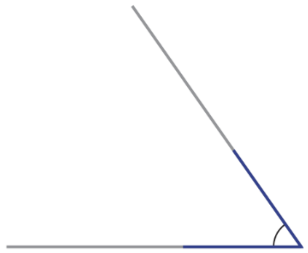

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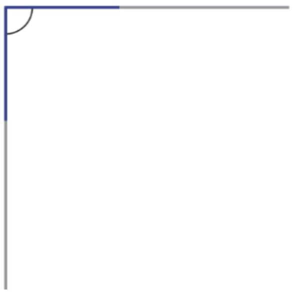

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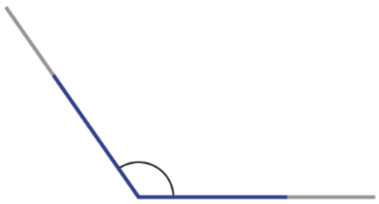

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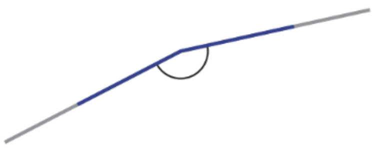

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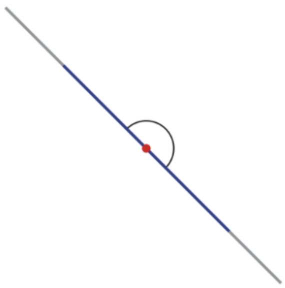

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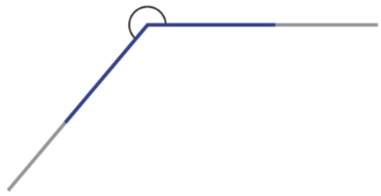

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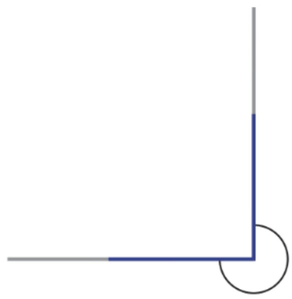

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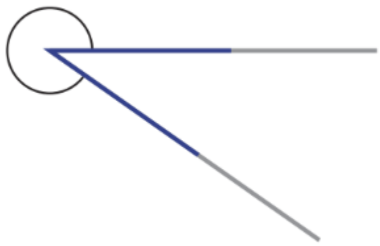


2. A clock face is numbered 1 to 12. Find the angle the minute hand turns in:

a. 30 minutes

b. 1 hour

c. 15 *minutes*

d. 45 minutes

e. 5 minutes

f. 20 minutes

g. 55 minutes

h. 1 minute

i. 9 minutes

j. 10.5 minutes

k. 42.5 minutes

l. 21.5 minutes

3. A clock face is numbered 1 to 12. Find the angle between the hour hand and the minute hand at:

a. 6:00 pm

b. 3:00 pm

c. 3:00 pm

d. 11:00 am

Chapter 2.3 Angles at a Point

Summary

- Angles at a point are formed by two rays that share a common endpoint, which is called the vertex.
- The total of all angles around a point always equals $360°$.
- If multiple angles are present around a point, their sum is $360°$.
- This property helps in geometric calculations and solving real-life problems.
- Angles around a point can be adjacent or non-adjacent.
- Adjacent angles share a common side and vertex, but non-adjacent angles do not.
- This concept is important in situations like designing and measuring angles in architectural plans and machinery.
- To find missing angles around a point, subtract the known angles from $360°$.
- In diagrams, it's common to label angles to help with solving problems.
- Example problem: If three angles around a point are $120°, 80°$, and $100°$, the fourth angle is $360° - (120° + 80° + 100°) = 60°$.
- Three angles at a point: $120° + 80° + 100° = 300°$, so the fourth angle is $360° - 300° = 60°$.
- Example: $90° + 90° + 90° + 90° = 360°$ (four right angles).
- If the angles around a point are $50°, 120°$, and $130°$, find the remaining angle: $360° - (50° + 120° + 130°) = 60°$.

Word Problems

Intermediate Questions

1. What is the total of all angles around a point?

2. If three angles are $50°$, $120°$, and $180°$, what is the fourth angle?

3. What is the sum of $100°$, $120°$, and $140°$?

4. How many degrees are in a complete circle?

5. What is the missing angle if two angles are $110°$ and $70°$?

6. If three angles at a point are $80°, 90°$, and $50°$, what is the last angle?

7. Write an equation to find the missing angle if the other angles are $30°$ and $120°$.

8. What is the total of three right angles around a point?

9. How do you calculate the missing angle if you have three known angles?

10. What is the sum of $10°, 15°$, and $20°$?

Hard Questions

1. If three angles around a point are $75°, 90°$, and $100°$, what is the fourth angle?

2. Explain why the sum of all angles around a point is $360°$.

3. If five angles around a point are $60°, 70°, 80°, 50°$, and $90°$, what is the sixth angle?

4. Solve: Three angles are $110°, 90°$, and $65°$; find the fourth angle.

5. If two angles around a point are $80°$ and $130°$, find the other two angles.

6. Calculate the missing angle if three angles are $45°, 120°$, and $110°$.

7. What is the fourth angle if the first three are $100°, 85°$, and $75°$?

8. If four angles around a point are equal, what is the size of each angle?

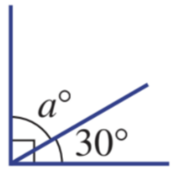
9. If three angles at a point are $95°, 105°$, and $40°$, find the missing angle.

10. How do you find the missing angle when three angles are given around a point?

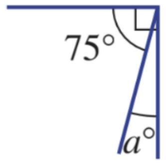
Practice Questions

1. Without using a protractor, find value of the pronumeral a. (The diagrams shown may not be drawn to scale.)

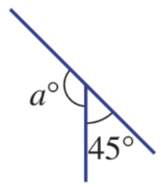
a.



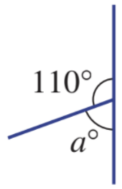
b.



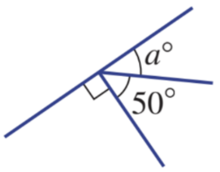
c.



d.



e.



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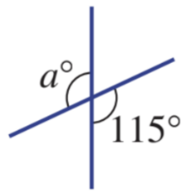

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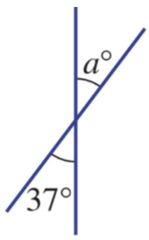

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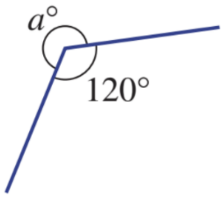

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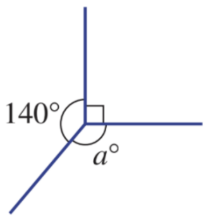
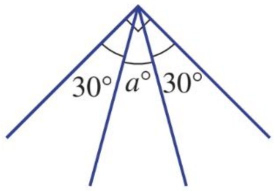
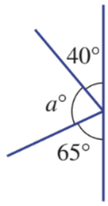


2. Without using a protractor, find the value of a in these diagrams.

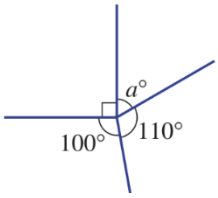
a.



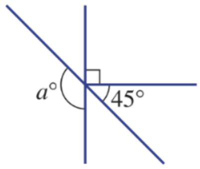
b.



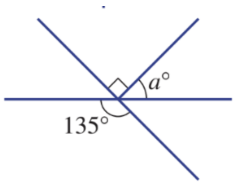
c.



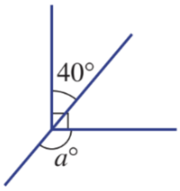
d.



e.

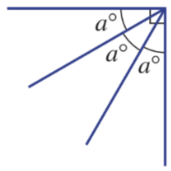


f.

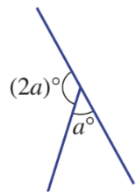


3. Find the value of a in these diagrams.

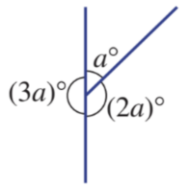
a.



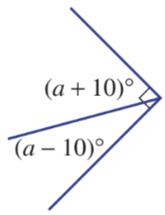
b.



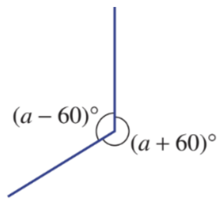
c.



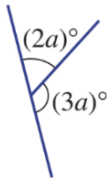
d.



e.



f.



Chapter 2.4 Transversal Lines and Parallel Lines

Summary

- A transversal line is a line that intersects two or more other lines at distinct points.
- When a transversal intersects parallel lines, several types of angles are formed.
- These include alternate interior angles, alternate exterior angles, corresponding angles, and consecutive interior angles.
- Parallel lines are lines that never meet and are equidistant from each other.
- When parallel lines are cut by a transversal, corresponding angles are congruent (equal).
- Alternate interior angles are congruent, and alternate exterior angles are also congruent.
- Consecutive interior angles are supplementary, meaning their sum is $180°$.
- Understanding these angle relationships helps solve geometry problems and prove statements about parallel lines.
- These properties are used in real-world applications like architecture, engineering, and design.
- To identify parallel lines, use markers like arrows or look for angle relationships.
- Example of parallel lines: Two train tracks.
- When a transversal cuts parallel lines, corresponding angles are equal.
- Example: If two parallel lines are intersected by a transversal, and one angle is $50°$, the corresponding angle is also $50°$.
- If alternate interior angles are $75°$, then both alternate angles will also measure $75°$.

Word Problems

Intermediate Questions

1. What is a transversal?

2. What happens when a transversal cuts parallel lines?

3. Name the types of angles formed when a transversal intersects parallel lines.

4. How can you identify parallel lines?

5. What is the sum of consecutive interior angles?

6. If one angle is $30°$ and its corresponding angle is equal, what is the second angle?

7. Explain the relationship between alternate interior angles.

8. How can parallel lines be shown in diagrams?

9. What do corresponding angles have in common?

10. How do consecutive interior angles behave?

Hard Questions

1. Explain why alternate exterior angles are congruent.

2. What is the value of the angle formed by a transversal cutting two parallel lines if one angle is $45°$?

3. What is the relationship between alternate interior and exterior angles?

4. If two angles are supplementary, what does that tell us about the lines they intersect?

5. How can you use the angle properties of parallel lines and a transversal to solve problems in design?

6. Given that two parallel lines are intersected by a transversal, and alternate exterior angles are $120°$, what are the other angles formed?

7. If one pair of consecutive interior angles is $120°$, what is the other consecutive angle?

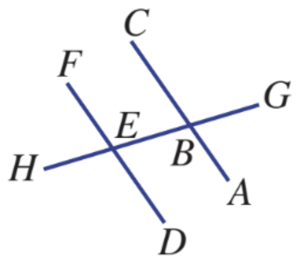
8. Explain how transversal lines help in determining if two lines are parallel.

9. If the corresponding angle is $60°$, what will the angle on the other parallel line be?

10. In a diagram, how do you prove that two lines are parallel using a transversal?

Practice Questions

1. Name the angle that is:



a. corresponding to $\angle ABE$

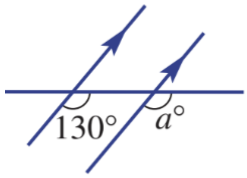
b. alternate to $\angle ABE$

c. cointerior to $\angle ABE$

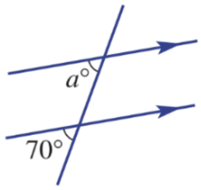
d. vertically opposite to $\angle ABE$

2. Find the value of *a* in these diagrams, giving a reason.

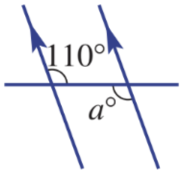
a.



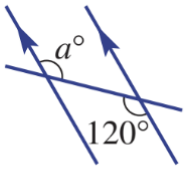
b.



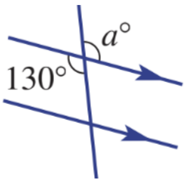
c.



d.



e.



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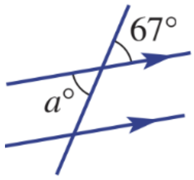
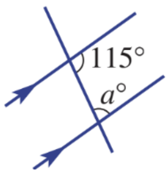
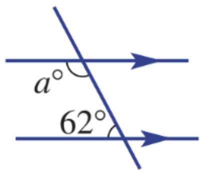


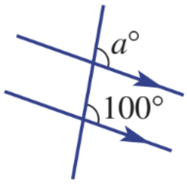
g.



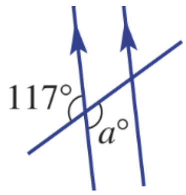
h.



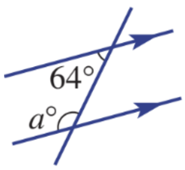
i.



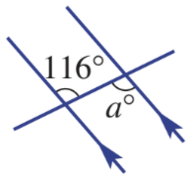
j.



k.

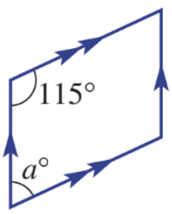


l.

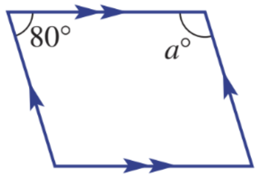


3. Find the value of a in these diagrams.

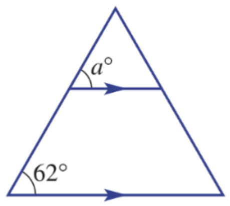
a.



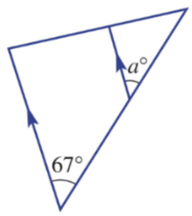
b.



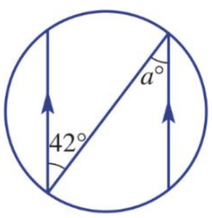
c.



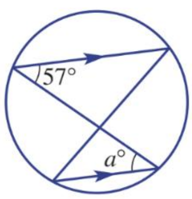
d.



e.



f.



Chapter 2.5 Problems with Parallel Lines

Summary

- Problems with parallel lines involve applying properties of parallel lines and transversals to solve for unknown angles or lengths.
- When two parallel lines are intersected by a transversal, multiple angle relationships are formed.
- Using properties like corresponding angles, alternate interior angles, and consecutive interior angles helps solve for missing values.
- Angle sum properties are key in solving problems involving parallel lines and transversals.
- For example, consecutive interior angles sum to $180°$ when two parallel lines are cut by a transversal.
- Similarly, alternate interior angles are equal, and corresponding angles are equal.
- Understanding these properties is crucial in real-world contexts like engineering, surveying, and architecture.
- Problems often involve finding the value of an unknown angle or length based on angle relationships.
- Diagramming and labeling angles carefully helps solve these problems accurately.
- When solving for unknown angles, always look for angle pairs that add up to 180° or are congruent.
- If alternate interior angles are $50°$, the opposite angle is also $50°$.
- Example problem: Two parallel lines are intersected by a transversal, and one angle is $30°$. What is its corresponding angle? The answer is $30°$.
- Given two parallel lines, if one angle is $60°$, what is the alternate interior angle? The answer is $60°$.
- Consecutive interior angles: If one is $120°$, the other must be $60°$ to add up to $180°$.

Word Problems

Intermediate Questions

1. What is the relationship between consecutive interior angles?

2. How do you find the corresponding angle when two parallel lines are cut by a transversal?

3. If one angle is $30°$ and the lines are parallel, what is the corresponding angle?

4. How do you solve for unknown angles in parallel line problems?

5. What is the sum of consecutive interior angles?

6. If two parallel lines are cut by a transversal, what can we say about the alternate interior angles?

7. How can you use angle relationships to solve for missing angles?

8. Explain why the sum of consecutive interior angles is always $180°$.

9. What is the relationship between corresponding angles and parallel lines?

10. If two parallel lines are cut by a transversal, and one angle is $80°$, what is the alternate angle?

Hard Questions

1. Two parallel lines are cut by a transversal. If one angle is $45°$, what is its corresponding angle?

2. Given parallel lines and a transversal, if one angle is $130°$, what is the angle that is supplementary to it?

3. Solve for the unknown angle in a parallel line diagram where three angles are given.

4. If alternate interior angles are $70°$ and $110°$, are the lines parallel? Explain why or why not.

5. How would you calculate unknown angles using the angle sum property in parallel line problems?

6. If two consecutive interior angles are $95°$ and $85°$, are the lines parallel?

7. What is the missing angle in a diagram where you know two consecutive angles are $90°$ and $60°$?

8. Explain how to solve for angles when there are multiple transversals cutting parallel lines.

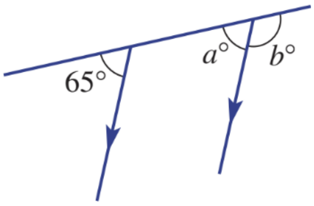
9. How do angle relationships help determine the measure of unknown angles in parallel line problems?

10. Why is it important to label all angles carefully when solving problems with parallel lines?

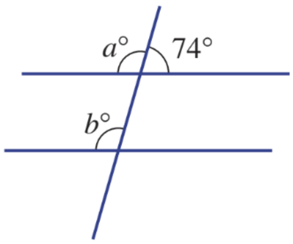
Practice Questions

1. In these diagrams, first find the value of a and then find the value of b.

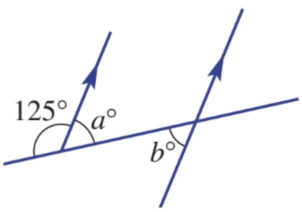
a.



b.

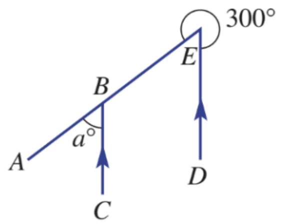


c.

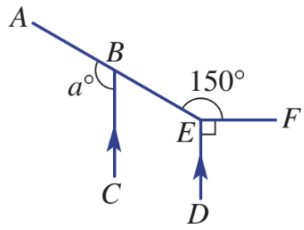


2. Find the value of a in these diagrams.

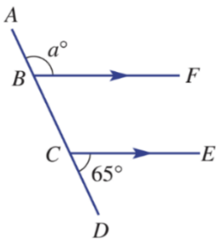
a.



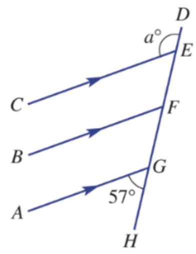
b.



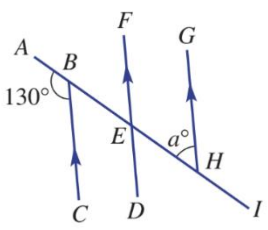
c.



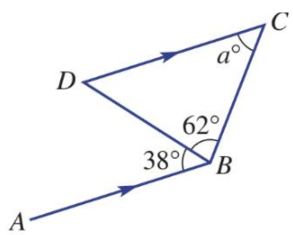
d.



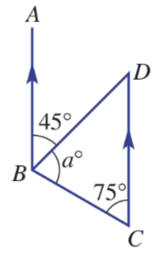
e.



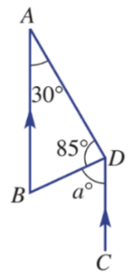
f.



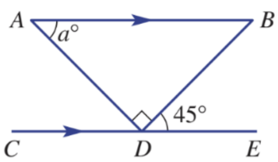
g.


A
D
45°
a°
B
75°
C

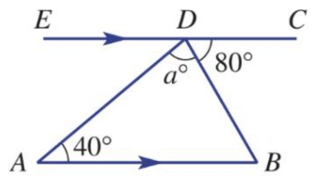
h.


A
30°
85°
D
B
a°
C

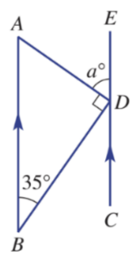
i.


A
a°
B
45°
C
D
E

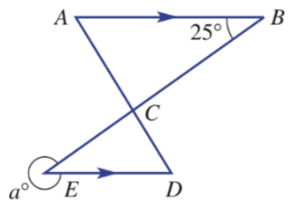
j.


E
D
C
a°
80°
40°
A
B

k.

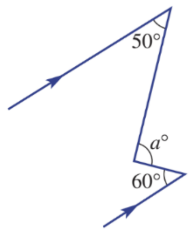

A
E
a°
D
35°
C
B

l.

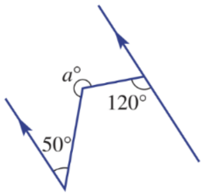

A
B
25°
C
a°
E
D

3. Find the value of a in these diagrams. You may wish to add one or more parallel lines to each diagram.

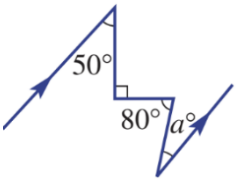
a.



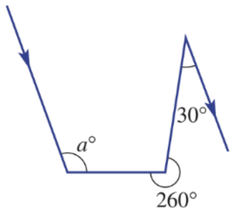
b.



c.



d.



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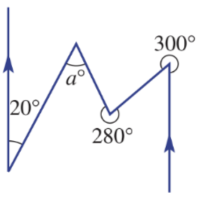

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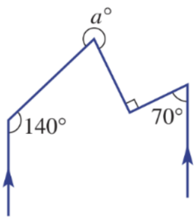


Chapter 2.6 Circles and Constructions

Summary

- Circles are round shapes defined by their center and radius. Every point on a circle is equidistant from the center.
- The radius is the distance from the center to any point on the circle.
- A diameter is twice the radius and passes through the center of the circle.
- A circumference is the perimeter of the circle, the total distance around it.
- The area of a circle is calculated by the formula $A = \pi r^2$, where r is the radius.
- Constructions in geometry involve drawing accurate shapes or figures using tools like a ruler, compass, and protractor.
- To construct a circle, use a compass to draw a circle with a given radius from a central point.
- A chord is a line segment that connects two points on the circle's circumference.
- An arc is any portion of the circumference of the circle.
- Angle relationships in circles include the central angle (formed by two radii) and inscribed angles (formed by two chords).
- A circle with radius $5\ \text{cm}$ has a diameter of $10\ \text{cm}$.
- To find the area of a circle with radius 4 cm: $A = \pi r^2 = \pi(4)^2 \approx 3.14 \times 16 \approx 50.24\ \text{cm}^2$.
- Draw a circle with a $3\ \text{cm}$ radius using a compass.
- If a chord divides a circle into two arcs, one arc will be greater than the other.
- The angle at the center of the circle is twice the angle at the circumference (for the same arc).

Word Problems

Intermediate Questions

1. What is the radius of a circle with a diameter of 10 cm?

2. How do you calculate the area of a circle?

3. What is the difference between a radius and a diameter?

4. How do you construct a circle using a compass?

5. What is an arc in a circle?

6. If a circle has a radius of 7 cm, what is its diameter?

7. What is the relationship between a central angle and an inscribed angle in a circle?

8. How do you measure the circumference of a circle?

9. What is the formula for the area of a circle?

10. What do you call a line segment that connects two points on a circle?

Hard Questions

1. Find the area of a circle with radius $5\ \text{cm}$, using $\pi = 3.14$.

2. If the radius of a circle is $6\ \text{cm}$, what is the circumference?

3. Explain how to construct a tangent to a circle.

4. If the central angle is $90°$, what is the corresponding inscribed angle?

5. How can you find the length of a chord in a circle?

6. What is the relationship between the central angle and arc length?

7. If a circle’s radius is doubled, how does the area change?

8. Explain the process of constructing a circle with a given diameter.

9. What is the sector area of a circle with radius 4 cm and central angle $60°$?

10. Calculate the area and circumference of a circle with radius 3.5 cm.

Chapter 2.7 Dynamic Geometry

Summary

- Dynamic geometry involves using interactive software or tools to explore and construct geometric shapes and transformations.
- It allows users to manipulate shapes, angles, and points to observe their relationships and properties.
- Dynamic geometry tools like Geogebra help visualize concepts like reflection, rotation, dilation, and translation.
- These tools can show real-time changes as shapes are moved or rotated, providing an engaging way to learn geometry.
- Dynamic geometry is often used in classrooms and professional design to model, test, and visualize complex geometric concepts.
- Key geometric transformations include:
  - Translation: Moving a shape without changing its size or orientation.
  - Rotation: Turning a shape around a fixed point.
  - Reflection: Flipping a shape across a line (mirror image).
  - Dilation: Changing the size of a shape while maintaining its proportions.
- Understanding dynamic geometry is crucial for fields like architecture, engineering, and computer graphics.
- Dynamic geometry tools can rotate a triangle around a fixed point.
- Use Geogebra to reflect a square over a line of symmetry.
- Translate a rectangle by moving it along a straight path.
- Use rotation to spin a polygon around its center by 90°.
- In dynamic geometry, dilating a triangle with a factor of 2 will double its size.

Word Problems

Intermediate Questions

1. What is dynamic geometry?

2. How does rotation affect a shape?

3. What is the difference between translation and dilation?

4. How does reflection create a mirror image?

5. What is the effect of a dilation factor greater than 1?

6. In dynamic geometry, how do you reflect a shape?

7. What happens to a shape during translation?

8. How is a rotation different from a reflection?

9. What happens to the area of a shape when it undergoes dilation?

10. How do you rotate a shape around a fixed point in dynamic geometry tools?

Hard Questions

1. Explain how to use Geogebra to reflect a quadrilateral across a line of symmetry.

2. How does dynamic geometry help visualize transformations in real-time?

3. In dynamic geometry, if you dilate a triangle by a factor of 0.5, what happens?

4. What effect does rotation have on the angles of a shape?

5. How does a transformation like dilation affect the perimeter of a shape?

6. Using dynamic geometry, how would you construct a square by reflecting a rectangle over its diagonal?

7. Explain how translation differs from reflection in dynamic geometry.

8. In dynamic geometry, what happens when you rotate a shape $180°$?

9. How would you use dynamic geometry to demonstrate congruence of two triangles?

10. Why is dynamic geometry essential for modern architecture and engineering designs?

CHAPTER 3 NUMBER PROPERTIES AND PATTERNS

Table of Contents

Chapter 3.1 Factors and Multiples

Summary

- Factors are numbers that divide exactly into a given number without leaving a remainder.
- Multiples are the numbers obtained by multiplying a number by whole numbers.
- For example, factors of 12 are 1, 2, 3, 4, 6, and 12.
- Multiples of 3 are 3, 6, 9, 12, 15, etc.
- Prime numbers have only two factors: 1 and themselves (e.g., 2, 3, 5, 7).
- Composite numbers have more than two factors (e.g., 6, 8, 12).
- Greatest common factor (GCF) is the largest number that divides two or more numbers.
- Least common multiple (LCM) is the smallest multiple that is common to two or more numbers.
- Finding the factors and multiples of numbers is important in problem solving, especially in fractions and ratios.

Examples

- Factors of 18: 1, 2, 3, 6, 9, 18.
- Multiples of 5: 5, 10, 15, 20, 25, ...
- GCF of 12 and 18: 6.
- LCM of 4 and 6: 12.

Word Problems

Intermediate Questions

1. List the factors of 30.

2. What are the first five multiples of 6?

3. Find the GCF of 15 and 25.

4. List the first five multiples of 8.

5. What is the LCM of 3 and 5?

6. Find the GCF of 24 and 36.

7. What is the smallest multiple of 7?

8. Find the factors of 48.

9. What is the LCM of 10 and 15?

10. Is 9 a factor of 72?

Hard Questions

1. Find the GCF of 56 and 84.

2. What is the LCM of 6, 8, and 12?

3. List all the factors of 72.

4. Find the GCF and LCM of 14 and 18.

5. Explain the difference between factors and multiples.

6. Find the LCM of 9 and 20.

7. What are the common factors of 36 and 60?

8. Explain how to find the LCM of two numbers using prime factorization.

9. Find the smallest common multiple of 11 and 17.

10. List the factors of 100.

Practice Questions

1. For each of the following numbers, state whether they are factors (F), multiples (M) or neither (N) of the number 60.

a. 120　　b. 14　　c. 15

d. 40　　e. 6　　f. 5

g. 240　　h. 2　　i. 22

j. 600　　k. 70　　l. 1

2. List the complete set of factors for each of the following numbers.

a. 10

b. 24

c. 17

d. 36

e. 60

f. 42

g. 80

h. 12

i. 28

3. List the complete set of factors for each of the following numbers.

a. 5

b. 8

c. 12

d. 7

e. 20

f. 75

g. 15

h. 100

i. 37

4. Express each of the following numbers as a product of two factors, both of which are greater than 10.

a. 192

b. 315

c. 180

d. 121

e. 336

f. 494

Chapter 3.2 Highest Common Factor and Lowest Common Multiple

Summary

- The highest common factor (HCF) is the largest number that divides exactly into two or more numbers.
- The lowest common multiple (LCM) is the smallest number that is a multiple of two or more numbers.
- To find the HCF, list the factors of each number and choose the largest factor they share.
- To find the LCM, list the multiples of each number and choose the smallest multiple they share.
- Alternatively, the HCF can be found using prime factorization and the LCM by taking the highest powers of each prime factor.
- The HCF is useful for simplifying fractions and ratios, while the LCM is used for finding common denominators.
- In real-life applications, the LCM can help in scheduling events that repeat at different intervals.
- HCF and LCM play an important role in problem-solving, especially in fractions, ratios, and number theory.
- Prime factorization is often a quick way to find the HCF and LCM of large numbers.

Examples

- Find the HCF of 24 and 36: HCF is 12.
- Find the LCM of 4 and 6: LCM is 12.
- To find the LCM of 5 and 7, list the multiples: 5, 10, 15, 20, 25...; 7, 14, 21, 28, 35...; the LCM is 35.

Word Problems

Intermediate Questions

1. Find the HCF of 18 and 24.

2. What is the LCM of 4 and 9?

3. Find the LCM of 5 and 12.

4. List the factors of 30.

5. What is the LCM of 6 and 15?

6. Find the HCF of 36 and 48.

7. What is the LCM of 8 and 10?

8. Find the HCF of 50 and 75.

9. What is the LCM of 12 and 15?

10. What is the smallest common multiple of 7 and 14?

Hard Questions

1. Find the HCF and LCM of 54 and 72.

2. List all the multiples of 18 and 24 and find the LCM.

3. Using prime factorization, find the LCM of 36 and 48.

4. Calculate the HCF of 132 and 198 using prime factorization.

5. What is the LCM of 5, 8, and 12?

6. Explain how to find the HCF using prime factorization.

7. Find the LCM of 40 and 50 using prime factorization.

8. If the HCF of two numbers is 3 and the LCM is 36, what are the two numbers?

9. Find the HCF of 84 and 126 using the prime factorization method.

10. How can you use the LCM to find the smallest common denominator?

Practice Questions

1. Find the HCF of the following pairs of numbers.

a. 4 and 5

b. 8 and 13

c. 2 and 12

d. 3 and 15

e. 16 and 20

f. 16 and 20

g. 50 and 150

h. 48 and 72

i. 80 and 120

j. 75 and 125

k. 42 and 63

l. 28 and 42

2. Find the HCF of the following groups of numbers.

a. 20, 40, 50

b. 6, 15, 42

c. 50, 100, 81

d. 18, 13, 21

e. 24, 72, 16

f. 120, 84, 144

3. Find the HCF of the following pairs of numbers and then use this information to help calculate the LCM of the same pair of numbers.

a. 15 and 20

b. 12 and 24

c. 14 and 21

d. 45 and 27

Chapter 3.3 Divisibility

Summary

- Divisibility rules are shortcuts that allow us to determine whether one number is divisible by another without performing division.
- The most common divisibility rules involve numbers like 2, 3, 5, 6, 9, and 10.
- For example, a number is divisible by 2 if its last digit is even (0, 2, 4, 6, 8).
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 5 if its last digit is 0 or 5.
- A number is divisible by 6 if it is divisible by both 2 and 3.
- For divisibility by 9, the sum of the digits of the number must be divisible by 9.
- For divisibility by 10, the last digit of the number must be 0.
- Divisibility rules are helpful in simplifying problems, especially when working with large numbers.
- These rules are also used in finding factors and multiples of numbers.

Examples

- 432 is divisible by 3 because the sum of its digits $(4 + 3 + 2 = 9)$ is divisible by 3.
- 246 is divisible by 2 because its last digit is 6 (an even number).
- 125 is divisible by 5 because its last digit is 5.
- 154 is divisible by 6 because it is divisible by both 2 and 3.

Word Problems

Intermediate Questions

1. Is 48 divisible by 2?

2. What rule can be used to check if a number is divisible by 3?

3. Is 615 divisible by 5?

4. What is the divisibility rule for 10?

5. Is 249 divisible by 3?

6. Check if 456 is divisible by 6.

7. Is 110 divisible by 5?

8. Use divisibility rules to check if 72 is divisible by 9.

9. Is 1,000 divisible by 2?

10. Check if 123 is divisible by 3.

Hard Questions

1. Check if 864 is divisible by 6 and explain why.

2. What is the rule for divisibility by 9?

3. Is 3,000 divisible by 10? Justify your answer.

4. Explain why a number divisible by both 2 and 3 is also divisible by 6.

5. Check if 1,428 is divisible by 2, 3, and 6.

6. Find a number divisible by 5, 6, and 10.

7. Using divisibility rules, check if 372 is divisible by 3 and 6.

8. Is 1,875 divisible by 9? Explain using the divisibility rule.

9. How would you check if 456 is divisible by both 2 and 5?

10. Check if 3,356 is divisible by 2, 5, and 10.

Practice Questions

1. Determine whether the following calculations are possible without leaving a remainder.

a. $23\ 562 \div 3$

b. $39\ 245\ 678 \div 4$

c. $213\ 456 \div 8$

d. $3\ 193\ 457 \div 6$

e. $51\ 345\ 678 \div 5$

f. $215\ 364 \div 6$

g. $25\ 756 \div 2$

h. $56\ 789 \div 9$

i. $2\ 345\ 176 \div 8$

j. $329\ 541 \div 10$

k. $356\ 781\ 276 \div 9$

l. $164\ 567 \div 8$

Chapter 3.4 Prime Numbers

Summary

- Prime numbers are numbers greater than 1 that have no divisors other than 1 and themselves.
- The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and so on.
- The number 1 is not considered a prime number.
- Every number can be factored into prime numbers, which is known as prime factorization.
- A composite number has more than two factors, unlike a prime number.
- Prime numbers play a key role in number theory and cryptography.
- The only even prime number is 2; all other even numbers are composite.
- Prime numbers are essential for constructing secure codes and encryptions.
- To check if a number is prime, divide it by all prime numbers up to its square root. If no division is exact, it is prime.

Examples

- 5 is prime because its only divisors are 1 and 5.
- 11 is prime because its only divisors are 1 and 11.
- 15 is not prime because it has divisors 1, 3, 5, and 15.

Word Problems

Intermediate Questions

1. Is 17 a prime number?

2. What are the first 5 prime numbers?

3. Is 20 a prime number?

4. List the factors of 29.

5. Is 13 a prime number?

6. What is the smallest prime number?

7. Find the next prime number after 23.

8. Is 19 a prime number?

9. Explain why 1 is not a prime number.

10. What is the prime number between 30 and 40?

Hard Questions

1. Is 97 a prime number? Explain how to check.

2. List the first 10 prime numbers.

3. Find the next prime number after 101.

4. Is 121 a prime number? Justify your answer.

5. Explain how to determine if a large number is prime.

6. What is the largest prime number less than 50?

7. Find the prime factors of 84.

8. How can prime numbers be used in cryptography?

9. Check if 71 is a prime number and explain why.

10. What is the prime number between 50 and 60?

Practice Questions

1. State whether each of the following is a prime (P) or composite (C) number.

a. 14

b. 23

c. 70

d. 37

e. 51

f. 27

g. 29

h. 3

i. 8

j. 49

k. 99

l. 59

m. 2

n. 31

o. 39

p. 89

2. The following are not prime numbers, yet they are the product (×) of two primes. Find the two primes for each of the following numbers.

a. 55

b. 91

c. 143

d. 187

e. 365

f. 133

Chapter 3.5 Powers

Summary

- Powers (also known as exponents) represent repeated multiplication of a number by itself.
- The notation $'a^n'$ means multiplying the base number 'a' by itself 'n' times.
- For example, 2^3 means $2 \times 2 \times 2 = 8$.
- The base is the number being multiplied, and the exponent (or power) shows how many times the base is multiplied.
- Powers of 10 are commonly used in scientific notation, where powers of 10 are used to express very large or very small numbers.
- Zero raised to any power is 0 (e.g., $0^n = 0$).
- Any non-zero number raised to the power of 0 is 1 (e.g., $5^0 = 1$).
- Powers with negative exponents represent the reciprocal $\left(\text{e.g.}, 2^{-3} = \frac{1}{2^3} = \frac{1}{8}\right)$.
- Understanding powers is essential for working with exponential growth and decay in real-life applications.
- Powers are used in areas such as finance, physics, and computer science.

Examples

- $2^3 = 2 \times 2 \times 2 = 8.$
- $10^4 = 10{,}000.$
- $5^0 = 1.$
- $3^{-2} = \frac{1}{9}.$
- $4^2 = 4 \times 4 = 16.$

Word Problems

Intermediate Questions

1. What is 3^2?

2. Calculate 5^3.

3. What is the value of 2^4?

4. Is 7^0 equal to 1?

5. Find the value of 10^3.

6. What is 4^2?

7. Calculate 3^{-1}.

8. What is the reciprocal of 2^3?

9. If a number is raised to the power of 0, what is the result?

10. What is 6^1 ?

Hard Questions

1. What is the value of 5^{-2}?

2. Calculate the result of 3^4.

3. Simplify 10^{-3}.

4. Find the value of 2^5 .

5. Calculate 8^{-2}.

6. What is 9^0?

7. Express $\frac{1}{16}$ as a power of 2.

8. Evaluate $4^3 \times 2^2$.

9. What is the value of $7^2 \times 7^3$?

10. How can you simplify the expression $2^5 \times 2^{-2}$?

Practice Questions

1. Simplify the following expressions by writing them as powers.

a. $3 \times 3 \times 5 \times 5$

b. $7 \times 7 \times 2 \times 2 \times 7$

c. $12 \times 9 \times 9 \times 12$

d. $8 \times 8 \times 5 \times 5 \times 5$

e. $6 \times 3 \times 6 \times 3 \times 6 \times 3$

f. $13 \times 7 \times 13 \times 7 \times 7 \times 7$

g. $4 \times 13 \times 4 \times 4 \times 7$

h. $10 \times 9 \times 10 \times 9 \times 9$

i. $2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 2$

2. Expand these terms. (Do not evaluate).

a. $3^5 \times 2^3$

b. $4^3 \times 3^4$

c. $7^2 \times 5^3$

d. $4^6 \times 9^3$

e. 5×7^4

f. $2^2 \times 3^3 \times 4^1$

g. $11^5 \times 9^2$

h. $20^3 \times 30^2$

3. Evaluate:

a. $3^2 + 4^2$

b. $2 \times 5^2 - 7^2$

c. $8^2 - 2 \times 3^3$

d. $(9 - 5)^3$

e. $2^4 \times 2^3$

f. $2^7 - 1 \times 2 \times 3 \times 4 \times 5$

g. $1^4 + 2^3 + 3^2 + 4^1$

h. $10^3 - 10^2$

i. $(1^{27} + 1^{23}) \times 2^2$

Chapter 3.6 Prime Decomposition

Summary

- Prime decomposition involves expressing a number as the product of prime numbers.
- Prime numbers are the building blocks of all numbers, as every number can be expressed as a product of primes.
- For example, the prime decomposition of 12 is $2 \times 2 \times 3$.
- To find the prime decomposition of a number, divide it by prime numbers starting from the smallest prime (2) until you can no longer divide.
- Prime factorization is used in simplifying fractions and solving problems in number theory.
- Every composite number has a unique prime factorization, which is known as the fundamental theorem of arithmetic.
- Prime decomposition is useful in finding the greatest common factor (GCF) and the least common multiple (LCM).
- In real life, prime decomposition can be used in cryptography and data encryption.
- The prime factorization of a number is useful for simplifying square roots and working with exponents.

Examples

- The prime decomposition of 18 is $2 \times 3 \times 3$.
- The prime decomposition of 30 is $2 \times 3 \times 5$.
- The prime decomposition of 56 is $2 \times 2 \times 2 \times 7$.

Word Problems

Intermediate Questions

1. Find the prime decomposition of 45.

2. What is the prime decomposition of 60?

3. List the prime factors of 56.

4. Find the prime decomposition of 72.

5. What is the prime factorization of 100?

6. Write the prime decomposition of 84.

7. What are the prime factors of 90?

8. Find the prime factorization of 64.

9. What is the prime decomposition of 48?

10. Write the prime decomposition of 120.

Hard Questions

1. Find the prime decomposition of 180.

2. Express 84 as a product of prime numbers.

3. How would you use prime decomposition to find the GCF of 40 and 60?

4. Find the prime factorization of 150.

5. What is the prime decomposition of 210?

6. Write the prime decomposition of 132.

7. How does prime factorization help in calculating the LCM of 50 and 75?

8. Simplify the square root of 144 using its prime decomposition.

9. What is the prime decomposition of 144?

10. Express 90 as the product of prime numbers.

Chapter 3.7 Squares and Square Roots

Summary

- A square is a number multiplied by itself. It is represented as a^2 (read as 'a squared').
- For example, $4^2 = 4 \times 4 = 16$.
- The square root is the inverse operation of squaring a number.
- The square root of a number is the value that, when multiplied by itself, gives the original number.
- For example, the square root of 16 is 4, because $4 \times 4 = 16$.
- Square roots are often written as $\sqrt{x}$, where x is the number to find the square root of.
- Perfect squares are numbers that have an integer square root, such as $1, 4, 9, 16, 25$.
- Not all numbers have perfect square roots, and some square roots result in decimals.
- The square root of non-perfect squares is an irrational number (e.g.,$\sqrt{2} \approx 1.4142$).
- Square roots are used in areas like geometry, algebra, and statistics.

Examples

- $4^2 = 16$.
- The square root of 25 is $\sqrt{25} = 5$.
- 9 is a perfect square because$\sqrt{9} = 3$.
- The square root of 50 is approximately 7.071.

Word Problems

Intermediate Questions

1. What is the square of 5?

2. What is the square root of 36?

3. Find the square of 12.

4. What is $\sqrt{64}$?

5. Find the square root of 49.

6. Is 64 a perfect square? Why?

7. What is the square of 3?

8. Find the square root of 81.

9. What is $\sqrt{25}$?

10. Is 10^2 equal to 100?

Hard Questions

1. What is the square root of 15?

2. Find the square of 17.

3. What is $\sqrt{100}$?

4. What is the square of 13?

5. Calculate the square root of 72.

6. What is the square root of 150?

7. Find the square of 21.

8. What is the square root of 200?

9. Calculate $\sqrt{2}$ and give its approximate value.

10. What is the value of $\sqrt{81} \times \sqrt{9}$?

Practice Questions

1. Evaluate

a. 8^2

b. 7^2

c. 1^2

d. 12^2

e. 3^2

f. 15^2

g. 5^2

h. 0^2

i. 11^2

j. 100^2

k. 17^2

l. 33^2

2. Evaluate.

a. $\sqrt{25}$

b. $\sqrt{9}$

c. $\sqrt{1}$

d. $\sqrt{121}$

e. $\sqrt{0}$

f. $\sqrt{81}$

g. $\sqrt{49}$

h. $\sqrt{16}$

i. $\sqrt{4}$

j. $\sqrt{144}$

k. $\sqrt{400}$

l. $\sqrt{169}$

3. Evaluate

a. $3^2 + 5^2 - \sqrt{16}$

b. 4×4^2

c. $8^2 - 0^2 + 1^2$

d. $1^2 \times 2^2 \times 3^2$

e. $\sqrt{5^2 - 3^2}$

f. $\sqrt{81} - 3^2$

g. $6^2 \div 2^2 \times 3^2$

h. $\sqrt{9} \times \sqrt{64} \div \sqrt{36}$

i. $\sqrt{12^2 + 5^2}$

Chapter 3.8 Number Patterns

Summary

- Number patterns are sequences of numbers that follow a specific rule or relationship.
- A pattern can be arithmetic, geometric, or another type of sequence.
- In an arithmetic sequence, the difference between consecutive terms is constant.
- For example, in the sequence 2, 4, 6, 8, 10, ... the difference between consecutive terms is 2.
- In a geometric sequence, each term is multiplied by the same number to get the next term.
- For example, in the sequence 2, 4, 8, 16, 32, ... each term is multiplied by 2.
- Number patterns help to predict future terms and understand relationships between numbers.
- Recognizing number patterns is useful for solving algebraic problems, such as finding terms in a sequence or solving equations.
- Real-life examples include calendars, financial growth, and population modeling.

Examples

- Arithmetic sequence: 5, 10, 15, 20, 25, ... (difference of 5).
- Geometric sequence: 3, 6, 12, 24, 48, ... (multiplying by 2).
- Find the next term in the sequence 1, 4, 7, 10, 13, ... (difference of 3).
- Find the next term in the sequence 2, 6, 18, 54, ... (multiplying by 3).

Word Problems

Intermediate Questions

1. Find the next term in the sequence 3, 6, 9, 12, 15.

2. Is 5, 10, 15, 20 an arithmetic sequence?

3. Find the common difference in the sequence 7, 10, 13, 16, 19.

4. What is the next term in the sequence 2, 5, 8, 11, 14?

5. Is 1, 3, 6, 10, 15 a number pattern? If so, what type?

6. Find the common ratio in the sequence 3, 6, 12, 24.

7. What is the next term in the sequence 8, 16, 32, 64?

8. Identify the pattern in the sequence 100, 90, 80, 70, 60.

9. What is the next term in the sequence 2, 4, 6, 8, 10?

10. Find the common difference in the sequence 1, 3, 5, 7, 9.

Hard Questions

1. What is the 10th term in the sequence 3, 6, 9, 12, ...?

2. Given the arithmetic sequence 5, 10, 15, 20, find the nth term.

3. What is the sum of the first 10 terms of the sequence 2, 4, 6, 8, ...?

4. Write the general formula for the nth term of the sequence 5, 10, 15, 20.

5. Find the 6th term in the geometric sequence 3, 9, 27, 81, ...

6. What is the 8th term in the sequence 1, 4, 7, 10, ...?

7. Find the common difference of the sequence 20, 18, 16, 14, ...

8. Explain how to find the nth term of a geometric sequence.

9. Find the 5th term in the sequence 1, 2, 4, 8, 16, ...

10. What is the sum of the first 6 terms in the sequence 4, 8, 16, 32, ...?

Practice Questions

1. Find the next three terms for the following number patterns that have a common difference.

a. $3, 8, 13, 18, \_\_\_\_, \_\_\_\_, \_\_\_\_$

b. $4, 14, 24, 34, \_\_\_\_, \_\_\_\_, \_\_\_$

c. $26, 23, 20, 17, \_\_\_\_, \_\_\_\_, \_\_\_\_$

d. $106, 108, 110, 112, \_\_\_\_, \_\_\_\_, \_\_\_$

e. $63, 54, 45, 36, \_\_\_\_, \_\_\_\_, \_\_\_\_$

f. $9, 8, 7, 6, \_\_\_\_, \_\_\_\_, \_\_\_\_$

g. $101, 202, 303, 404, \_\_\_\_, \_\_\_\_, \_\_\_\_$

h. $75, 69, 63, 57, \_\_\_\_, \_\_\_\_, \_\_\_\_$

2. Find the missing numbers in each of the following number patterns.

a. 62,56, ___, 44, 38 , ____ , ____

b. 15, ___ , 35, ____ , ____ , 65,75

c. 4, 8 16, ____ , ____ , 128, ____

d. 3,6, ___ , 12, ____ , 18, ___

e. 88, 77, 66, ____ , ____ , ___ ,22

f. 2997, 999, ____ , ____ , 37

g. 14, 42, ____ , ____ , 126, ____ , 182

h. 14, 42, ____ , ____ , 1134, ___ , 10 206

3. Generate the next three terms for the following number sequences and give an appropriate name to the sequence.

a. $1, 4, 9, 16, 25, 36,$ ____ , ____ , ____

b. $1, 1, 2, 3, 5, 8, 13,$ ____ , ____ , ____

c. $1, 8, 27, 64, 125,$ ____ , ____ , ____

d. $2, 3, 5, 7, 11, 13, 17,$ ____ , ____ , ____

e. $4, 6, 8, 9, 10, 12, 14, 15,$ ____ , ____ , ____

f. $121, 131, 141, 151,$ ____ , ____ , ____

Chapter 3.9 Tables and Rules

Summary

- Tables and rules are used to organize and simplify mathematical relationships.
- A table can represent a pattern or relationship between numbers, where one variable depends on another.
- Rules define how one number in the table is related to others.
- For example, in a table for the relationship $y = 2x$, the rule states that y is always double x.
- Using tables helps to find missing values and predict outcomes based on known relationships.
- In real-life applications, tables are used for budgeting, planning, and organizing data.
- Tables can represent arithmetic sequences, geometric sequences, and even real-world problems like temperature conversions.
- The rule helps determine how the variables interact and predict the next values in the sequence.
- A table of values can make it easier to see patterns and solve equations.

Examples

- Example: A table showing $y = 2x$: x: $1, 2, 3, 4$; y: $2, 4, 6, 8$.
- A table for $y = 3x$: x: $1, 2, 3, 4$; y: $3, 6, 9, 12$.
- In a budget table, income and expenses are related by a specific rule, like 'expenses = income × 0.7'.

Word Problems

Intermediate Questions

1. Fill in the missing values in the table for $y = 3x$: x: $1, 2, 3, 4$; y: $?, ?, ?, ?$.

2. Write the rule for the table: x: $1, 2, 3, 4$; y: $2, 4, 6, 8$.

3. What is the value of y when $x = 5$ in the rule $y = 4x$?

4. Find the missing value in the table for $y = 5x$: x: $1, 2, 3, 4$; y: $?, ?, ?, ?$.

5. Write a rule based on the table: x: $1, 2, 3, 4$; y: $4, 8, 12, 16$.

Hard Questions

1. Solve for x in the equation $y = 6x + 2$ when $y = 20$.

2. Create a table for the rule $y = 2x + 3$ and find y when $x = 6$.

3. Write the rule for the table of values: x: $1, 2, 3, 4$; y: $10, 15, 20, 25$.

4. Given the rule $y = 3x - 1$, what is the value of y when $x = 8$?

5. In a table for $y = 5x + 1$, find the missing y-values when $x = 2, 3, 4$.

Chapter 3.10: The Number Plane and Graphs

Summary

- The number plane (also known as the Cartesian plane) is a system for graphing points and representing equations using two axes: the x-axis (horizontal) and y-axis (vertical).
- Points on the number plane are written as ordered pairs (x, y), where x is the value on the x-axis and y is the value on the y-axis.
- The number plane is divided into four quadrants: Quadrant I (positive x and y), Quadrant II (negative x, positive y), Quadrant III (negative x and y), and Quadrant IV (positive x, negative y).
- The origin (0, 0) is the point where the x-axis and y-axis intersect.
- Graphs are visual representations of relationships between variables, often used to show patterns or trends.
- Linear graphs are graphs of straight lines, and they represent linear equations of the form $y = mx + b$.
- Graphing points on the number plane is used in various fields like physics, economics, and engineering.
- To plot a point on the number plane, move from the origin: first along the x-axis, then along the y-axis.
- Using graphs, we can visualize relationships between quantities, such as time vs. distance or price vs. quantity.

Examples

- Plotting the point (3, 4): Move 3 units along the x-axis and 4 units up on the y-axis.
- Plotting the point (-2, -3): Move 2 units to the left on the x-axis and 3 units down on the y-axis.
- The line $y = 2x + 1$ is a straight line that can be graphed by plotting several points and connecting them.

Word Problems

Intermediate Questions

1. Plot the point (4, 5) on the number plane.

2. What is the point (0, 0) called?

3. What does the point (0, 3) represent on the number plane?

4. In which quadrant does the point (3, -2) lie?

5. What is the x-coordinate of the point (6, -4)?

6. Plot the point (-2, 6) on the graph.

7. What is the y-coordinate of the point (-3, 2)?

8. In which quadrant does the point (-5, -7) lie?

9. What is the origin on the number plane?

10. How do you plot the point (2, 3) on the graph?

Hard Questions

1. Plot the points (-1, 2), (3, -4), and (5, 6). Which quadrant does each point lie in?

2. What is the equation of the line passing through the points (2, 3) and (4, 5)?

3. Explain how to find the slope of a line from its graph.

4. Plot the points (3, 5), (-4, 6), and (-1, -3) and determine which quadrant each lies in.

5. Given the equation $y = 2x - 1$, plot the graph of this linear equation.

6. What is the slope of the line passing through the points (1, 2) and (3, 6)?

7. Find the equation of a line passing through the points (0, 2) and (2, 6).

8. Plot the points (1, 2), (3, 4), and (5, 6), and determine if they form a straight line.

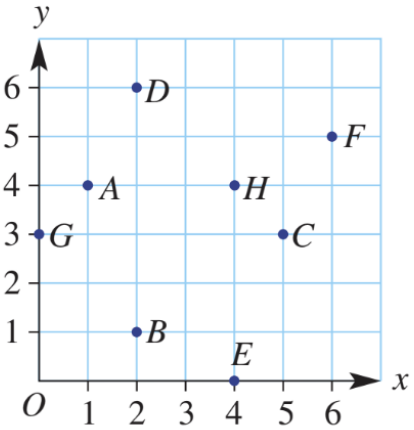
9. What does the y-intercept represent in the equation $y = mx + b$?

10. Explain how to identify the slope of a line from its graph.

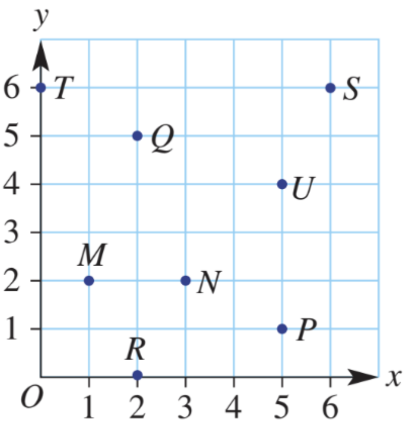
Practice Questions

1. Write down the coordinates of each of these labelled points.

a.



b.



2. Draw a Cartesian plane from 0 to 8 on both axes. Plot the following points on the grid and join them in the order they are given.

$(2,7),(6,7),(5,5),(7,5),(6,2),(5,2),(4,1),(3,2),(2,2),(1,5),(3,5),(2,7)$

CHAPTER 4 FRACTIONS AND PERCENTAGES

Table of Contents

Chapter 4.1 Equivalent Fractions and Simplified Fractions

Summary

- Equivalent fractions are different fractions that represent the same value.
- To find equivalent fractions, multiply or divide both numerator and denominator by the same number.
- Simplified fractions (also known as lowest terms) have no common factors other than 1 between the numerator and denominator.
- To simplify a fraction, divide the numerator and denominator by their greatest common factor (GCF).
- Simplifying fractions makes calculations easier and clearer.
- Equivalent fractions are useful for comparing fractions and performing arithmetic operations with fractions.
- Finding equivalent fractions helps in adding, subtracting, and comparing fractions with different denominators.
- Fractions in simplest form are easier to interpret in real-life contexts, such as recipes or measurements.

Examples

- Equivalent fractions to $\frac{1}{2}$: $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$.
- Simplify $\frac{4}{8}$ to $\frac{1}{2}$ by dividing numerator and denominator by 4.
- Simplify $\frac{18}{24}$ to $\frac{3}{4}$ by dividing numerator and denominator by 6.

Word Problems

Intermediate Questions

1. Simplify the fraction $\frac{6}{9}$.

2. Find two equivalent fractions for $\frac{3}{5}$.

3. Simplify $\frac{15}{20}$ to its lowest terms.

4. Are $\frac{2}{4}$ and $\frac{1}{2}$ equivalent fractions?

5. Simplify the fraction $\frac{8}{10}$.

6. Write an equivalent fraction for $\frac{2}{3}$.

7. Simplify $\frac{10}{25}$.

8. Find an equivalent fraction for $\frac{4}{6}$.

9. Simplify $\frac{14}{21}$.

10. Are $\frac{6}{8}$ and $\frac{3}{4}$ equivalent?

Hard Questions

1. Simplify $\frac{36}{48}$ to its lowest terms.

2. Find three equivalent fractions for $\frac{5}{8}$.

3. Simplify $\frac{45}{60}$.

4. How do you find the simplest form of $\frac{24}{36}$?

5. Write two equivalent fractions for $\frac{7}{9}$.

6. Simplify $\frac{28}{35}$.

7. Find an equivalent fraction to $\frac{12}{15}$.

8. Simplify $\frac{56}{70}$.

9. Explain how you simplify the fraction $\frac{18}{27}$.

10. Write three equivalent fractions for $\frac{9}{12}$.

Practice Questions

1. Which of the following fractions are equivalent to $\frac{8}{20}$?

$\frac{4}{10}, \frac{1}{5}, \frac{6}{20}, \frac{8}{10}, \frac{16}{40}, \frac{2}{5}, \frac{4}{12}, \frac{12}{40}, \frac{80}{200}, \frac{1}{4}$

2. Write four equivalent fractions for each of the fractions listed.

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{2}{5}$

d. $\frac{3}{5}$

e. $\frac{2}{9}$

f. $\frac{3}{7}$

g. $\frac{5}{12}$

h. $\frac{3}{11}$

3. Write the following fractions in simplest form.

a. $\frac{15}{20}$

b. $\frac{12}{18}$

c. $\frac{10}{30}$

d. $\frac{8}{22}$

e. $\frac{14}{35}$

f. $\frac{2}{22}$

g. $\frac{8}{56}$

h. $\frac{9}{27}$

i. $\frac{35}{45}$

j. $\frac{36}{96}$

k. $\frac{120}{144}$

l. $\frac{700}{140}$

Chapter 4.2 Mixed Numbers

Summary

- Mixed numbers consist of a whole number and a fraction combined.
- They are often used to express numbers greater than one that include a fractional part.
- Mixed numbers can be converted to improper fractions by multiplying the whole number by the denominator and adding the numerator.
- An improper fraction is a fraction where the numerator is greater than or equal to the denominator.
- Mixed numbers are useful in measuring quantities, such as lengths, areas, or capacities in real-life situations.
- To convert improper fractions to mixed numbers, divide the numerator by the denominator.
- Understanding mixed numbers is essential for accurately interpreting real-world measurements and calculations.

Examples

- Convert the mixed number $3\frac{1}{2}$ to an improper fraction: $(3 \times 2) + 1 = \frac{7}{2}$.
- Convert the improper fraction $\frac{11}{4}$ to a mixed number: $11 \div 4 = 2$ remainder 3, so $2\frac{3}{4}$.
- Express $7\frac{2}{3}$ as an improper fraction: $(7 \times 3) + 2 = \frac{23}{3}$.

Word Problems

Intermediate Questions

1. Convert $5\frac{1}{4}$ to an improper fraction.

2. Change $\frac{9}{2}$ into a mixed number.

3. Express $4\frac{3}{5}$ as an improper fraction.

4. Convert $\frac{15}{4}$ to a mixed number.

5. What is $3\frac{2}{7}$ as an improper fraction?

6. Change $\frac{11}{3}$ into a mixed number.

7. Express $2\frac{5}{8}$ as an improper fraction.

8. Convert the improper fraction $\frac{17}{5}$ to a mixed number.

9. Express $6\frac{1}{3}$ as an improper fraction.

10. Convert $\frac{22}{6}$ to a mixed number.

Hard Questions

1. Convert $8\frac{5}{12}$ to an improper fraction.

2. Change the improper fraction $\frac{29}{8}$ to a mixed number.

3. Express $7\frac{7}{9}$ as an improper fraction.

4. Convert $\frac{19}{7}$ to a mixed number.

5. How do you change $14\frac{13}{4}$ into an improper fraction?

6. Convert $\frac{55}{9}$ into a mixed number.

7. Express $9\frac{2}{5}$ as an improper fraction.

8. Change $\frac{33}{5}$ into a mixed number.

9. Explain how to convert $4\frac{7}{10}$ into an improper fraction.

10. Express $6\frac{3}{8}$ as an improper fraction.

Practice Questions

1. Convert these mixed numbers to improper fractions.

a. $2\frac{1}{5}$

b. $1\frac{3}{5}$

c. $2\frac{1}{2}$

d. $6\frac{1}{2}$

e. $6\frac{1}{9}$

f. $2\frac{7}{9}$

g. $4\frac{5}{12}$

h. $9\frac{7}{12}$

2. Convert these improper fractions to mixed numbers.

a. $\frac{5}{3}$

b. $\frac{16}{7}$

c. $\frac{20}{3}$

d. $\frac{48}{7}$

e. $\frac{93}{10}$

f. $\frac{135}{11}$

3. Convert these improper fractions to mixed numbers in their simplest form.

a. $\frac{28}{10}$

b. $\frac{40}{15}$

c. $\frac{60}{25}$

Chapter 4.3 Adding Fractions and Subtracting Fractions

Summary

- When adding or subtracting fractions with the same denominator, simply add or subtract the numerators and keep the denominator the same.
- If fractions have different denominators, first convert them to equivalent fractions with a common denominator.
- A common denominator can be found by determining the least common multiple (LCM) of the denominators.
- After finding the common denominator, rewrite the fractions with the new denominator before adding or subtracting.
- Always simplify your answer to the simplest form (lowest terms) after adding or subtracting.
- Adding and subtracting fractions is important for solving real-world problems involving measurements, cooking recipes, and financial calculations.
- Fractions must have the same denominator to perform addition or subtraction directly.

Examples

- $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$, simplified to $\frac{1}{2}$.
- $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$.
- $\frac{3}{8}-\frac{1}{8}=\frac{2}{8}$, simplified to $\frac{1}{4}$.
- $\frac{1}{2}+\frac{1}{3}$: Convert to $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Word Problems

Intermediate Questions

1. Add $\frac{1}{4}$ and $\frac{2}{4}$.

2. Subtract $\frac{3}{5}$ from $\frac{4}{5}$.

3. What is $\frac{2}{7}+\frac{3}{7}$?

4. Add $\frac{1}{6}$ and $\frac{4}{6}$.

5. Subtract $\frac{2}{8}$ from $\frac{5}{8}$.

6. Add $\frac{1}{3}$ and $\frac{1}{9}$ (use common denominators).

7. Subtract $\frac{3}{10}$ from $\frac{7}{10}$.

8. What is the sum of $\frac{1}{8}$ and $\frac{3}{8}$?

9. Subtract $\frac{1}{6}$ from $\frac{5}{6}$.

10. Add $\frac{2}{5}$ and $\frac{1}{10}$ (find a common denominator first).

Hard Questions

1. Add $\frac{3}{4}$ and $\frac{1}{6}$ (use a common denominator).

2. Subtract $\frac{5}{8}$ from $\frac{7}{8}$.

3. Calculate $\frac{1}{3} + \frac{2}{5}$ using common denominators.

4. Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

5. Find the sum of $\frac{4}{9}$ and $\frac{2}{3}$.

6. Add $\frac{5}{12}$ and $\frac{1}{3}$.

7. Subtract $\frac{1}{2}$ from $\frac{3}{4}$.

8. Find the result of $\frac{2}{3} - \frac{3}{9}$.

9. Calculate $\frac{3}{5} + \frac{2}{7}$.

10. Subtract $\frac{7}{10}$ from $\frac{9}{10}$.

Practice Questions

1. The following sums have been completed, but only six of them are correct. Copy them into your workbook, then place a tick beside the six correct answers and a cross beside the six incorrect answers.

a. $\frac{1}{6}+\frac{3}{6}=\frac{4}{6}$

b. $\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$

c. $\frac{2}{5}+\frac{4}{5}=\frac{6}{10}$

d. $\frac{1}{11}+\frac{3}{11}=\frac{4}{11}$

e. $\frac{3}{5}+\frac{4}{5}=\frac{7}{10}$

f. $\frac{2}{7}+\frac{2}{7}=\frac{2}{7}$

g. $\frac{7}{12} + \frac{4}{12} = \frac{11}{12}$

h. $\frac{4}{9} + \frac{4}{5} = \frac{4}{14}$

i. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$

j. $\frac{1}{2} + \frac{2}{5} = \frac{3}{7}$

k. $2\frac{2}{7} + 3\frac{1}{7} = 5\frac{3}{7}$

l. $1\frac{2}{3} + 2\frac{1}{5} = 3\frac{3}{8}$

2. Add the following fractions.

a. $\frac{1}{2} + \frac{1}{4}$

b. $\frac{1}{3} + \frac{3}{5}$

c. $\frac{1}{2} + \frac{1}{6}$

d. $\frac{1}{4} + \frac{1}{3}$

e. $\frac{2}{5} + \frac{1}{4}$

f. $\frac{1}{5} + \frac{3}{4}$

g. $\frac{2}{7}+\frac{1}{3}$

h. $\frac{3}{8}+\frac{1}{5}$

i. $\frac{3}{5}+\frac{5}{6}$

j. $\frac{4}{7}+\frac{3}{4}$

k. $\frac{8}{11}+\frac{2}{3}$

l. $\frac{2}{3}+\frac{3}{4}$

3. Simplify.

a. $2\frac{2}{3} + 1\frac{3}{4}$

b. $5\frac{2}{5} + 1\frac{5}{6}$

c. $3\frac{1}{2} + 8\frac{2}{3}$

d. $5\frac{4}{7} + 7\frac{3}{4}$

e. $8\frac{1}{2} + 6\frac{3}{5}$

f. $12\frac{2}{3} + 6\frac{4}{9}$

g. $17\frac{8}{11} + 7\frac{3}{4}$

h. $9\frac{7}{12} + 5\frac{5}{8}$

4. Simplify.

a. $\frac{5}{7} - \frac{3}{7}$

b. $\frac{4}{11} - \frac{1}{11}$

c. $\frac{12}{18} - \frac{5}{18}$

d. $\frac{2}{3} - \frac{1}{3}$

e. $\frac{3}{5} - \frac{3}{5}$

f. $\frac{6}{9} - \frac{2}{9}$

g. $\frac{5}{19} - \frac{2}{19}$

h. $\frac{17}{23} - \frac{9}{23}$

i. $\frac{84}{100} - \frac{53}{100}$

j. $\frac{41}{50} - \frac{17}{50}$

k. $\frac{23}{25} - \frac{7}{25}$

l. $\frac{7}{10} - \frac{3}{10}$

5. Simplify.

a. $3\frac{4}{5} - 2\frac{1}{5}$

b. $23\frac{5}{7} - 15\frac{2}{7}$

c. $8\frac{11}{14} - 7\frac{9}{14}$

d. $3\frac{5}{9} - \frac{3}{9}$

e. $6\frac{2}{3} - 4\frac{1}{4}$

f. $5\frac{3}{7} - 2\frac{1}{4}$

g. $9\frac{5}{6} - 5\frac{4}{9}$

h. $14\frac{3}{4} - 7\frac{7}{10}$

Chapter 4.4 Ordering Fractions

Summary

- Ordering fractions involves arranging fractions in ascending (smallest to largest) or descending (largest to smallest) order.
- To order fractions with different denominators, first find a common denominator.
- Once fractions have a common denominator, compare their numerators to determine their order.
- Another method involves converting fractions to decimals to easily compare their values.
- Ordering fractions is useful in real-life situations, such as ranking measurements or comparing quantities.
- Fractions can also be compared by cross-multiplying, where you multiply diagonally and compare products.

Examples

- Order $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}: \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$.
- Order $\frac{2}{5}, \frac{3}{10}, \frac{1}{2}$ by converting to decimals: $0.4, 0.3, 0.5 \rightarrow 3/10, 2/5, 1/2$.
- Order $\frac{1}{3}, \frac{2}{9}, \frac{4}{9}$ by finding a common denominator: $\frac{3}{9}, \frac{2}{9}, \frac{4}{9} \rightarrow \frac{2}{9}, \frac{1}{3}, \frac{4}{9}$.

Word Problems

Intermediate Questions

1. Order the fractions $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{4}{5}$.

2. Arrange $\frac{3}{8}$, $\frac{1}{8}$, and $\frac{5}{8}$ from smallest to largest.

3. Order the fractions $\frac{2}{3}$, $\frac{1}{3}$, and $\frac{3}{3}$.

4. Arrange $\frac{1}{6}$, $\frac{4}{6}$, and $\frac{5}{6}$ in descending order.

5. Order $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{2}{4}$.

6. Arrange the fractions $\frac{1}{7}, \frac{2}{7}$, and $\frac{5}{7}$ from largest to smallest.

7. Order the fractions $\frac{1}{3}, \frac{2}{6}$, and $\frac{1}{2}$.

8. Arrange $\frac{1}{10}, \frac{3}{10}$, and $\frac{7}{10}$ in ascending order.

9. Order the fractions $\frac{2}{8}, \frac{3}{8}$, and $\frac{5}{8}$.

10. Arrange the fractions $\frac{1}{9}, \frac{5}{9}$, and $\frac{4}{9}$ from smallest to largest.

Hard Questions

1. Order $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ from smallest to largest.

2. Arrange $\frac{2}{5}$, $\frac{4}{10}$, and $\frac{3}{5}$ by converting to decimals.

3. Order $\frac{1}{3}$, $\frac{3}{9}$, and $\frac{2}{6}$ using common denominators.

4. Arrange $\frac{5}{12}$, $\frac{1}{3}$, and $\frac{7}{12}$ in ascending order.

5. Order the fractions $\frac{3}{7}$, $\frac{4}{7}$, and $\frac{2}{7}$ from largest to smallest.

6. Arrange $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{8}$ in descending order.

7. Order $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{7}{9}$.

8. Arrange the fractions $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{2}{5}$ from smallest to largest.

9. Order $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$ using decimal equivalents.

10. Arrange $\frac{4}{6}$, $\frac{2}{3}$, and $\frac{5}{9}$ in ascending order using cross-multiplying.

Chapter 4.5 Multiplying Fractions and Dividing Fractions

Summary

- To multiply fractions, multiply the numerators together and the denominators together.
- Simplify the result to its lowest terms if possible.
- When dividing fractions, multiply by the reciprocal (invert the second fraction and multiply).
- Always simplify fractions after multiplying or dividing to the lowest terms.
- Multiplying fractions is used to find parts of quantities, such as finding half of a quarter.
- Dividing fractions is used to determine how many times one fraction fits into another, useful in recipes, dividing resources, and measurements.

Examples

- Multiplying $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.
- Dividing $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4.$
- Multiplying $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$, simplified to $\frac{2}{5}$.

Word Problems

Intermediate Questions

1. Multiply $\frac{1}{2}$ by $\frac{3}{4}$.

2. Divide $\frac{3}{5}$ by $\frac{1}{5}$.

3. Multiply $\frac{2}{3}$ by $\frac{3}{5}$.

4. Divide $\frac{1}{2}$ by $1/4$.

5. Multiply $\frac{4}{7}$ by $\frac{1}{2}$.

6. Divide $\frac{3}{8}$ by $\frac{1}{2}$.

7. Find the product of $\frac{1}{3}$ and $\frac{2}{3}$.

8. Divide $\frac{2}{5}$ by $\frac{1}{5}$.

9. Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

10. Divide $\frac{5}{6}$ by $\frac{1}{3}$.

Hard Questions

1. Multiply $\frac{7}{8}$ by $\frac{2}{3}$ and simplify.

2. Divide $\frac{4}{5}$ by $\frac{2}{3}$ and simplify.

3. Calculate $\frac{5}{6} \times \frac{3}{4}$.

4. Divide $\frac{7}{9}$ by $\frac{2}{3}$ and simplify your answer.

5. Find the product of $\frac{3}{8}$ and $\frac{4}{5}$.

6. Divide $\frac{9}{10}$ by $\frac{3}{5}$.

7. Multiply $\frac{2}{7}$ by $\frac{5}{6}$ and simplify.

8. Divide $\frac{5}{8}$ by $\frac{1}{4}$ and simplify.

9. Calculate $\frac{4}{9} \times \frac{3}{8}$.

10. Explain how to divide $\frac{2}{5}$ by $\frac{4}{7}$.

Practice Questions

1. Evaluate:

a. $\frac{2}{3} \times \frac{5}{7}$

b. $\frac{4}{9} \times \frac{2}{5}$

c. $\frac{3}{4} \times \frac{1}{3}$

d. $\frac{5}{9} \times \frac{9}{11}$

e. $\frac{8}{11} \times \frac{3}{4}$

f. $\frac{2}{5} \times \frac{10}{11}$

g. $\frac{5}{10}$ of $\frac{4}{7}$

h. $\frac{6}{9}$ of $\frac{3}{12}$

2. Find:

a. $\frac{1}{3}$ of 18

b. $\frac{1}{5}$ of 45

c. $\frac{2}{3}$ of 24

d. $\frac{2}{7}$ of 42

e. $\frac{1}{4}$ of 16

g. $\frac{4}{5}$ of 100

3. Find:

a. $1\frac{3}{5} \times 2\frac{1}{3}$

b. $1\frac{1}{7} \times 1\frac{2}{9}$

c. $3\frac{1}{4} \times 2\frac{2}{5}$

d. $4\frac{2}{3} \times 5\frac{1}{7}$

4. Find:

a. $\frac{3}{4} \div 2$

b. $\frac{5}{11} \div 3$

c. $\frac{8}{5} \div 4$

d. $\frac{15}{7} \div 3$

e. $2\frac{1}{4} \div 3$

f. $5\frac{1}{3} \div 4$

g. $12\frac{4}{5} \div 8$

h. $1\frac{13}{14} \div 9$

5. Find:

a. $\frac{2}{7} \div \frac{2}{5}$

b. $\frac{1}{5} \div \frac{1}{4}$

c. $\frac{3}{7} \div \frac{6}{11}$

d. $\frac{2}{3} \div \frac{8}{9}$

e. $2\frac{1}{4} \div 1\frac{1}{3}$

f. $4\frac{1}{5} \div 3\frac{3}{10}$

g. $12\frac{1}{2} \div 3\frac{3}{4}$

h. $9\frac{3}{7} \div 12\frac{4}{7}$

Chapter 4.6 Fractions and Percentages

Summary

- Fractions and percentages are two ways of representing parts of a whole.
- Percentages are fractions expressed out of 100.
- To convert a fraction to a percentage, multiply it by 100.
- To convert a percentage to a fraction, divide by 100 and simplify.
- Understanding the relationship between fractions and percentages is useful in financial calculations, discounts, and data interpretation.
- Percentages are widely used in comparing quantities, determining proportions, and analyzing statistical data.

Examples

- Convert $\frac{1}{2}$ to a percentage: $\frac{1}{2} \times 100 = 50\%$.
- Convert 25% to a fraction: $\frac{25}{100} = \frac{1}{4}$.
- Convert $\frac{3}{4}$ to a percentage: $\frac{3}{4} \times 100 = 75\%$.

Word Problems

Intermediate Questions

1. Convert $\frac{3}{5}$ to a percentage.

2. Express 50% as a fraction.

3. Convert $\frac{4}{10}$ to a percentage.

4. Express 20% as a simplified fraction.

5. Convert $\frac{1}{4}$ to a percentage.

6. What percentage is equivalent to $\frac{7}{10}$?

7. Express 60% as a fraction.

8. Convert $\frac{2}{5}$ to a percentage.

9. What fraction is equivalent to 40%?

10. Convert $\frac{8}{20}$ to a percentage.

Hard Questions

1. Convert $\frac{7}{8}$ to a percentage.

2. Express 75% as a simplified fraction.

3. Convert $\frac{9}{25}$ to a percentage.

4. Express 12% as a simplified fraction.

5. Convert $\frac{5}{8}$ to a percentage.

6. Express 80% as a simplified fraction.

7. Convert $\frac{11}{20}$ to a percentage.

8. Express 36% as a simplified fraction.

9. Convert $\frac{13}{50}$ to a percentage.

10. Explain how to convert $\frac{7}{10}$ into a percentage.

Practice Questions

1. Express these percentages as mixed numbers in their simplest form.

a. 120%

b. 180%

c. 237%

d. 401%

e. 175%

f. 110%

g. 316%

h. 840%

2. Convert these fractions to percentages, using equivalent fractions.

a. $\frac{8}{100}$

b. $\frac{7}{20}$

c. $\frac{56}{50}$

d. $\frac{97}{100}$

e. $\frac{43}{50}$

e. $\frac{20}{5}$

3. Write each of the following percentages as fractions.

a. $2\frac{1}{2}\%$

b. $8\frac{1}{4}\%$

c. $12\frac{1}{2}\%$

d. $33\frac{1}{3}\%$

Chapter 4.7 Percentage of a Number

Summary

- Finding the percentage of a number involves multiplying the number by the percentage and dividing by 100.
- This calculation is useful in various real-life scenarios such as calculating discounts, interest, and statistics.
- To find a percentage of a number, convert the percentage into a decimal and multiply it by the number.
- Understanding how to calculate percentages helps in budgeting, shopping, and making financial decisions.

Examples

- Calculate 20% of 50: $(20/100) \times 50 = 10$.
- Find 15% of 200: $(15/100) \times 200 = 30$.
- What is 10% of $120?$ $(10/100) \times 120 = 12$.

Word Problems

Intermediate Questions

1. What is 25% of 80?

2. Calculate 10% of 150.

3. Find 50% of 64.

4. What is 30% of 90?

5. Calculate 20% of 75.

6. What is 15% of 60?

7. Find 40% of 200.

8. Calculate 5% of 300.

9. What is 70% of 20?

10. Find 80% of 50.

Hard Questions

1. Calculate 35% of 180.

2. Find 12.5% of 400.

3. What is 22% of 150?

4. Calculate 17% of 250.

5. Find 60% of 45.

6. What is 33.3% of 300?

7. Calculate 45% of 120.

8. Find 75% of 88.

9. What is 27% of 90?

10. Explain how you calculate 40% of 220.

Practice Questions

1. Find:

a. 50% of 140

b. 10% of 360

c. 25% of 40

d. 25% of 28

e. 5% of 80

f. 4% of 1200

g. 11% of 200

h. 21% of 400

2. Find.

a. 110% of 60

b. 400% of 25

c. 146% of 50

d. 3000% of 20

3. Find:

a. 30% of $\$140$

b. 10% of 240 $millimetres$

c. 15% of 60 $kilograms$

d. 2% of 4500 $tonnes$

e. 20% of 40 $minutes$

f. 80% of 500 $centimetres$

g. 5% of 30 $grams$

h. 25% of 12 $hectares$

i. 120% of 120 $seconds$

Chapter 4.8 Expressing a Quantity as a Proportion

Summary

- A proportion expresses the relationship between two quantities and is often written as a fraction or ratio.
- Expressing quantities as proportions helps to compare quantities relative to each other.
- To express a quantity as a proportion, divide the part by the whole and simplify the fraction.
- Proportions are used in various scenarios, including recipes, scale models, maps, and statistics.
- Understanding proportions is essential in solving problems involving similar figures, mixing ingredients, and comparing statistical data.

Examples

- Express 20 out of 50 as a proportion: $\frac{20}{50} = \frac{2}{5}$.
- In a class of 30 students, if 15 are girls, the proportion of girls is $\frac{15}{30}$, simplified to $\frac{1}{2}$.
- If you have 8 apples out of a total of 40 fruits, the proportion is $\frac{8}{40} = \frac{1}{5}$.

Word Problems

Intermediate Questions

1. Express 15 out of 45 as a proportion.

2. If 10 out of 25 students passed a test, what is the proportion of students who passed?

3. Express 12 out of 36 as a simplified proportion.

4. What proportion is 9 out of 18?

5. Express 20 out of 100 as a proportion.

6. In a bag with 50 candies, if 10 are chocolates, express the proportion of chocolates.

7. Express 6 out of 30 as a proportion.

8. What proportion is 25 out of 75?

9. Express 4 out of 16 as a simplified proportion.

10. Find the proportion of red balls if there are 3 red balls out of 15 total balls.

Hard Questions

1. Express 45 out of 180 as a proportion.

2. In a class of 40 students, 12 play football. What is the proportion of football players?

3. If 28 out of 70 people chose pizza, express the proportion.

4. Express 16 out of 64 as a proportion.

5. What is the proportion if 7 out of 49 candies are blue?

6. Express 18 out of 60 as a proportion.

7. In a fruit basket, if 5 out of 20 fruits are bananas, find the proportion.

8. Express 35 out of 140 as a simplified proportion.

9. Find the proportion of students who scored above 80 if 24 out of 120 students achieved this.

10. Explain how to express 30 out of 120 as a simplified proportion.

Practice Questions

1. Express the following as both a fraction and a percentage of the total.

a. 30 out of a total of 100

b. 3 out of a total of 5

c. $10 out of a total of $50

d. $60 out of a total of $80

e. 2 kg out of a total of 40 kg

f. 14 g out of a total of 28 g

g. 3 L out of a total of 12 L

h. 30 mL out of a total of 200 mL

2. A jug of lemonade is made up of 2 parts of lemon juice to 18 parts of water.

a. Express the amount of lemon juice as a fraction of the total.

b. Express the amount of lemon juice as a percentage of the total.

3. An orchard of 80 apple trees is tested for diseases. 20 of the trees have blight disease, 16 have brown rot disease and some trees have both. A total of 48 trees have neither blight nor brown rot.

   a. What percentage of the trees has both diseases?

   b. What fraction of the trees has blight but does not have brown rot?

CHAPTER 5 ALGEBRA

Table of Contents

Chapter 5.1 Introduction to Algebra

Summary

- Algebra uses symbols and letters to represent numbers and express mathematical relationships.
- The symbols (usually letters) in algebra are called variables because their values can vary.
- Expressions are combinations of numbers, variables, and operations (addition, subtraction, multiplication, division).
- An algebraic equation has an equals sign and states that two expressions are equal.
- Algebra helps in finding unknown values and solving problems logically and systematically.
- Algebraic expressions can be used to describe real-world situations like total costs, distance traveled, and scientific formulas.

Examples

- $x + 5 = 9$ (Solving for x gives $x = 4$).
- $3a + 4$ represents 'three times a number plus four'.
- The expression $2x - 3$ represents twice a number minus three.

Word Problems

Intermediate Questions

1. What is the value of x if $x + 7 = 10$?

2. Express 'five more than twice a number' algebraically.

3. Simplify the expression $4y - y$.

4. If $x = 2$, what is the value of $3x + 4$?

5. Write an algebraic expression for 'seven less than a number'.

6. Evaluate $2a - 3$ when $a = 4$.

7. Simplify the expression $x + x + x$.

8. Write an algebraic expression for 'a number divided by five'.

9. Evaluate $\frac{x}{2}$ when $x = 8$.

10. What is the simplified form of $2y + 3y$?

Hard Questions

1. If $2x + 5 = 13$, what is the value of x?

2. Express 'four times a number decreased by two' algebraically.

3. Simplify the expression $5a + 3a - 2a$.

4. Evaluate $4x - 2$ when $x = 3$.

5. Write an algebraic expression for 'twice the sum of a number and three'.

6. If $3y - 4 = 8$, find the value of y.

7. Simplify the expression $7b - 3b + b$.

8. Express 'half a number plus six' algebraically.

9. Evaluate $x - 2$ when $x = 7$.

10. Simplify $4m + 6m - 3m$.

Practice Questions

1. Write an expression for each of the following without using the $\times$ or $\div$ symbols.

   a. 5 is added to x, then the result is doubled.

   b. a is tripled, then 4 is added.

   c. k is multiplied by 8, then 3 is subtracted.

   d. 3 is subtracted from k, then the result is multiplied by 8.

e. The sum of x and y is multiplied by 6.

f. x is multiplied by 7 and the result is halved.

g. p is halved and then 2 is added.

h. The product of x and y is subtracted from 12.

2. If x is a whole number between 10 and 99, classify each of these statements as true or false.

   a. x must be smaller than $2 \times x$.

   b. x must be smaller than $x + 2$.

   c. $x - 3$ must be greater than 10.

   d. $4 \times x$ must be an even number.

   e. $3 \times x$ must be an odd number.

Chapter 5.2 Substituting and Evaluating

Summary

- Substituting involves replacing variables in an algebraic expression with numerical values.
- Evaluating is the process of calculating the numerical value of an expression after substitution.
- To evaluate an algebraic expression, follow the order of operations (BODMAS/BIDMAS).
- Substituting and evaluating help in solving algebraic problems and checking the solutions of equations.
- These skills are essential in using algebra for real-world problem-solving and scientific calculations.

Examples

- Evaluate $2x + 3$ when $x = 4$: Substitute to get $2(4) + 3 = 8 + 3 = 11$.
- Evaluate $3a - b$ when $a = 2$ and $b = 1$: Substitute to get $3(2) - 1 = 6 - 1 = 5$.
- Evaluate $x^2 + 2x$ when $x = 3$: Substitute to get $3^2 + 2(3) = 9 + 6 = 15$.

Word Problems

Intermediate Questions

1. Evaluate $2x$ when $x = 5$.

2. Find the value of $3a - 2$ when $a = 4$.

3. Evaluate $4x + 2$ when $x = 3$.

4. Calculate y^2 when $y = 6$.

5. Evaluate $x - 3$ when $x = 7$.

6. Find the value of $2y + 5$ when $y = 2$.

7. Evaluate $5a$ when $a = 3$.

8. Calculate $4x^2$ when $x = 2$.

9. Evaluate $3y + 2y$ when $y = 4$.

10. Find the value of $\frac{x}{4}$ when $x = 20$.

Hard Questions

1. Evaluate $3x^2 - 2x$ when $x = 4$.

2. Find the value of $4a + 3b$ when $a = 2$ and $b = 3$.

3. Evaluate $5y - y^2$ when $y = 2$.

4. Calculate the value of $x^2 - 4x + 1$ when $x = 3$.

5. Evaluate $2(a^2 + b)$ when $a = 3$ and $b = 2$.

6. Find the value of $6x - x^2$ when $x = 5$.

7. Evaluate $4y^2 - 3y$ when $y = 2$.

8. Calculate the value of $3a^2 + 2a - 1$ when $a = 1$.

9. Evaluate $x^2 + 3x + 2$ when $x = 2$.

10. Find the value of $2(x + 3)$ when $x = 4$.

Practice Questions

1. If $x = 5$, evaluate each of the following.

a. $x + 3$

b. $14 - x$

c. $3x + 2 - x$

d. $2(x + 2) + x$

e. $\frac{20}{3} + 3$

f. $\frac{x+7}{4}$

g. $7x + 3(x - 1)$

h. $x + x(x + 1)$

i. $100 - 4(3 + 4x)$

2. Substitute $a = 2$ and $b = 3$ into each of these expressions and evaluate.

a. $2a + 4$

b. $a + b$

c. $5a - 2b$

d. $ab - 4 + b$

e. $100 - (10a + 10b)$

f. $\frac{ab}{3} + b$

3. Evaluate the expression $5x + 2y$ when:

a. $x = 3$ and $y = 6$

b. $x = 7$ and $y = 3$

c. $x = 2$ and $y = 0$

4. Evaluate each of the following, given that $a = 9, b = 3$ and $c = 5$.

a. $a^2 - 3^3$

b. $2b^2 + \frac{a}{3} - 2c$

c. $24 + \frac{2b^3}{6}$

d. $(2c)^2 - a^2$

Chapter 5.3 Equivalent Expressions

Summary

- Equivalent expressions are expressions that represent the same value, even though they look different.
- Expressions can be simplified or expanded to show they are equivalent.
- To verify if two expressions are equivalent, simplify both to their simplest form or substitute numbers to test equivalence.
- Understanding equivalent expressions helps in simplifying algebraic expressions and solving equations.

Examples

- The expressions $2(x + 3)$ and $2x + 6$ are equivalent because expanding the first gives the second.
- Expressions $3(x + 4)$ and $3x + 12$ are equivalent.
- Expressions $4(x - 2)$ and $4x - 8$ are equivalent.

Word Problems

Intermediate Questions

1. Are $2(x + 1)$ and $2x + 2$ equivalent?

2. Simplify to check equivalence: $3(x - 2)$ and $3x - 6$.

3. Check if $4(x + 5)$ and $4x + 20$ are equivalent.

4. Simplify $5(x + 3)$ to find an equivalent expression.

5. Are expressions $2(x - 3)$ and $2x - 6$ equivalent?

6. Check equivalence of $x + x + x$ and $3x$.

7. Simplify $2(2y + 3)$ to find its equivalent.

8. Are $3(2a + 1)$ and $6a + 3$ equivalent?

9. Expand $4(y - 1)$ to check equivalence with $4y - 4$.

10. Are $5(z + 2)$ and $5z + 10$ equivalent?

Hard Questions

1. Check if expressions $3(x - 2) + x$ and $4x - 6$ are equivalent.

2. Determine if $2(3y - 1)$ and $6y - 2$ are equivalent.

3. Simplify and check equivalence: $4(a + 2) + 2a$ and $6a + 8$.

4. Are expressions $5(x + 4) - 3x$ and $2x + 20$ equivalent?

5. Check equivalence of $3(2y + 4)$ and $6y + 12$.

6. Simplify expressions $2(x + 3) + 3x$ and $5x + 6$ to check equivalence.

7. Expand and simplify $3(a + 2) - a$ and check if equivalent to $2a + 6$.

8. Are expressions $4(2x - 1)$ and $8x - 4$ equivalent?

9. Check equivalence of $3(x + 2) + 2(x - 1)$ and $5x + 4$.

10. Simplify to verify if expressions $4(y + 3) - 2y$ and $2y + 12$ are equivalent.

Practice Questions

1. Match up the equivalent expressions below.

a. $3x + 2x$

A. $6 - 3x$

b. $4 - 3x + 2$

B. $2x + 4x + x$

c. $2x + 5 + x$

C. $5x$

d. $x + x - 5 + x$

D. $4 - x$

e. $7x$

E. $3x + 5$

f. $4 - 3x + 2x$

F. $3x - 5$

Chapter 5.4 Like Terms

Summary

- Like terms are algebraic terms that have the same variables raised to the same power.
- Only like terms can be combined by addition or subtraction.
- Combining like terms simplifies expressions, making them easier to work with.
- Terms that have different variables or exponents are called unlike terms and cannot be directly combined.

Examples

- $2x$ and $5x$ are like terms; $2x + 5x = 7x$.
- $3y^2$ and $-7y^2$ are like terms; $3y^2 - 7y^2 = -4y^2$.
- $4a$ and $4b$ are unlike terms and cannot be combined.

Word Problems

Intermediate Questions

1. Combine the like terms: $3x + 2x$.

2. Simplify the expression: $4y - y$.

3. Combine like terms: $5a + 2a + a$.

4. Simplify: $6b^2 - 2b^2$.

5. Combine $4x$ and $3x$.

6. Simplify $7y - 3y + y$.

7. Are $2x$ and $2y$ like terms?

8. Combine like terms: $3a^2 + 5a^2$.

9. Simplify: $6x - x + 2x$.

10. Combine like terms: $5m - 2m$.

Hard Questions

1. Simplify by combining like terms: $3x^2 - x^2 + 4x$.

2. Combine the like terms: $5y + 2y - 3y$.

3. Simplify the expression: $6a + 2b - 3a + b$.

4. Combine like terms in the expression: $7x - 3x + 2x$.

5. Simplify by combining like terms: $4y^2 + 3y^2 - 5y^2$.

6. Combine like terms: $8a - 4a + 2a$.

7. Simplify the expression: $3x + 5y - x + 2y$.

8. Combine like terms: $9b^2 - 4b^2 + b^2$.

9. Simplify by combining: $10x - 3x + 4x$.

10. Explain why $5x$ and $2y$ are not like terms.

Practice Questions

1. Simplify the following by collecting like terms.

a. $2a + a + 4b + b$

b. $5a + 2a + b + 8b$

c. $3x - 2x + 2y + 4y$

d. $4a + 2 + 3a$

e. $7 + 2b + 5b$

f. $3k - 2 + 3k$

g. $7f + 4 - 2f + 8$

h. $4a - 4 + 5b + b$

i. $3x + 7x + 3y - 4x + y$

j. $10a + 3 + 4b - 2a$

k. $4 + 10h - 3h$

l. $10x + 4x + 31y - y$

m. $10 + 7y - 3x + 5x + 2y$

n. $11a + 4 - 3a + 9$

o. $3b + 4b + c + 5b - c$

p. $7ab + 4 + 2ab$

q. $9xy + 2x - 3xy + 3x$

r. $2cd + 5dc - 3d + 2c$

s. $5uv + 12v + 4uv - 5v$

t. $7pR + 2p + 4Rp - R$

u. $7ab + 32 - ab + 4$

2. Simplify the following by collecting like terms.

a. $3xy + 4xy + 5xy$

b. $4ab + 5 + 2ab$

c. $5ab + 3ba + 2ab$

d. $10xy - 4yx + 3$

e. $10 - 3xy + 8xy + 4$

f. $3cde + 5ecd + 2ced$

g. $4 + x + 4xy + 2xy + 5x$

h. $12ab + 7 - 3ab + 2$

i. $3xy - 2y + 4yx$

Chapter 5.5 Division

Summary

- When multiplying algebraic expressions, multiply the coefficients (numbers) and add the exponents of like variables.
- When dividing algebraic expressions, divide the coefficients and subtract the exponents of like variables.
- Multiplying and dividing expressions simplify complex algebraic expressions and are fundamental in solving algebraic equations.
- Remember to simplify the final expression whenever possible.

Examples

- Multiply $2x$ by $3x$: $2 \times 3 = 6$ and $x \times x = x^2$, giving $6x^2$.
- Divide $6x^2$ by $2x$: $6 \div 2 = 3$ and $x^2 \div x = x$, giving $3x$.
- Multiply $4y$ by $5y^2$: $4 \times 5 = 20$ and $y \times y^2 = y^3$, giving $20y^3$.

Word Problems

Intermediate Questions

1. Multiply $3x$ by $2x$.

2. Divide $8y^2$ by $4y$.

3. Simplify $5a \times 4a$.

4. Divide $10x^2$ by $5x$.

5. Multiply $2m$ by $3m^2$.

6. Simplify $6b \times 2b$.

7. Divide $9y^3$ by $3y^2$.

8. Multiply $7x$ by x.

9. Simplify $4y \times 5y$.

10. Divide $12a^2$ by $6a$.

Hard Questions

1. Multiply $2x^2$ by $3x^3$.

2. Divide $15y^3$ by $5y$.

3. Simplify $4a^2 \times 3a$.

4. Divide $20x^4$ by $4x^2$.

5. Multiply $5y$ by $3y^2$.

6. Divide $18b^3$ by $6b^2$.

7. Simplify $3x \times 7x^2$.

8. Divide $24a^4$ by $8a$.

9. Multiply $6y^2$ by $2y^3$.

10. Explain how to divide $16x^3$ by $4x$.

Practice Questions

1. Write each of these expressions without any multiplication signs.

a. $2 \times x$

b. $5 \times p$

c. $8 \times a \times b$

d. $7 \times 4 \times f$

e. $5 \times 2 \times a \times b$

f. $2 \times 8 \times x \times y$

g. $2 \times b \times 5$

h. $x \times 7 \times z \times 4$

2. Write each expression without a division sign.

a. $x \div 5$

b. $z \div 2$

c. $a \div 12$

d. $b \div 5$

e. $2 \div x$

f. $5 \div d$

g. $x \div y$

h. $a \div b$

i. $(4x + 1) \div 5$

j. $(2x + y) \div 5$

k. $(2 + x) \div (1 + y)$

l. $(x - 5) \div (3 + b)$

3. Simplify the following expressions by dividing by any common factors. Remember that $\frac{a}{1} = a$.

a. $\frac{2x}{5x}$

b. $\frac{5a}{9a}$

c. $\frac{9ab}{4b}$

d. $\frac{2ab}{5a}$

e. $\frac{2x}{4}$

f. $\frac{9x}{12}$

g. $\frac{10a}{15a}$

h. $\frac{30y}{40y}$

i. $\frac{4a}{2}$

j. $\frac{21x}{7x}$

k. $\frac{4xy}{2x}$

l. $\frac{9x}{3xy}$

Chapter 5.6 Expanding Brackets

Summary

- Expanding brackets involves multiplying terms inside the brackets by terms outside the brackets.
- It helps simplify algebraic expressions and solve equations.
- The distributive property is used to expand brackets, which states $a(b + c) = ab + ac$.
- Always simplify expressions fully after expanding.

Examples

- Expand $2(x + 3)$: $2 \times x + 2 \times 3 = 2x + 6$.
- Expand $3(a - 4)$: $3a - 12$.
- Expand $-4(y + 2)$: $-4y - 8$.

Word Problems

Intermediate Questions

1. Expand $3(x + 2)$.

2. Expand $2(y - 4)$.

3. Expand $4(a + 1)$.

4. Simplify by expanding: $5(b - 2)$.

5. Expand and simplify $3(x + 4)$.

6. Expand $2(a - 3)$.

7. Simplify $4(x - 1)$.

8. Expand $-3(y + 5)$.

9. Expand and simplify $5(a - 1)$.

10. Expand $6(m + 2)$.

Hard Questions

1. Expand $4(x + 2) - 3(x - 1)$.

2. Simplify by expanding brackets: $2(a + 3) + 3(a - 2)$.

3. Expand and simplify $3(x - 4) - 2(x + 1)$.

4. Expand and simplify $5(y + 2) + 4(y - 3)$.

5. Simplify by expanding: $3(a - 2) + 4(a + 1)$.

6. Expand $2(x - 3) - 5(x + 2)$.

7. Simplify: $4(y + 2) - 3(y - 1)$.

8. Expand and simplify: $2(a + 5) - (a + 3)$.

9. Expand and simplify: $6(x - 1) + 2(x + 4)$.

10. Explain how to expand and simplify: $3(y - 2) + 4(y + 1)$.

Practice Questions

1. Use the distributive law to expand the following.

a. $6(y + 8)$

b. $7(l + 4)$

c. $8(s + 7)$

d. $4(2 + a)$

e. $7(x + 5)$

f. $3(6 + a)$

g. $9(9 - x)$

h. $5(K - 4)$

i. $8(y - 8)$

j. $8(e - 7)$

k. $6(e - 3)$

l. $10(8 - y)$

2. Use the distributive law to expand the following.

a. $10(6g - 7)$

b. $5(3e + 8)$

c. $5(7w + 10)$

d. $5(2u + 5)$

e. $7(8x - 2)$

f. $3(9v - 4)$

g. $7(R - 7)$

h. $4(5c - v)$

i. $2(2u + 6)$

j. $6(8l + 8)$

k. $5(k - 10)$

l. $9(o + 7)$

3. Use the distributive law to expand the following.

a. $6i(t - v)$

b. $2d(v + m)$

c. $5c(2w - t)$

d. $6e(s + p)$

e. $d(x + 9s)$

f. $5a(2x + 3v)$

g. $5K(r + 7p)$

h. $i(n + 4w)$

i. $8d(s - 3t)$

j. $f(2u + v)$

k. $7k(2v + 5y)$

l. $4e(m + 10y)$

Chapter 5.7 Algebraic Modelling Investigation

Summary

- Algebraic modelling uses algebra to represent real-world situations mathematically.
- Models help predict, analyze, and solve real-life problems.
- Creating algebraic models involves defining variables, writing equations, and solving these equations to find answers.
- Real-world examples include financial forecasting, population growth, scientific predictions, and geometry problems.

Examples

- If a taxi charges \$2 plus \$0.50 per kilometer, the algebraic model is: Cost = $2 + 0.5x$, where x is kilometers traveled.
- A rectangle's perimeter model: $P = 2(l + w)$, where l and w represent length and width respectively.
- If a mobile phone company charges a \$20 fixed fee plus \$0.10 per minute, the algebraic model is: Cost = $20 + 0.1m$, where m is the number of minutes.

Word Problems

Intermediate Questions

1. Write an algebraic model for total cost if an entry ticket is \$5 plus \$2 per ride.

2. Create an algebraic model for a taxi fare that charges \$3 plus \$1 per kilometer.

3. Write a model for the perimeter of a square with side length x.

4. Create an algebraic model for total income if someone earns \$50 per day plus \$10 per sale.

5. Write a model for cost if pizza is \$8 plus \$1 per extra topping.

6. Create an algebraic model for the distance traveled at $60\ \mathrm{km/h}$ for x hours.

7. Write a model for a rectangle's area given length l and width w.

8. Create an algebraic model for a salary of $\$200$ plus $\$15$ per hour worked.

9. Write a model for total payment if membership costs $\$100$ annually plus $\$5$ per month.

10. Create a model for the cost if hiring equipment costs $\$10$ plus $\$2$ per hour

Hard Questions

1. Write an algebraic model for profit if revenue is $\$50$ per item minus $\$100$ fixed cost.

2. Create a model for total distance if you travel at $80\ \mathrm{km/h}$ for x hours.

3. Develop an algebraic model for total savings if saving $\$20$ weekly plus $\$5$ extra per month.

4. Write a model for the volume of a box with length l, width w, and height h.

5. Create a model for expenses if monthly bills total $\$200$ plus $\$3$ per day for food.

6. Develop an algebraic model for the cost of electricity if a company charges $\$30$ monthly plus $\$0.15$ per kilowatt-hour used.

7. Write a model for the area of a triangle with base b and height h.

8. Create an algebraic model for earnings if paid $\$12$ per hour plus $\$50$ weekly bonus.

9. Develop a model for the total cost of a rental car charging $\$40$ per day plus $\$0.25$ per kilometer.

10. Explain how to create an algebraic model for monthly phone bills charging a fixed fee of $\$25$ plus $\$0.10$ per call.

Practice Questions

1. In a closing-down sale, a shop sells all CDs for $\$c$ each, books cost $\$b$ each and DVDs cost $\$d$ each. Claudia buys 5 books, 2 CDs and 6 DVDs.

   a. What is the cost of Claudia's order? Give your answer as an expression involving b, c and d.

   b. Write an expression for the cost of Claudia's order if CDs doubled in price and DVDs halved in price.

   c. As it happens, the total price Claudia ends up paying is the same in both situations. Given that CDs cost \$12 and books cost \$20 (so $c = 12$ and $b = 20$), how much do DVDs cost?

2. A shop charges $\$c$ for a box of tissues.

   a. Write an expression for the total cost, in dollars, of buying n boxes of tissues.

   b. If the original price is tripled, write an expression for the total cost, in dollars, of buying n boxes of tissues.

   c. If the original price is tripled and twice as many boxes are bought, write an expression for the total cost in dollars.

3. Hiring a basketball court costs $\$10$ for a booking fee, plus $\$30$ per hour.

a. Write an expression for the total cost in dollars to hire the court for x hours.

b. For the cost of $\$40$, you could hire the court for 1 hour. How long could you hire the court for the cost of $\$80$?

c. Explain why it is ***not*** the case that hiring the court for twice as long costs twice as much.

d. Find the average cost per hour if the court is hired for a 5 hour basketball tournament.

e. Describe what would happen to the ***average*** cost per hour if the court is hired for many hours (e.g. more than 50 hours).

CHAPTER 6 DECIMALS

Table of Contents

Chapter 6.1 Decimals and Place Value

Summary

- Decimals represent fractions or parts of a whole number, using a decimal point.
- Place value indicates the position of a digit and its value within the decimal.
- Decimal places from left to right after the decimal point are tenths, hundredths, thousandths, etc.
- Understanding decimal place value is essential for reading, writing, and comparing decimal numbers accurately.

Examples

- In 3.47, '4' is in the tenths place and '7' is in the hundredths place.
- The number 0.528 has 5 in the tenths, 2 in the hundredths, and 8 in the thousandths place.
- Write 4.206 with correct place values: 4 units, 2 tenths, 0 hundredths, and 6 thousandths.

Word Problems

Intermediate Questions

1. Identify the digit in the tenths place: 3.64.

2. What is the place value of 7 in 5.78?

3. Express 0.25 in words.

4. Which digit is in the hundredths place in 8.219?

5. Write the decimal for 4 units, 3 tenths, and 5 hundredths.

6. Identify the thousandths digit in 0.607.

7. Express the number 0.04 in words.

8. What place value is the digit 9 in 2.391?

9. Write the decimal number for 7 units, 2 tenths, and 9 thousandths.

10. Identify the place value of the digit 6 in 0.346.

Hard Questions

1. What is the value of the digit 5 in 4.758?

2. Express the decimal 6.045 in expanded form.

3. Identify the digit in the thousandths place in 9.367.

4. Write in decimal form: two units, six hundredths, and eight thousandths.

5. Express 0.306 in words.

6. Identify the hundredths digit in 12.804.

7. Write the decimal number for three units, zero tenths, four hundredths, and five thousandths.

8. Express the number 15.023 in expanded form.

9. What is the place value of the digit 1 in 0.017?

10. Explain the place value of each digit in 7.491.

Practice Questions

1. What is the value of the digit 6 in the following numbers?

a. 23.612

b. 17.46

c. 80.016

d. 0.693

e. 16.4

f. $8.568\ 13$

g. 2.3641

h. 11.926

2. Express each of the following proper fractions as a decimal.

a. $\frac{3}{10}$

b. $\frac{8}{10}$

c. $\frac{15}{100}$

d. $\frac{23}{100}$

e. $\frac{9}{10}$

f. $\frac{2}{100}$

g. $\frac{121}{1000}$

h. $\frac{74}{1000}$

3. Express each of the following mixed numerals as a decimal.

a. $6\frac{4}{10}$

b. $5\frac{7}{10}$

c. $212\frac{3}{10}$

d. $1\frac{16}{100}$

e. $14\frac{83}{100}$

f. $7\frac{51}{100}$

g. $5\frac{7}{100}$

h. $18\frac{612}{1000}$

Chapter 6.2 Rounding Decimals

Summary

- Rounding decimals means approximating a decimal number to a certain number of decimal places.
- To round decimals, look at the digit immediately after the place you are rounding to.
- If this digit is 5 or more, round up the digit before it; if it is 4 or less, leave it as it is.
- Rounding decimals is useful for estimating and simplifying calculations in everyday life and practical scenarios.

Examples

- Round 3.456 to two decimal places: 3.46.
- Round 5.243 to one decimal place: 5.2.
- Round 2.998 to the nearest whole number: 3.

Word Problems

Intermediate Questions

1. Round 4.36 to one decimal place.

2. Round 5.678 to two decimal places.

3. Round 2.35 to one decimal place.

4. Round 8.445 to two decimal places.

5. Round 3.94 to the nearest whole number.

6. Round 7.803 to one decimal place.

7. Round 9.667 to two decimal places.

8. Round 0.435 to one decimal place.

9. Round 6.99 to the nearest whole number.

10. Round 2.781 to two decimal places.

Hard Questions

1. Round 12.3456 to three decimal places.

2. Round 0.98765 to four decimal places.

3. Round 7.4999 to the nearest whole number.

4. Round 8.0567 to two decimal places.

5. Round 3.14159 to three decimal places.

6. Round 0.0045 to three decimal places.

7. Round 15.2389 to two decimal places.

8. Round 6.555 to two decimal places.

9. Explain how to round 4.5678 to one decimal place.

10. Round 9.995 to the nearest whole number.

Practice Questions

1. Round each of the following to 1 decimal place.

a. 14.82

b. 7.38

c. 15.62

d. 0.87

e. 6.85

f. 9.94

g. 55.55

h. 7.98

2. Round each of the following to the specified number of decimal places, given as the number in the brackets.

a. $15.913\ (1)$

b. $7.8923\ (2)$

c. $235.62\ (0)$

d. $0.5111\ (0)$

e. $231.86\ (1)$

f. $9.3951\ (1)$

g. $9.3951\ (2)$

h. $34.712\ 89\ (3)$

Chapter 6.3 Addition and Subtraction of Decimals

Summary

- To add or subtract decimals, align the decimal points vertically before calculating.
- Place values (tenths, hundredths, thousandths) must be matched correctly.
- If necessary, add zeros to ensure all decimals have the same number of decimal places before adding or subtracting.
- Addition and subtraction of decimals are important for financial calculations, measurements, and scientific data.

Examples

- Add $2.34 + 1.2$ by aligning decimal points: $2.34 + 1.20 = 3.54$.
- Subtract $5.6 - 2.35$: $5.60 - 2.35 = 3.25$.
- Add $4.056 + 3.9$: $4.056 + 3.900 = 7.956$.

Word Problems

Intermediate Questions

1. Add 2.5 and 3.45.

2. Subtract 4.8 from 9.3.

3. Add 1.25 and 0.75.

4. Subtract 3.67 from 5.8.

5. Calculate $4.56 + 2.3$.

6. Subtract 1.005 from 2.01.

7. Add $6.7 + 2.89$.

8. Calculate $3.4 - 1.67$.

9. Find the sum of 2.45 and 3.555.

10. Subtract 0.89 from 4.23.

Hard Questions

1. Calculate $7.456 + 3.004$.

2. Subtract 2.889 from 6.78.

3. Find the sum of 0.345 and 0.567.

4. Calculate $10.02 - 8.567$.

5. Add $4.5689 + 3.023$.

6. Subtract 3.999 from 5.5.

7. Calculate $7.123 - 2.456$.

8. Find the sum of 5.01 and 3.9.

9. Explain how to add 4.56 and 2.4 correctly.

10. Calculate $9.345 - 1.234$.

Practice Questions

1. Find each of the following.

a. $12.45 + 3.61$

b. $5.37 + 13.81 + 2.15$

c. $5.37 + 13.81 + 2.15$

d. $1.567 + 3.4 + 32.6$

e. $5.882 + 3.01 + 12.7$

f. $323.71 + 3.4506 + 12.9$

2. Find:

a. $14.8 - 2.5$

b. $234.6 - 103.2$

c. $25.9 - 3.67$

d. $31.657 - 18.2$

e. $412.1 - 368.83$

f. $5312.271 - 364.93$

3. Find the missing numbers in the following sums.

a.
$$\begin{array}{r} 3.\square \\ +\ 4.6 \\ \hline \square.3 \end{array}$$

b.
$$\begin{array}{r} 8.\square 9 \\ +\ \square.75 \\ \hline \square 4.4\square \end{array}$$

c.
$$\begin{array}{r} 1.\ \ 1 \\ +\ \square\square.11 \\ \hline 11.1\square \end{array}$$

d.
$$\begin{array}{r} \square.3\square 6 \\ 2.\square 43 \\ +\ 1.89\square \\ \hline \square 1.395 \end{array}$$

Chapter 6.4 Multiplying and Dividing by Powers of 10

Summary

- Multiplying by powers of 10 moves the decimal point to the right; dividing moves it to the left.
- For multiplication by 10, 100, 1000, etc., move the decimal point right by 1, 2, or 3 places respectively.
- For division by 10, 100, 1000, etc., move the decimal point left by 1, 2, or 3 places respectively.
- This technique simplifies calculations involving decimals, making mental arithmetic easier.

Examples

- Multiply 4.32 by 100: move the decimal point 2 places right → 432.
- Divide 56.8 by 10: move the decimal point 1 place left → 5.68.
- Multiply 0.789 by 1000: move the decimal point 3 places right → 789.

Word Problems

Intermediate Questions

1. Multiply 2.56 by 10.

2. Divide 43.2 by 100.

3. Multiply 0.07 by 1000.

4. Divide 5.9 by 10.

5. Multiply 3.14 by 100.

6. Divide 67.8 by 10.

7. Multiply 0.05 by 100.

8. Divide 123 by 1000.

9. Multiply 9.8 by 10.

10. Divide 2.3 by 100.

Hard Questions

1. Multiply 0.1234 by 1000.

2. Divide 567.89 by 100.

3. Multiply 8.076 by 100.

4. Divide 34.56 by 10.

5. Multiply 0.0345 by 10,000.

6. Divide 780.2 by 1000.

7. Calculate 0.005 × 1000.

8. Divide 1234.5 by 100.

9. Explain how to multiply 4.567 by 100.

10. Calculate 2.003 × 100.

Practice Questions

1. Calculate:

a. 4.87×10

b. 35.283×10

c. 14.304×100

d. $5.699\,23 \times 1000$

e. 12.7×1000

f. 154.23×1000

g. 213.2×10

h. $867.1 \times 100\,000$

2. Calculate:

a. $353.1 \div 10$

b. $24.422 \div 10$

c. $12\ 135.18 \div 1000$

d. $93\ 261.1 \div 10\ 000$

e. $13.62 \div 10\ 000$

f. $0.54 \div 1000$

g. $0.02 \div 10\ 000$

h. $1000.04 \div 100\ 000$

3. Calculate:

a. 22.913×100

b. $0.031\,67 \times 1000$

c. $22.2 \div 100$

d. $6348.9 \times 10\,000$

4. Calculate:

a. $2251 \div 10$

b. $2134 \div 100$

c. $34 \div 10\,000$

5. Calculate the following, using the order of operations.

a. $1.56 \times 100 + 24 \div 10$

b. $3 + 10(24 \div 100 + 8)$

c. $35.4 + 4.2 \times 10 - 63.4 \div 10$

d. $14 \div 100 + 1897 \div 1000$

Chapter 6.5 Multiplication of Decimals

Summary

- Multiplying decimals involves first multiplying the numbers as if they were whole numbers.
- Count the total number of decimal places in both decimals combined.
- Place the decimal point in the answer so that it has the same total number of decimal places.
- Multiplication of decimals is important in calculations involving money, measurements, and percentages.

Examples

- Multiply 0.4 × 0.2: 4 × 2 = 8; two decimal places → 0.08.
- Multiply 1.2 × 0.3: 12 × 3 = 36; two decimal places → 0.36.
- Multiply 0.05 × 0.2: 5 × 2 = 10; three decimal places → 0.010.

Word Problems

Intermediate Questions

1. Calculate 0.3×0.5.

2. Multiply 2.5 by 0.2.

3. Find the product of 0.7 and 0.4.

4. Multiply 1.1 by 0.3.

5. Calculate 0.05×2.

6. Multiply 0.6 by 0.1.

7. Find the product of 1.2 and 0.2.

8. Calculate 0.4×0.4.

9. Multiply 0.8 by 0.5.

10. Find the product of 0.03 and 3.

Hard Questions

1. Calculate 1.23×0.04.

2. Multiply 0.56 by 0.78.

3. Find the product of 3.45 and 0.2.

4. Multiply 0.89 by 1.3.

5. Calculate 2.34×0.07.

6. Multiply 0.005 by 3.2.

7. Find the product of 0.067 and 0.4.

8. Calculate 0.45×0.45.

9. Explain how to multiply 2.4 by 0.15.

10. Multiply 0.32 by 0.25.

Practice Questions

1. Calculate:

a. 5.21×4

b. 3.8×7

c. 22.93×8

d. 14×7.2

e. 3×72.82

f. 1.293×12

g. 3.4×6.8

h. 5.4×2.3

i. 0.34×16

j. 43.21×7.2

k. 0.023×0.042

l. 18.61×0.071

2. Calculate:

a. 31.75×800

b. $7.291 \times 50\,000$

c. 3.004×30

3. Calculate and then round your answer to the nearest dollar.

a. $3 \times \$7.55$

b. $4 \times \$18.70$

c. $\$30.25 \times 4.8$

d. $7.2 \times \$5200$

e. $0.063 \times \$70.00$

f. $0.085 \times \$212.50$

Chapter 6.6 Division of Decimals

Summary

- To divide decimals, if the divisor is a whole number, divide as usual and place the decimal point directly above in the quotient.
- If the divisor is a decimal, multiply both the dividend and divisor by 10, 100, etc. to make the divisor a whole number.
- Then perform the division as with whole numbers, placing the decimal in the correct position.
- Decimal division is used in real-world contexts such as dividing money, distances, and measurements.

Examples

- Divide $3.6 \div 2 = 1.8$.
- Divide $4.5 \div 0.5$ → Multiply both by 10: $45 \div 5 = 9$.
- Divide $0.84 \div 0.2$ → Multiply both by 10: $8.4 \div 2 = 4.2$.

Word Problems

Intermediate Questions

1. Divide 4.2 by 2.

2. Calculate $6.4 \div 4$.

3. Divide 3.6 by 0.3.

4. Divide 8.1 by 3.

5. Calculate $7.5 \div 2.5$.

6. Divide 0.6 by 0.2.

7. Calculate $9.0 \div 3$.

8. Divide 1.2 by 0.4.

9. Calculate $2.5 \div 5$.

10. Divide 0.7 by 0.1.

Hard Questions

1. Divide 5.46 by 0.3.

2. Calculate $8.64 \div 0.12$.

3. Divide 12.35 by 5.

4. Divide 0.625 by 0.25.

5. Calculate $4.8 \div 0.06$.

6. Divide 7.92 by 0.04.

7. Find the quotient: $1.44 \div 0.12$.

8. Divide 3.75 by 0.5.

9. Calculate $0.81 \div 0.09$.

10. Explain how to divide 2.45 by 0.5.

Practice Questions

1. Calculate:

a. $30.5 \div 5$

b. $64.02 \div 3$

c. $4.713 \div 3$

d. $2.156 \div 7$

e. $1491.6 \div 12$

f. $0.0144 \div 6$

g. $3.417 \div 5$

h. $0.010\,25 \div 4$

2. Calculate:

a. $6.14 \div 0.2$

b. $23.25 \div 0.3$

c. $5.1 \div 6$

d. $0.3996 \div 0.009$

e. $0.0032 \div 0.04$

f. $0.04034 \div 0.8$

g. $4.003 \div 0.005$

h. $0.948 \div 1.2$

3. Calculate:

a. $236.14 \div 200$

b. $413.35 \div 50$

c. $3.712\ 44 \div 300$

d. $0.846 \div 200$

e. $482.435 \div 5000$

f. $0.0313 \div 40$

Chapter 6.7 Decimals and Fractions

Summary

- Decimals and fractions both represent parts of a whole and can be converted into one another.
- To convert a decimal to a fraction, write it over a power of ten and simplify.
- To convert a fraction to a decimal, divide the numerator by the denominator.
- Understanding how to convert between decimals and fractions is essential in many areas of maths and real-world contexts.

Examples

- Convert 0.75 to a fraction: $\frac{75}{100} = \frac{3}{4}$.
- Convert $\frac{1}{2}$ to a decimal: $1 \div 2 = 0.5$.
- Convert 0.2 to a fraction: $\frac{2}{10} = \frac{1}{5}$.

Word Problems

Intermediate Questions

1. Convert 0.4 to a fraction.

2. Write $\frac{3}{4}$ as a decimal.

3. Convert 0.2 to a fraction.

4. Express $\frac{1}{5}$ as a decimal.

5. Convert 0.8 to a fraction.

6. Write $\frac{2}{5}$ as a decimal.

7. Convert 0.25 to a fraction.

8. Write $\frac{1}{4}$ as a decimal.

9. Convert 0.6 to a fraction.

10. Write $\frac{3}{10}$ as a decimal.

Hard Questions

1. Convert 0.375 to a fraction in simplest form.

2. Write $\frac{5}{8}$ as a decimal.

3. Convert 0.125 to a fraction.

4. Write $\frac{7}{16}$ as a decimal.

5. Convert 0.05 to a fraction.

6. Write $\frac{11}{20}$ as a decimal.

7. Convert 0.625 to a fraction.

8. Write $\frac{13}{40}$ as a decimal.

9. Explain how to convert 0.66 to a fraction.

10. Convert 0.875 to a fraction.

Practice Questions

1. Convert each of these fractions to decimals.

a. $\frac{7}{10}$

b. $\frac{9}{10}$

c. $\frac{31}{100}$

d. $\frac{79}{100}$

e. $\frac{121}{100}$

f. $3\frac{29}{100}$

g. $\frac{123}{100}$

h. $\frac{3}{100}$

2. Convert the following fractions to decimals, by dividing the numerator by the denominator.

a. $\frac{1}{2}$

b. $\frac{3}{6}$

c. $\frac{3}{4}$

d. $\frac{2}{5}$

e. $\frac{1}{3}$

f. $\frac{3}{8}$

g. $\frac{5}{12}$

h. $\frac{3}{7}$

Chapter 6.8 Decimals and Percentages

Summary

- Decimals and percentages are related: a percentage is a decimal multiplied by 100.
- To convert a decimal to a percentage, multiply by 100 and add the $\%$ symbol.
- To convert a percentage to a decimal, divide by 100.
- Understanding this relationship helps in interpreting data and solving real-world problems involving proportions.

Examples

- Convert 0.75 to a percentage: $0.75 \times 100 = 75\%$.
- Convert 25% to a decimal: $25 \div 100 = 0.25$.
- Convert 0.6 to a percentage: $0.6 \times 100 = 60\%$.

Word Problems

Intermediate Questions

1. Convert 0.5 to a percentage.

2. Convert 40% to a decimal.

3. Convert 0.25 to a percentage.

4. Write 80% as a decimal.

5. Convert 0.1 to a percentage.

6. Convert 10% to a decimal.

7. Convert 0.2 to a percentage.

8. Convert 75% to a decimal.

9. Convert 0.05 to a percentage.

10. Write 90% as a decimal.

Hard Questions

1. Convert 0.375 to a percentage.

2. Write 12.5% as a decimal.

3. Convert 0.005 to a percentage.

4. Convert 87.5% to a decimal.

5. Convert 0.625 to a percentage.

6. Write 66.6% as a decimal.

7. Convert 0.099 to a percentage.

8. Write 33.3% as a decimal.

9. Explain how to convert 0.48 to a percentage.

10. Convert 88.8% to a decimal.

Practice Questions

1. Express the following percentages as decimals.

a. 32%

b. 27%

c. 68%

d. 54%

e. 6%

f. 142%

g. 100%

h. 1%

i. 218%

j. 142%

k. 75%

l. 199%

2. Express the following decimals as percentages.

a. 0.8

b. 0.3

c. 0.45

d. 0.71

e. 0.416

f. 0.375

g. 2.5

h. 2.314

i. 0.025

j. 0.0014

k. 12.7

l. 1.004

3. Place the following values in order from highest to lowest.

a. $86\%, 0.5\%, 0.6, 0.125, 22\%, 75\%, 2\%, 0.78$

b. $124\%, 2.45, 1.99\%, 0.02\%, 1.8, 55\%, 7.2, 50$

CHAPTER 7 NEGATIVE NUMBERS

Table of Contents

Chapter 7.1 Integers

Summary

- Integers include all whole numbers and their negative counterparts (e.g. -3, -2, -1, 0, 1, 2, 3...).
- Zero is an integer and it is neither positive nor negative.
- Integers are used to describe situations involving gains and losses, temperature changes, elevation, and more.
- They can be represented on a number line, with positive integers to the right of 0 and negative integers to the left.

Examples

- Examples of integers: -5, -2, 0, 4, 10.
- On a number line, -3 is to the left of 2, so -3 < 2.
- In temperature, -2°C is colder than 3°C.

Word Problems

Intermediate Questions

1. List 5 examples of integers.

2. Identify the smallest integer among: -3, 0, 2, -5.

3. Place the following integers in order: -2, 4, 0, -1, 3.

4. What is the opposite of 7?

5. What integer lies between -2 and 0?

6. Compare: Which is greater, -1 or -4?

7. Represent -3 on a number line.

8. Is 0 a positive or negative integer?

9. What is the absolute value of -8?

10. Write all integers between -3 and 3.

Hard Questions

1. Compare and order: -7, -2, 0, 5, -4.

2. Explain the meaning of absolute value with two examples.

3. How would integers be used to represent sea level and mountain elevation?

4. Identify all integers between -10 and -5.

5. Which is further from zero: -9 or 6?

6. Write a real-world situation involving both positive and negative integers.

7. Graph the integers -3, 0, and 2 on a number line.

8. Which is greater: the absolute value of -12 or 7?

9. Explain why -1 is greater than -3.

10. Write a word problem involving temperature rise and fall using integers.

Practice Questions

1. Insert the symbol $<$ (less than) or $>$ (greater than) into these statements to make them true.

a. $7 \square 9$

b. $3 \square 2$

c. $-1 \square -5$

d. $-7 \square -6$

e. $-3 \square 3$

f. $3 \square -3$

Chapter 7.2 Adding and Subtracting Positive Integers

Summary

- Adding positive integers increases the value (e.g. $3 + 4 = 7$).
- Subtracting positive integers means moving left on a number line (e.g. $7 - 3 = 4$).
- These operations are basic arithmetic skills needed for all levels of maths.
- Positive integers behave like whole numbers and are used frequently in everyday life.

Examples

• Add $5 + 6 = 11$.

• Subtract $10 - 4 = 6$.

• $7 + 2 = 9, 8 - 3 = 5$.

Word Problems

Intermediate Questions

1. Calculate: $3 + 6$

2. Find the sum of 7 and 4.

3. Subtract: $10 - 3$

4. What is $15 - 6$?

5. Add 8 and 2.

6. Subtract $12 - 5$.

7. Calculate $9 + 3$.

8. What is $11 - 7$?

9. Add: $13 + 6$

10. Subtract: $18 - 9$

Hard Questions

1. Solve: $25 + 13 - 8$

2. Evaluate: $50 - 25 + 10$

3. Add: $34 + 21$

4. Subtract: $89 - 47$

5. Add and subtract: $16 + 22 - 9$

6. Calculate: $72 - 19 + 4$

7. Find the result: $33 + 17 - 15$

8. Add: $56 + 44$

9. Subtract: $100 - 75$

10. Explain the process to solve: $40 + 22 - 18$

Practice Questions

1. Calculate the answer to these additions. Check your answers using a calculator.

a. $-3 + 5$

b. $-10 + 11$

c. $-11 + 9$

d. $-20 + 18$

e. $-30 + 29$

f. $-39 + 41$

g. $-57 + 63$

h. $-99 + 68$

2. Calculate the answer to these subtractions. Check your answers using a calculator.

a. $4-6$

b. $7-8$

c. $-3-1$

d. $-5-5$

e. $-37-4$

f. $39-51$

g. $-100-200$

h. $100-200$

3. Find the missing number.

a. $-2+\square=3$

b. $-4+\square=-2$

c. $-9-\square=-12$

d. $-20-\square=30$

e. $\square+1=-3$

f. $\square+7=2$

g. $\square+6=-24$

h. $\square-100=-213$

4. For an experiment, a chemical solution starts at a temperature of $25°C$, falls to – $3°C$, rises to $15°C$ and then falls again to – $8°C$. What is the total change in temperature? Add all the changes together for each rise and fall.

5. An ocean sensor is raised and lowered to different depths in the sea. Note that –100 m means 100 m below sea level.

   a. If the sensor is initially at –100 m and then raised to –41 m, how far does the sensor rise?

   b. If the sensor is initially at –37 m and then lowered to –93 m, how far is the sensor lowered?

Chapter 7.3 Adding and Subtracting Negative Integers

Summary

- Solve: $25 + 13 - 8$
- Evaluate: $50 - 25 + 10$
- Add: $34 + 21$
- Subtract: $89 - 47$
- Add and subtract: $16 + 22 - 9$
- Calculate: $72 - 19 + 4$
- Find the result: $33 + 17 - 15$
- Add: $56 + 44$
- Subtract: $100 - 75$
- Explain the process to solve: $40 + 22 - 18$

Examples

- $7 + (-4) = 3.$
- $6 - (-2) = 8.$
- $-3 + (-5) = -8.$

Word Problems

Intermediate Questions

1. Calculate: $-2 + 5$

2. Find: $3 + (-6)$

3. Evaluate: $-4 + (-7)$

4. What is: $-5 - (-3)$?

5. Calculate: $6 - (-4)$

6. Find the sum: $-8 + 3$

7. Calculate: $-7 + (-2)$

8. Evaluate: $-1 - (-6)$

9. What is: $-3 - 4$?

10. Calculate: $-10 + 8$

Hard Questions

1. Solve: $-5 + (-3) - (-2)$

2. Evaluate: $10 - (-4) + (-6)$

3. Calculate: $-12 + 15 - 3$

4. Solve: $-7 - (-5) + (-2)$

5. Evaluate: $-6 - 9 + 10$

6. Find the result: $-15 + 7 - (-8)$

7. Add and subtract: $-11 + 4 + (-9)$

8. Solve: $6 - 14 + (-2)$

9. Calculate: $-4 - (-8) - 7$

10. Explain how to solve: $-5 + (-6) - (-2)$

Practice Questions

1. Calculate the answer to these additions. Check your answer using a calculator.

a. $8 + (-3)$

b. $9 + (-7)$

c. $6 + (-11)$

d. $0 + (-4)$

e. $-7 + (-15)$

f. $-28 + (-52)$

g. $-20 + (-9)$

h. $-103 + (-9)$

2. Find the missing number.

a. $3+\square=-7$

b. $-2+\square=-6$

c. $\square+(-4)=0$

d. $5-\square=6$

e. $\square-(-3)=7$

f. $\square-(-10)=12$

g. $\square-(-2)=-3$

h. $-2-\square=-4$

Chapter 7.4 Multiplication and Division of I

Summary

- Multiplying or dividing integers follows the sign rules:
  - Positive × Positive = Positive
  - Negative × Negative = Positive
  - Positive × Negative = Negative
  - Negative × Positive = Negative
- The same rules apply for division.
- These operations are essential in algebra, measurement, and problem solving.

Examples

- $4 \times -3 = -12.$
- $-5 \times -2 = 10.$
- $6 \div -2 = -3.$
- $-12 \div -4 = 3.$

Word Problems

Intermediate Questions

1. Calculate: 3×-4

2. What is -2×5?

3. Solve: $-6 \div 2$

4. What is $12 \div -3$?

5. Multiply: -5×-3

6. Divide: $-15 \div -5$

7. Evaluate: 4×-6

8. Find: $-20 \div 4$

9. Multiply: 7×-2

10. Divide: $-8 \div 2$

Hard Questions

1. Solve: $-4 \times -3 \div 2$

2. Calculate: $10 \times -2 \div -5$

3. Multiply and divide: $-6 \times 3 \div 2$

4. Evaluate: $-12 \div 3 \times -2$

5. Solve: $5 \times -3 \times -2$

6. Divide: $-24 \div -6 \times -1$

7. What is: $-18 \div 2 \times 3$

8. Calculate: $-5 \times 6 \div -3$

9. Solve: $7 \times -2 \times -4$

10. Explain: Why is -2×-3 positive?

Practice Questions

1. Calculate the answer to these products.

a. $1 \times (-10)$

b. -9×6

c. $-2 \times (-14)$

d. -11×9

e. -11×9

f. -36×3

g. $5 \times (-9)$

h. $-36 \times (-2)$

2. Calculate the answer to these quotients.

a. $-40 \div 20$

b. $-25 \div 5$

c. $-136 \div 2$

d. $78 \div (-6)$

3. Work from left to right to find the answer. Check your answer using a calculator.

a. $2 \times (-3) \times (-4)$

b. $-15 \div (-3) \times 1$

c. $48 \div (-2) \times (-3)$

d. $-8 \div (-8) \div (-1)$

4. Write down the missing number in these calculations.

a. $16 \div \square = -4$

b. $-32 \div \square = -4$

c. $-5000 \times \square = -10\ 000$

d. $-87 \times \square = 261$

e. $-92 \times \square = 184$

f. $-800 \div \square = -20$

Chapter 7.5 Order of Operations

Summary

- The order of operations tells us the correct sequence to evaluate a mathematical expression.
- The standard order is: Brackets, Orders (powers, roots), Division and Multiplication (from left to right), Addition and Subtraction (from left to right).
- This is often remembered by the acronym BODMAS or BIDMAS.
- Following this order ensures that expressions are simplified correctly.

Examples

- Evaluate: $3 + 2 \times 5 \rightarrow 2 \times 5 = 10 \rightarrow 3 + 10 = 13.$
- Evaluate: $(3 + 2) \times 5 = 5 \times 5 = 25.$
- Evaluate: $6 + 4 \div 2 = 6 + 2 = 8.$

Word Problems

Intermediate Questions

1. Evaluate: $4 + 3 \times 2$

2. Solve: $6 + (2 \times 5)$

3. Calculate: $(3 + 4) \times 2$

4. Evaluate: $8 \div 4 + 6$

5. Find the value of: $10 - 3 \times 2$

6. Solve: $5 + 6 \div 2$

7. Calculate: $(7 - 2) \times 3$

8. Evaluate: $12 \div (4 + 2)$

9. Find the result of: $2 + 3 \times 4$

10. Solve: $(6 + 2) \times 3$

Hard Questions

1. Evaluate: $2 + 3 \times (4 + 5)$

2. Solve: $(8 - 3) \times 2 + 1$

3. Calculate: $6 + 12 \div 3 \times 2$

4. Evaluate: $(5 + 2) \times (3 - 1)$

5. Solve: $9 - (2 + 3) \times 2$

6. Calculate: $18 \div (3 + 3) + 2$

7. Evaluate: $6 \times (4 + 5 - 3)$

8. Find the result: $20 \div (2 \times 2 + 2)$

9. Solve: $(4 + 6) \div 2 + 3 \times 2$

10. Explain why $2 + 3 \times 4 \neq (2 + 3) \times 4$

Practice Questions

1. Use order of operations to evaluate the following. Check your answer using a calculator.

a. $9 + 10 \div (-5)$

b. $20 + (-4) \div 4$

c. $10 - 2 \times (-3)$

d. $10 - 1 \times (-4)$

e. $-2 \times 4 + 8 \times (-3)$

f. $-3 \times (-1) + 4 \times (-2)$

g. $-30 \div 5 - 6 \times 2$

h. $-1 \times 0 - (-4) \times 1$

2. Use order of operations to evaluate the following. Check your answer using a calculator.

a. $(3 + 2) \times (-2)$

b. $(8 - 4) \div (-2)$

c. $-1 \times (7 - 8)$

d. $10 \div (4 - (-1))$

e. $(24 - 12) \div (16 + (-4))$

f. $(3 - 7) \div (-1 + 0)$

g. $(-3 + (-5)) \times (-2 - (-1))$

h. $(-3 + (-5)) \times (-2 - (-1))$

i. $-2 \times (8 - 4) + (-6)$

j. $1 - 2 \times (-3) \div (-3 - (-2))$

k. $-5 - (8 + (-2)) + 9 \div (-9)$

Chapter 7.6 Substituting Integers

Summary

- Substituting integers means replacing variables in algebraic expressions with integer values.
- Follow the correct order of operations after substitution.
- Substitution helps solve expressions and equations involving unknowns.
- Integers can be positive or negative, so signs must be handled carefully.

Examples

- Evaluate $2x + 3$ when $x = -2$: $2(-2) + 3 = -4 + 3 = -1$.
- Evaluate $a^2 - b$ when $a = 3$ and $b = 4$: $3^2 - 4 = 9 - 4 = 5$.
- Evaluate $x - y$ when $x = -5$ and $y = -2$: $-5 - (-2) = -3$.

Word Problems

Intermediate Questions

1. Evaluate $3x$ when $x = -2$.

2. Find the value of $x + 4$ when $x = -3$.

3. Evaluate $a - b$ when $a = 6$ and $b = 2$.

4. Find the result of $2y + 1$ when $y = -1$.

5. Evaluate x^2 when $x = -4$.

6. Substitute and evaluate $5x$ when $x = 3$.

7. Find the value of $x - y$ when $x = 5$ and $y = -2$.

8. Evaluate $4a + 2$ when $a = -1$.

9. Find the result of $3x - 1$ when $x = 2$.

10. Evaluate $y + 5$ when $y = -7$.

Hard Questions

1. Evaluate $2x - y$ when $x = -2$ and $y = 3$.

2. Find the result of $x^2 + 2x - 1$ when $x = -3$.

3. Evaluate $a^2 - b^2$ when $a = 4$ and $b = 5$.

4. Calculate x^3 when $x = -2$.

5. Evaluate $3x - 4y$ when $x = -1$ and $y = 2$.

6. Substitute and simplify: $4(x + 1)$ when $x = -2$.

7. Evaluate $-x^2 + 5x - 6$ when $x = -1$.

8. Find the value of $2a - 3b$ when $a = 2$ and $b = -1$.

9. Evaluate $2(x + y) - 3$ when $x = -3$ and $y = 4$.

10. Explain how to evaluate $3a - 2b$ when $a = -2$ and $b = -5$.

Practice Questions

1. Evaluate these expressions for the given pronumeral values.

a. $26 - 4x\ (x = -3)$

b. $-2 - 7k\ (k = -1)$

c. $10 \div n + 6(n = -5)$

d. $-3x + 2y\ (x = 3, y = -2)$

e. $18 \div y - x\ (x = -2, y = -3)$

f. $-36 \div a - ab\ (a = -18, b = -1)$

2. These expressions contain brackets. Evaluate them for the given pronumeral values. (Remember that ab means $a \times b$.)

a. $2 \times (a + b)$ $(a = -1, b = 6)$

b. $10 \div (a - b) + 1$ $(a = -6, b = -1)$

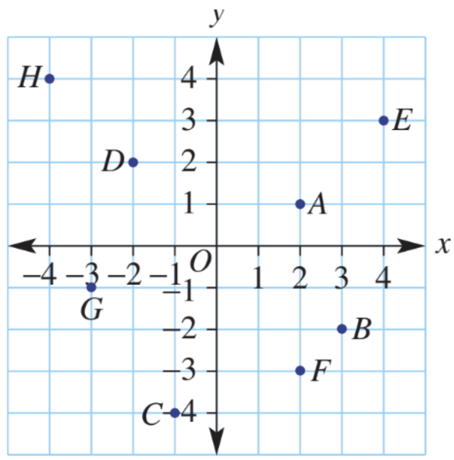
c. $ab \times (b - 1)$ $(a = -4, b = 3)$

d. $(a - b) \times bc$ $(a = 1, b = -1, c = 3)$

Chapter 7.7 The Number Plane

Practice Questions

1. For the Cartesian plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.



2. a. Draw a set of axes using 1 cm spacings. Use –4 to 4 on both axes.

b. Now plot these points.
   i. (–3, 2)
   ii. (1, 4)
   iii. (2, –1)
   iv. (–2, –4)
   v. (2, 2)
   vi. (–1, 4)
   vii. (–3, –1)
   viii. (1, –2)

CHAPTER 8 STATISTICS AND PROBABILITY

Table of Contents

Chapter 8.1 Collecting and Classifying Data

Summary

- Data collection involves gathering information to answer questions or solve problems.
- Data can be collected through surveys, observations, experiments, or online sources.
- There are two types of data: qualitative (categorical) and quantitative (numerical).
- Classifying data helps us organise it into meaningful categories or groups for better analysis.

Examples

- Surveying students about their favourite sport (categorical data).
- Measuring the height of plants in cm (numerical data).
- Recording the number of pets students have (discrete numerical data).
- Classifying eye colours in a class survey (categorical data).

Word Problems

Intermediate Questions

1. What is data collection?

2. List two ways to collect data.

3. What is categorical data?

4. What is numerical data?

5. Give an example of discrete data.

6. Give an example of continuous data.

7. How is data classified?

8. Give one example of a survey question.

9. What is the purpose of classifying data?

10. List one difference between qualitative and quantitative data.

Hard Questions

1. What is data collection?

2. List two ways to collect data.

3. What is categorical data?

4. What is numerical data?

5. Give an example of discrete data.

6. Give an example of continuous data.

7. How is data classified?

8. Give one example of a survey question.

9. What is the purpose of classifying data?

10. List one difference between qualitative and quantitative data.

Chapter 8.2 Summarising Data Numerically

Summary

- Numerical summaries provide key statistics about data such as the mean, median, mode, and range.
- Mean (average) is calculated by dividing the total of all values by the number of values.
- Median is the middle value when data is ordered from smallest to largest.
- Mode is the most frequently occurring value(s).
- Range is the difference between the highest and lowest values.

Examples

- Data: $3, 4, 4, 5, 6$ → Mean = $(3 + 4 + 4 + 5 + 6)/5 = 4.4$.
- Data: $8, 2, 5, 7, 1$ → Ordered: $1, 2, 5, 7, 8$ → Median = 5.
- Data: $6, 6, 7, 9$ → Mode = 6.
- Range of $2, 7, 4 = 7 - 2 = 5$.

Word Problems

Intermediate Questions

1. Find the mean of 3, 6, 9.

2. Calculate the median of 1, 3, 5.

3. Find the mode: 2, 4, 4, 5.

4. What is the range of 7, 3, 10?

5. Calculate the mean of 10, 20, 30.

6. Find the median of 2, 6, 4.

7. Find the mode: 5, 5, 6, 7.

8. Determine the range: 9, 1, 4.

9. What is the median of 3, 8, 5, 9, 6?

10. Find the mean of 2, 2, 4, 6.

Hard Questions

1. Calculate the mean and range of 12, 15, 9, 10.

2. Find the median of 7, 3, 1, 9, 5.

3. Find the mode of 4, 5, 4, 5, 6.

4. Explain the difference between mean and median.

5. Why might mode be useful in analysing data?

6. Calculate the mean of 4.5, 3.2, 5.3.

7. Find the median and range of 11, 15, 13, 9, 7.

8. Identify all the modes in 2, 2, 3, 3, 4.

9. Find the range and mean of 10, 15, 20.

10. Explain why outliers affect the mean more than the median.

Practice Questions

1. For each of the following sets of data, calculate the:

   i. range   ii. mean   iii. Median   iv. mode

   a. $1, 7, 1, 2, 4$

   b. $2, 2, 10, 8, 13$

   c. $3, 11, 11, 14, 21$

   d. $25, 25, 20, 37, 25, 24$

   e. $1, 22, 10, 20, 33, 10$

   f. $55, 24, 55, 19, 15, 36$

   g. $114, 84, 83, 81, 39, 12, 84$

   h. $97, 31, 18, 54, 18, 63, 6$

2. Alysha's tennis coach records how many double faults Alysha has served per match over a number of matches. Her coach presents the results in a table.

Number of double faults	0	1	2	3	4
Number of matches with this many double faults	2	3	1	4	2

a. In how many matches does Alysha have no double faults?

b. In how many matches does Alysha have 3 double faults?

c. How many matches are included in the coach's study?

d. What is the total number of double faults scored over the study period?

e. Calculate the mean of this set of data, correct to 1 decimal place.

f. What is the range of the data?

Chapter 8.3 Dot Plots and Column Graphs

Summary

- Dot plots and column graphs are used to display and compare sets of data visually.
- Dot plots show frequency by placing dots above each category or number.
- Column graphs use vertical or horizontal bars to show data values.
- Both are useful for showing trends, comparisons, and patterns in categorical or numerical data.

Examples

- Dot plot: Showing how many students got each score out of 10 in a test.
- Column graph: Showing number of pets per household using vertical bars.
- Dot plot: Recording daily number of customers in a week using dots for each customer.
- Column graph: Comparing monthly rainfall in centimetres across four cities.

Word Problems

Intermediate Questions

1. What is a dot plot used for?

2. What is a column graph?

3. When should you use a dot plot?

4. What type of data suits column graphs best?

5. How are frequencies shown in a dot plot?

6. What does each bar in a column graph represent?

7. Give one advantage of a dot plot.

8. List a similarity between dot plots and column graphs.

9. Draw a column graph for this data: Apples (3), Bananas (5), Oranges (2).

10. Draw a dot plot for scores: 2, 2, 3, 4, 4, 4.

Hard Questions

1. Explain how to create a column graph from survey data.

2. Describe a real-life use of dot plots in schools.

3. What are the limitations of column graphs?

4. Compare dot plots with column graphs using examples.

5. Draw a dot plot for: 1, 2, 2, 3, 3, 3, 4.

6. Draw a column graph for these values: Red (4), Blue (2), Green (5).

7. Explain how data accuracy affects visual graphs.

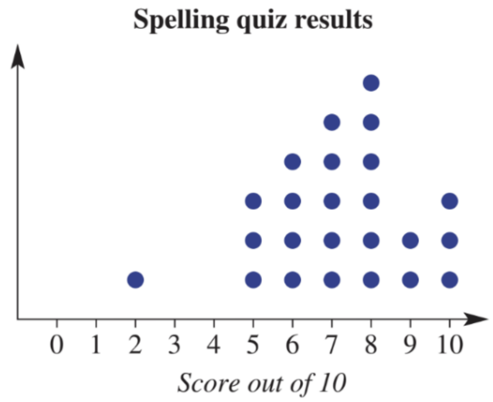
8. Describe how you could mislead someone with a poorly scaled column graph.

9. Create a real-world scenario suitable for a dot plot.

10. Create a column graph for weekly hours studied: Mon (2), Tue (3), Wed (4), Thu (1), Fri (5).

Practice Questions

1. In a Year 4 class, the results of a spelling quiz are presented as a dot plot.



a. What is the most common score in the class?

b. How many students participated in the quiz?

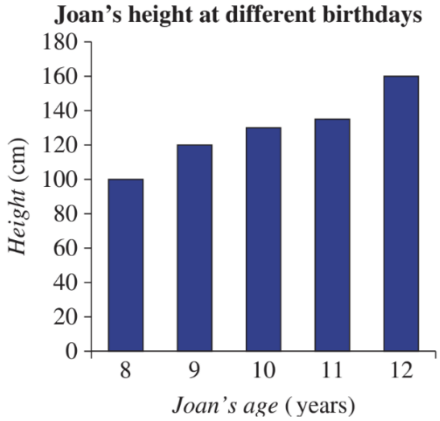
c. What is the range of scores achieved?

d. What is the median score?

e. Identify the outlier.

2. Joan has graphed her height at each of her past five birthdays.

a. How tall was Joan on her 9th birthday?



b. How much did she grow between her 8th birthday and 9th birthday?

c. How much did Joan grow between her 8th and 12th birthdays?

d. How old was Joan when she had her biggest growth spurt?

3. a. Draw a column graph to show the results of the following survey of the number of boys and girls born at a certain hospital. Put time (years) on the horizontal axis.

	2000	2001	2002	2003	2004	2005
Number of boys born	40	42	58	45	30	42
Number of girls born	50	40	53	41	26	35

b. During which year(s) were there more girls born than boys?

c. Which year had the fewest number of births?

d. Which year had the greatest number of births?

e. During the entire period of the survey, were there more boys or girls born?

Chapter 8.4 Line Graphs

Summary

- Line graphs are used to display data that changes over time.
- Data points are plotted on a grid and connected using straight lines.
- Line graphs show trends, patterns, and fluctuations in numerical data.
- They are especially useful for comparing changes across different groups or time periods.

Examples

- Plotting daily temperatures over a week to show weather trends.
- Displaying population growth across several years.
- Showing the progress of a student’s test scores over a semester.
- Comparing monthly sales figures of two different products.

Word Problems

Intermediate Questions

1. What does a line graph show?

2. When is it useful to use a line graph?

3. What are plotted and connected in a line graph?

4. What does the slope of a line represent?

5. Label the axes of a line graph showing height over time.

6. Give one reason to use a line graph over a column graph.

7. What is needed to plot a line graph?

8. List two examples of data suited to line graphs.

9. What is the difference between a dot plot and a line graph?

10. How many axes does a line graph have?

Hard Questions

1. Explain how to draw a line graph from a data table.

2. Describe a real-life situation suitable for a line graph.

3. Compare a line graph and a column graph for the same data.

4. Draw a line graph for this data: Day 1 (2), Day 2 (3), Day 3 (5), Day 4 (4).

5. Create a data set that could be used in a line graph showing internet usage.

6. Explain what it means if a line graph is flat between two points.

7. How can you use a line graph to make predictions?

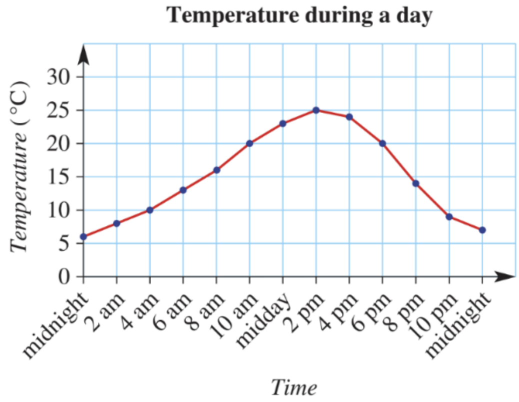
8. Describe how misleading line graphs can distort data interpretation.

9. Explain why spacing intervals consistently is important in line graphs.

10. Draw a line graph showing temperature changes from 6 AM to 6 PM.

Practice Questions

1. Consider the following graph, which shows the outside temperature over a 24-hour period that starts at midnight.



a. What was the temperature at midday?

b. When was the hottest time of the day?

c. When was the coolest time of the day?

d. Use the graph to estimate the temperature at these times of the day:

i. 4:00 am

ii. 9:00 am

iii. 1:00 pm

iv. 3:15 pm

S

2. Oliver measures his pet dog's weight over the course of a year, by weighing it at the start of each month. He obtains the following results.

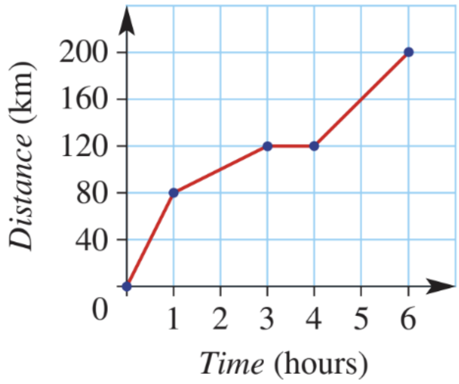
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Weight (kg)	7	7.5	8.5	9	9.5	9	9.2	7.8	7.8	7.5	8.3	8.5

a. Draw a line graph showing this information, making sure the vertical axis has an equal scale from 0 kg to 10 kg.

b. Describe any trends or patterns that you see.

c. Oliver put his dog on a weight loss diet for a period of 3 months. When do you think the dog started the diet? Justify your answer.

3. This travel graph shows the distance travelled by a van over 6 hours.



a. How far did the van travel in total?

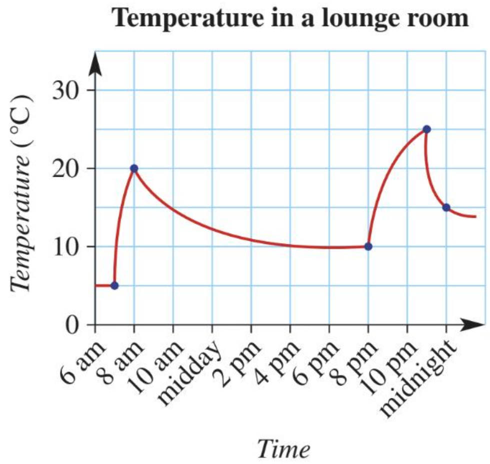
b. How far did the van travel in the first hour?

c. What is happening in the fourth hour?

d. When is the van travelling the fastest?

e. In the sixth hour, how far does the van travel?

4. The temperature in a lounge room is measured frequently throughout a particular day. The results are presented in a line graph, as shown below. The individual points are not indicated on this graph to reduce clutter.



a. Twice during the day the heating was switched on. At what times do you think this happened? Explain your reasoning.

b. When was the heating switched off? Explain your reasoning.

c. The house has a single occupant, who works during the day. Describe when you think that person is:

i. waking up ii. going to work iii. coming home iv. going to bed

d. These temperatures were recorded during a cold winter month. Draw a graph that shows what the lounge room temperature might look like during a hot summer month. Assume that the room has an air conditioner, which the person is happy to use when at home.

Chapter 8.5 Stem-and-Leaf Plots

Summary

- Stem-and-leaf plots are a way to organise numerical data while preserving original values.
- Each number is split into a 'stem' (e.g. tens place) and a 'leaf' (e.g. ones place).
- The stem is listed in a vertical column, and the leaves are listed next to them.
- This plot makes it easy to see the shape of the data and identify clusters, gaps, and outliers.

Examples

- Data: 12, 14, 15, 18 → Stem-and-leaf: 1 | 2 4 5 8
- Data: 21, 22, 25, 25, 29 → Stem-and-leaf: 2 | 1 2 5 5 9
- Data: 43, 46, 47 → Stem-and-leaf: 4 | 3 6 7

Word Problems

Intermediate Questions

1. What is a stem-and-leaf plot used for?

2. In the plot 3 | 1 2 4, what numbers are represented?

3. What does the stem represent in 5 | 2 3 7?

4. Create a plot for: 14, 16, 17, 18.

5. Write the values in the stem-and-leaf: 6 | 2 3 5 7.

6. How many values are shown in 2 | 4 4 5 7?

7. Why are stem-and-leaf plots useful?

8. Convert this data into a plot: 31, 33, 35.

9. What is the range of 3 | 0 2 6?

10. List the numbers from 7 | 1 2 2 4 5.

Hard Questions

1. Construct a stem-and-leaf plot from 45, 42, 47, 41, 49.

2. Interpret: 5 | 2 5 6 6 9 → How many data points are there?

3. Explain how stem-and-leaf plots show data shape.

4. Compare stem-and-leaf plots and column graphs.

5. Find the median in: 4 | 1 3 6 7 9.

6. Explain how to find mode using a stem-and-leaf plot.

7. Draw a plot for: 52, 54, 56, 58, 59, 59.

8. What does a gap in stems suggest about the data?

9. How can outliers be identified in stem-and-leaf plots?

10. Use a plot to estimate mean of: 6 | 3 4 5 7 8.

Practice Questions

1. For each of the stem-and-leaf plots below, state the range and the median.

a.

Stem	Leaf
0	9
1	3 5 6 7 7 8 9
2	0 1 9

b.

Stem	Leaf
1	1 4 8
2	1 2 4 4 6 8
3	0 3 4 7 9
4	2

c.

Stem	Leaf
3	1 1 2 3 4 4 8 8 9
4	0 1 1 2 3 5 7 8
5	0 0 0

2. Represent each of the following sets of data as a stem-and-leaf plot.

a. $11, 12, 13, 14, 14, 15, 17, 20, 24, 28, 29, 31, 32, 33, 35$

b. $20, 22, 39, 45, 47, 49, 49, 51, 52, 52, 53, 55, 56, 58, 58$

c. $21, 35, 24, 31, 16, 28, 48, 18, 49, 41, 50, 33, 29, 16, 32$

d. $32, 27, 38, 60, 29, 78, 87, 60, 37, 81, 38, 11, 73, 12, 14$

Chapter 8.6 Pie Charts and Divided Bar Graphs

Summary

- Pie charts show how a whole is divided into parts using a circular graph.
- Each sector represents a category and its proportion of the total.
- Divided bar graphs are similar but use rectangular bars segmented into proportional sections.
- Both are useful for comparing parts of a whole and visualising percentages or fractions.

Examples

- Pie chart: Dividing class survey results on favourite colours.
- Divided bar graph: Showing time allocation to different school subjects.
- Pie chart: Representing budget spending categories as a circle.
- Bar graph: Displaying proportion of different transport modes used by students.

Word Problems

Intermediate Questions

1. What does a pie chart represent?

2. How many degrees are there in a pie chart?

3. What is a sector in a pie chart?

4. What is a divided bar graph?

5. Why use a pie chart instead of a column graph?

6. Name a situation where a divided bar graph is better than a pie chart.

7. How can percentages be shown in a pie chart?

8. List one similarity between pie charts and divided bar graphs.

9. Draw a pie chart with categories: A (50%), B (30%), C (20%).

10. Draw a divided bar graph with categories: Maths (4 hrs), Science (3 hrs), English (5 hrs).

Hard Questions

1. Explain how to calculate angles for a pie chart from percentages.

2. Convert this data to a pie chart: 25%, 35%, 40%.

3. Design a divided bar graph from weekly spending data.

4. Explain the advantages of pie charts over line graphs.

5. Describe when a divided bar graph may be misleading.

6. Create a real-world scenario for a pie chart.

7. Compare pie charts and divided bar graphs in detail.

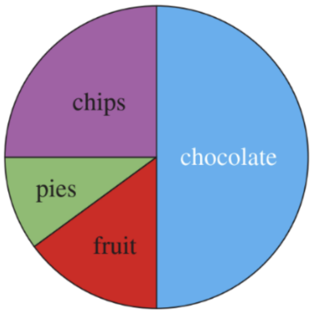
8. Calculate angles in a pie chart for: Red (25), Blue (15), Green (10) out of 50 total.

9. Draw a divided bar graph for a diet chart showing protein (20%), carbs (50%), fats (30%).

10. Explain how pie charts show proportion even when not labelled numerically.

Practice Questions

1. A group of Year 7 students was polled on their favourite foods, and the results are shown in this pie chart.



   a. If 40 students participated in the survey, find how many of them chose:

   i. chocolate

   ii. chips

   iii. fruit

   iv. pies

   b. Health experts are worried about what these results mean. They would like fruit to appear more prominently in the pie graph, and to not have the chocolate sector next to the chips. Redraw the pie chart so this is the case.

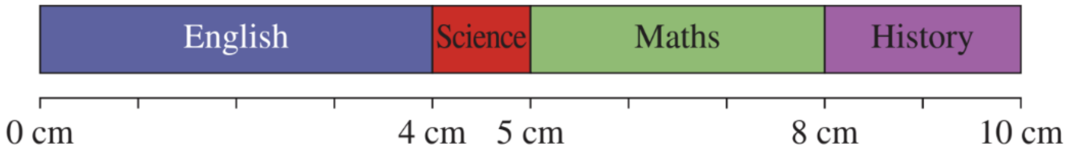
c. Another 20 students were surveyed. Ten of these students chose chocolate and the other 10 chose chips. Their results are to be included in the pie graph. Of the four sectors in the graph, state which sector will:

i. increase in size

ii. decrease in size

iii. stay the same size

2. Yakob has asked his friends what is their favourite school subject, and he has created the following divided bar graph from the information.



a. If Yakob surveyed 30 friends, state how many of them like:

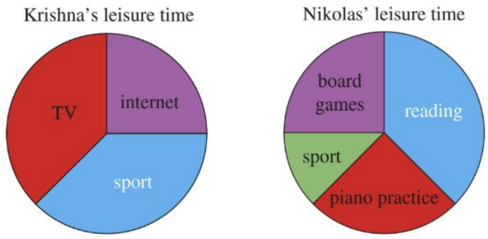
i. Maths best

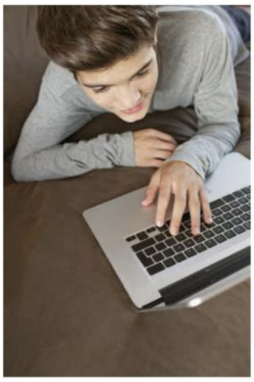
ii. History best

iii. either English or Science best

b. Redraw these results as a pie chart.

3. Friends Krishna and Nikolas have each graphed their leisure habits, as shown below.





a. Which of the two friends spends more of their time playing sport?

b. Which of the two friends does more intellectual activities in their leisure time?

c. Krishna has only 2 hours of leisure time each day because he spends the rest of his time doing homework. Nikolas has 8 hours of leisure time each day. How does this affect your answers to parts a and b above?

Chapter 8.7 Describing Chance

Summary

- Describing chance involves estimating how likely an event is to happen.
- We use language such as 'likely', 'unlikely', 'certain', 'impossible', and 'even chance'.
- Probability can be described using numbers between 0 (impossible) and 1 (certain).
- Understanding chance helps in making predictions and decisions in real-life situations.

Examples

- It is impossible to roll a 7 on a standard six-sided die.
- It is certain that the sun will rise tomorrow.
- There is an even chance of flipping a head or tail on a coin.
- It is likely to rain in winter, unlikely in summer (in some areas).

Word Problems

Intermediate Questions

1. What does 'even chance' mean?

2. Give an example of an impossible event.

3. What is the probability of flipping heads on a fair coin?

4. What number represents a certain event in probability?

5. Which is more likely: rolling a 6 or rolling a 1 on a fair die?

6. What does a probability of 0 mean?

7. What does a probability of 1 mean?

8. Describe an event that is unlikely.

9. Describe an event that is likely.

10. Write the probability of rolling a number less than 7 on a standard die.

Hard Questions

1. Explain why probability is always between 0 and 1.

2. Describe an everyday situation using the terms 'likely' and 'unlikely'.

3. Compare certain, impossible, and even chance events.

4. What is the chance of choosing a red ball from a bag with 5 red, 3 blue, and 2 green balls?

5. Describe a scenario where you would assign a 0.75 chance.

6. What are the limitations of using language to describe chance?

7. Explain how probability can help in weather forecasting.

8. List all outcomes when rolling a six-sided die and flipping a coin.

9. Use probability to evaluate a simple game of chance.

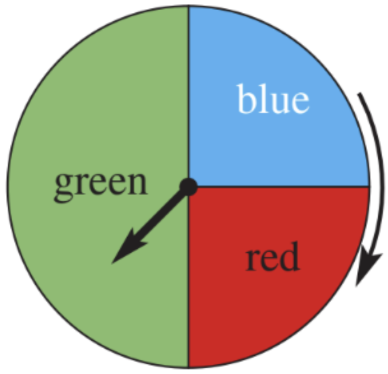
10. Why is flipping a coin considered a fair chance event?

Practice Questions

1. Consider the spinner shown, which is spun and could land with the arrow pointing to any of the three colours. (If it lands on a boundary, it is re-spun until it lands on a colour.)

   a. State whether each of the following is true or false.

   i. There is an even chance that the spinner will point to green.



   ii. It is likely that the spinner will point to red.

   iii. It is certain that the spinner will point to purple.

   iv. It is equally likely that the spinner will point to red or blue.

   v. Green is twice as likely to occur as blue.

b. Use the spinner to give an example of:

i. an impossible event

ii. a likely event

iii. a certain event

iv. two events that are equally likely

2. Draw spinners to match each of the following descriptions, using blue, red and green as the possible colours.

   a. Blue is likely, red is unlikely and green is impossible.

   b. Red is certain.

   c. Blue has an even chance, red and green are equally likely.

   d. Blue, red and green are all equally likely.

e. Blue is twice as likely as red, but red and green are equally likely.

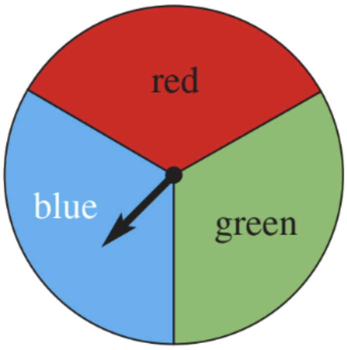
f. Red and green are equally likely and blue is impossible.

g. Blue, red and green are all unlikely, but no two colours are equally likely.

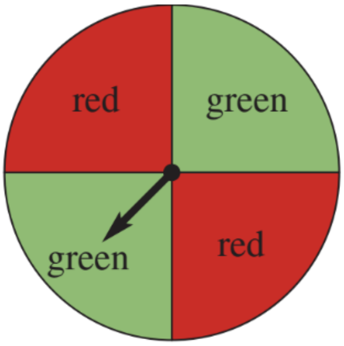
h. Blue is three times as likely as green, but red is impossible.

3. For each of the following spinners, give a description of the chances involved so that someone could determine which spinner is being described. Use the colour names and the language of chance (i.e. ‘likely’, ‘impossible’ etc.) in your descriptions.

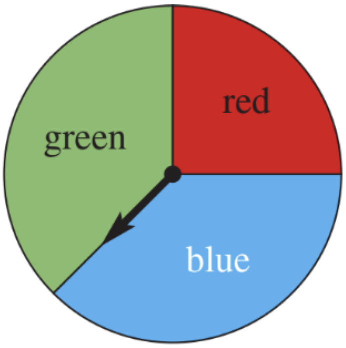
a.



b.



c.



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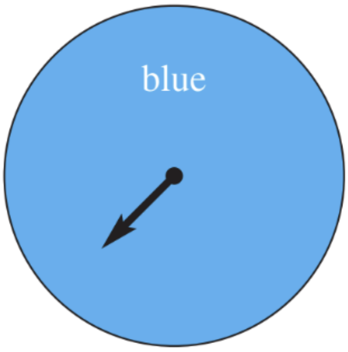
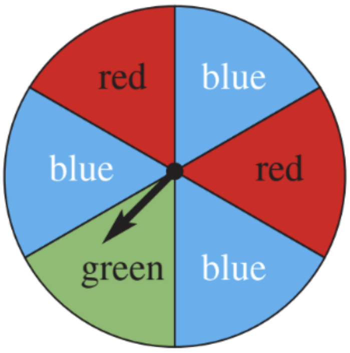


e.



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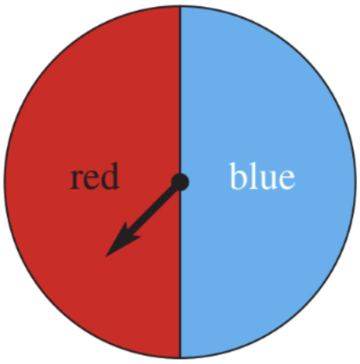


Chapter 8.8 Theoretical Probability

Summary

- Theoretical probability is calculated based on reasoning and known outcomes.
- It uses the formula: Probability = (Number of favourable outcomes) / (Total number of possible outcomes).
- It assumes all outcomes are equally likely.
- It helps predict the likelihood of events such as rolling dice or flipping coins.

Examples

- Probability of rolling a 3 on a standard die = $\frac{1}{6}$.
- Probability of getting a head on a coin = $\frac{1}{2}$.
- Probability of picking a red card from a standard deck = $\frac{26}{52} = \frac{1}{2}$.
- Probability of picking an even number from 1 to 10 $= \frac{5}{10} = \frac{1}{2}$.

Word Problems

Intermediate Questions

1. What is theoretical probability?

2. What is the formula for theoretical probability?

3. Calculate the probability of flipping a tail.

4. What is the chance of rolling an odd number on a die?

5. How many possible outcomes are there when rolling a die?

6. What is the probability of drawing a heart from a standard deck?

7. What is the probability of rolling a number less than 5 on a die?

8. Calculate the probability of selecting a vowel from the word ‘MATHS’.

9. Find the probability of picking a red marble from 3 red and 2 blue.

10. What is the probability of choosing a weekday from a week?

Hard Questions

1. Calculate the probability of rolling a number greater than 2 on a die.

2. If a bag has 4 red, 3 blue, and 3 green balls, what is the chance of blue?

3. Explain why theoretical probability is useful.

4. Find the probability of drawing a king from a deck of cards.

5. What is the probability of getting a 5 or 6 when rolling a die?

6. Explain the difference between theoretical and experimental probability.

7. Calculate the probability of getting a prime number on a die.

8. What is the chance of randomly choosing an even digit from 0 to 9?

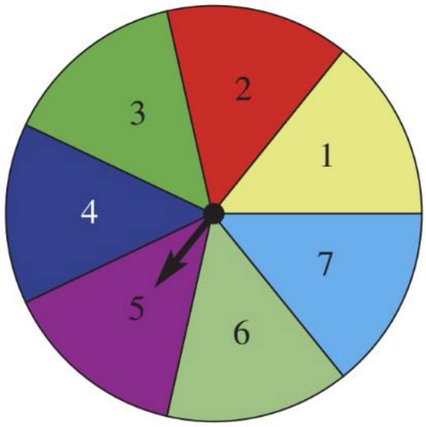
9. If a spinner has 8 equal sections (2 red, 3 blue, 3 green), what's the probability of red?

10. Draw a probability tree diagram for flipping two coins.

Practice Questions

1. A spinner with the numbers 1 to 7 is spun. The numbers are evenly spaced.

   a. List the sample space.



   b. Find $\Pr(6)$.

   c. Find $\Pr(8)$.

   d. Find $\Pr(2 \text{ or } 4)$.

e. Find $\Pr(\text{even})$.

f. Find $\Pr(\text{odd})$.

g. Give an example of an event having the probability of 1.

2. The letters in the word PROBABILIT: are written on 11 cards and then one is drawn from a hat.

a Find $\Pr(\mathrm{P})$.

b Find $\Pr(\mathrm{P\ or\ L})$.

c Find $\Pr(\text{letter chosen is in the word BIT})$.

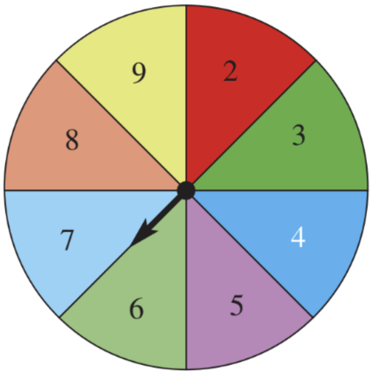
d Find $\Pr(\text{not a B})$.

e Find $\Pr(\text{a vowel is chosen})$.

f. Give an example of an event with the probability of $\frac{3}{11}$.

3. Consider the spinner opposite, numbered 2 to 9.

a. List the sample space.



b. Find the probability that a prime number will be spun, giving your answer as a decimal. (Remember that 2 is a prime number.)

c. Giving your answers as decimals, state the probability of getting a prime number if each number in the spinner opposite is:
(Hint: It will help if you draw the new spinner.)

i. increased by 1

ii. increased by 2

iii. doubled

d. Design a new spinner for which the $\Pr(\text{prime}) = 1$.

Chapter 8.9 Experimental Probability

Summary

- Experimental probability is based on actual results from experiments or observations.
- It uses the formula: Probability = (Number of successful outcomes) / (Total number of trials).
- Unlike theoretical probability, it may vary depending on the number of trials.
- It becomes more accurate as the number of trials increases.

Examples

- Flipping a coin 100 times and getting 47 heads → Probability = $\frac{47}{100}$.
- Rolling a die 60 times and getting a 4 fifteen times → Probability = $\frac{15}{60} = \frac{1}{4}$.
- Drawing a red card 13 times in 50 draws → Probability = $\frac{13}{50}$.

Word Problems

Intermediate Questions

1. What is experimental probability?

2. How is experimental probability different from theoretical probability?

3. What is the formula for experimental probability?

4. If 4 out of 10 trials are successful, what's the probability?

5. What happens when you increase the number of trials?

6. Find the probability of getting tails in 20 coin tosses with 12 tails.

7. What is the probability of rolling a 3 five times out of 30 rolls?

8. What affects the accuracy of experimental probability?

9. Why might results vary in experiments?

10. What is the experimental probability of rain if it rained 7 days out of 10?

Hard Questions

1. Compare experimental and theoretical probabilities with an example.

2. Explain why repeated trials improve experimental probability.

3. Design an experiment to test the probability of landing heads.

4. If a spinner lands on blue 18 times in 60 spins, what's the probability?

5. Why might experimental probability differ from theoretical results?

6. Find the probability of a light bulb failing 3 times in 40 uses.

7. Record and calculate probability from a real-world event.

8. How would you present experimental data visually?

9. Explain the role of sample size in experimental probability.

10. Calculate experimental probability and compare it to theory: 20 coin flips, 11 heads.

Practice Questions

1. A fair coin is tossed.

   a. How many times would you expect it to show tails in 1000 trials?

   b. How many times would you expect it to show heads in 3500 trials?

   c. Initially, you toss the coin 10 times to find the probability of the coin showing tails.
      i. Explain how you could get an experimental probability of 0.7.

      ii. If you toss the coin 100 times, are you more or less likely to get an experimental probability close to 0.5?

2. Each time a basketball player takes a free throw there is a 4 in 6 chance that the shot will go in. This can be simulated by rolling a 6-sided die and using numbers 1 to 4 to represent 'shot goes in' and numbers 5 and 6 to represent 'shot misses'.

   a. Use a 6-sided die over 10 trials to find the experimental probability that the shot goes in.

   b. Use a 6-sided die over 50 trials to find the experimental probability that the shot goes in.

   c. Working with a group, use a 6-sided die over 100 trials to find the experimental probability that the shot goes in.

   d. Use a 6-sided die over just one trial to find the experimental probability that the shot goes in. (Your answer should be either 0 or 1.)

   e. Which of the answers to parts a to d above is closest to the theoretical probability of 66.67%? Justify your answer.

3. The colour of the cars in a school car park is recorded.

Colour	Red	Black	White	Blue	Purple	Green
Number of cars	21	24	25	20	3	7

Based on this sample:

a. What is the probability that a randomly chosen car is white?

b. What is the probability that a randomly chosen car is purple?

c. What is the probability that a randomly chosen car is green or black?

d. How many purple cars would you expect to see in a shopping centre car park with 2000 cars?

CHAPTER 9 POLYGONS, SOLIDS AND TRANSFORMATIONS

Table of Contents

Chapter 9.1 Polygons

Summary

- Polygons are 2D shapes with straight sides and closed figures.
- The number of sides determines the name of the polygon (e.g. triangle = 3 sides, quadrilateral = 4 sides).
- Regular polygons have all sides and angles equal, while irregular polygons do not.
- The sum of interior angles of a polygon can be calculated using the formula: $(n - 2) \times 180°$, where n is the number of sides.
- Polygons are classified based on sides (triangles, quadrilaterals, pentagons, etc.) and symmetry.

Examples

- A triangle has 3 sides and interior angle sum $= (3 - 2) \times 180 = 180°$.
- A hexagon has 6 sides and interior angle sum $= (6 - 2) \times 180 = 720°$.
- A regular pentagon has 5 equal sides and angles (each interior angle $= 108°$).
- Irregular quadrilateral: 4 sides of different lengths and angles.

Word Problems

Intermediate Questions

1. What is a polygon?

2. Name a polygon with 3 sides.

3. How many sides does a pentagon have?

4. What’s the sum of interior angles in a quadrilateral?

5. Give an example of a regular polygon.

6. What does irregular polygon mean?

7. Calculate the angle sum of a hexagon.

8. Is a square a regular polygon? Why?

9. What is the angle sum of an octagon?

10. Name a 2D shape that is not a polygon.

Hard Questions

1. Use the formula to find the angle sum of a 12-sided polygon.

2. If a regular polygon has interior angles of $135°$, how many sides does it have?

3. Explain how polygons are classified.

4. Compare regular and irregular polygons with examples.

5. Draw a regular hexagon and label its sides.

6. Calculate one interior angle of a regular octagon.

7. Explain the role of symmetry in regular polygons.

8. How many diagonals can be drawn in a pentagon?

9. Find the name of a polygon with 9 sides and total angle sum.

10. Why is a circle not considered a polygon?

Chapter 9.2 Constructing Triangles and Triangle Angle Sum

Summary

- A triangle is a polygon with three sides and three angles.
- The sum of the interior angles of any triangle is always $180°$.
- Triangles can be classified as equilateral, isosceles, or scalene based on side lengths.
- Triangles can also be classified by angles: acute (all angles $< 90°$), right (one angle $= 90°$), obtuse (one angle $> 90°$).
- Constructing triangles accurately involves using a ruler, protractor, and compass.

Examples

- Equilateral triangle: 3 equal sides, each angle $= 60°$.
- Isosceles triangle: 2 equal sides, 2 equal angles.
- Scalene triangle: All sides and angles are different.
- Right triangle: One angle is exactly $90°$.
- Construct a triangle with angles $60°, 60°$, and $60°$ using a protractor.

Word Problems

Intermediate Questions

1. What is the angle sum of a triangle?

2. Name the three types of triangles based on sides.

3. How many angles does a triangle have?

4. What is a right triangle?

5. Draw a triangle with angles $60°, 60°, 60°$.

6. What tools are needed to construct a triangle?

7. Name a triangle with one angle greater than $90°$.

8. What is the smallest number of equal sides in an isosceles triangle?

9. How is an equilateral triangle different from a scalene triangle?

10. Classify a triangle with sides 5 cm, 5 cm, and 8 cm.

Hard Questions

1. Construct a triangle with sides 5 cm, 6 cm, and 7 cm using a compass and ruler.

2. Explain why the angle sum in every triangle is always $180°$.

3. Find the missing angle in a triangle with angles $85°$ and $40°$.

4. Compare acute, right, and obtuse triangles.

5. Design a triangle that has two equal angles and one $90°$ angle.

6. Identify and classify triangles based on side and angle properties.

7. Why can't a triangle have two obtuse angles?

8. Construct a triangle given one side and two angles.

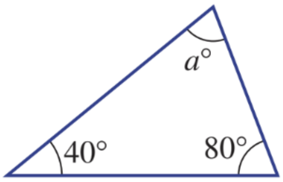
9. Explain how triangle classification helps in geometry.

10. Use a protractor to draw a triangle with angles $45°, 90°$, and $45°$.

Practice Questions

1. Find the value of a in each of these triangles.

a.



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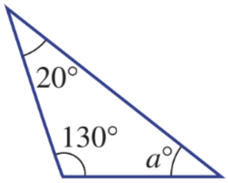

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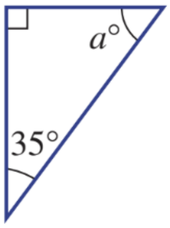

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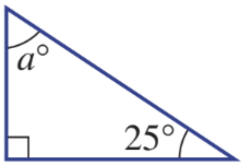

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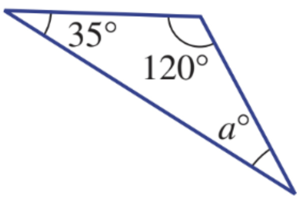

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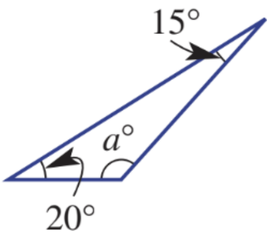
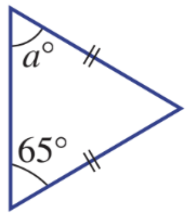


2. Find the value of a in each of these isosceles triangles.

a.



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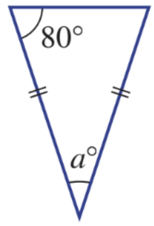
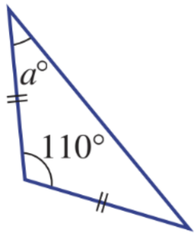
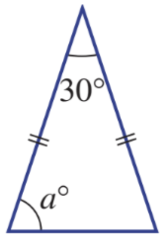


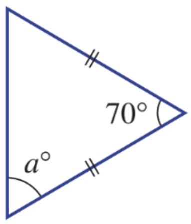
c.



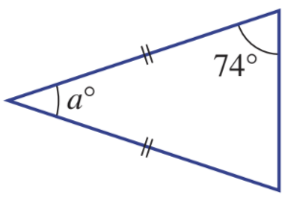
d.



e.

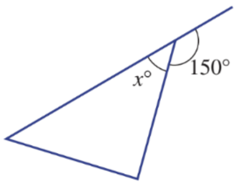


f.

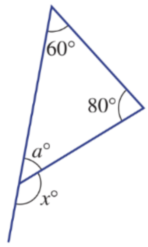


3. The triangles below have exterior angles. Find the value of x. For parts **b** to **f**, you will need to first calculate the value of a.

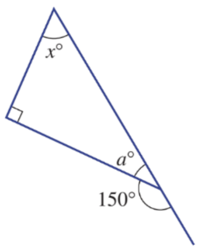
a.



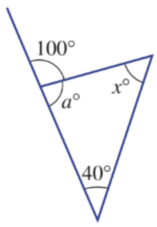
b.



c.



d.



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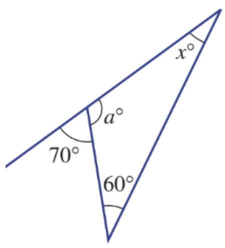

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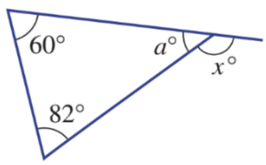


Chapter 9.3 Quadrilaterals and Quadrilateral Angle Sum

Summary

- Quadrilaterals are polygons with four sides and four angles.
- The sum of the interior angles of any quadrilateral is always 360°.
- Common types of quadrilaterals include squares, rectangles, parallelograms, rhombuses, trapeziums, and kites.
- These shapes differ in their properties such as equal sides, parallel sides, and right angles.
- Understanding these properties helps in classifying and constructing quadrilaterals accurately.

Examples

- Square: All sides equal, all angles 90°, opposite sides parallel.
- Rectangle: Opposite sides equal and parallel, all angles 90°.
- Parallelogram: Opposite sides equal and parallel, opposite angles equal.
- Rhombus: All sides equal, opposite angles equal, sides slanted.
- Trapezium: Only one pair of opposite sides is parallel.
- Kite: Two pairs of adjacent sides equal, one pair of opposite angles equal.

Word Problems

Intermediate Questions

1. What is a quadrilateral?

2. How many angles does a quadrilateral have?

3. What is the angle sum of a quadrilateral?

4. Name a quadrilateral with all equal sides.

5. What is the difference between a square and a rectangle?

6. Is a rhombus a parallelogram? Why or why not?

7. List 3 properties of a parallelogram.

8. Which quadrilateral has only one pair of parallel sides?

9. Give one real-world example of a kite shape.

10. Draw a quadrilateral with 2 pairs of adjacent equal sides.

Hard Questions

1. Construct a rectangle using a ruler and protractor.

2. Explain the differences between a rhombus and a kite.

3. Find the missing angle in a quadrilateral with angles $85°$, $95°$, and $100°$.

4. Compare the properties of a parallelogram and a trapezium.

5. Draw and label all quadrilaterals with their properties.

6. How do you classify a quadrilateral with no equal sides or angles?

7. Why is a square both a rectangle and a rhombus?

8. Describe how to construct a parallelogram using compass and ruler.

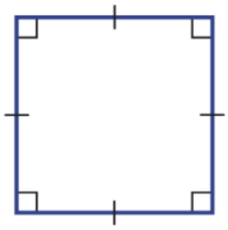
9. What happens if one angle in a quadrilateral is greater than $180°$?

10. Explain the importance of understanding quadrilateral properties in geometry.

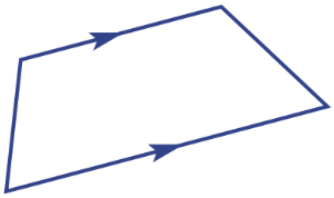
Practice Questions

1. State the type of special quadrilateral given below.

a.



b.



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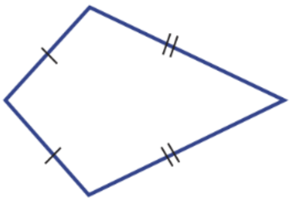
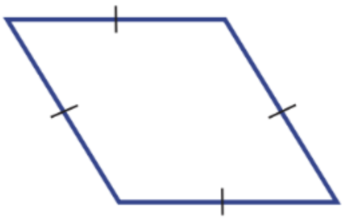
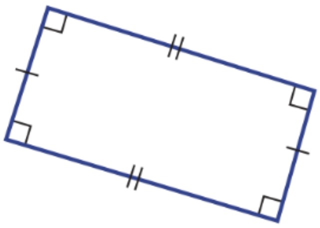


d.



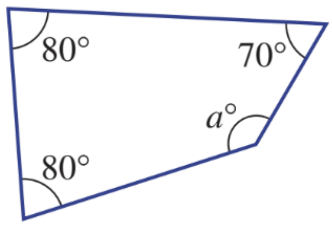
e.



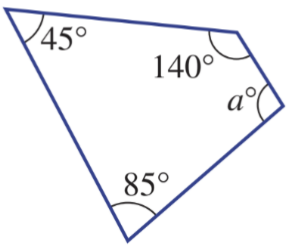
f.

2. For each of these quadrilaterals, find the value of a.

a.



b.



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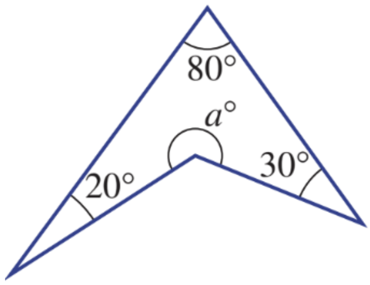
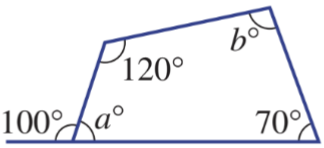
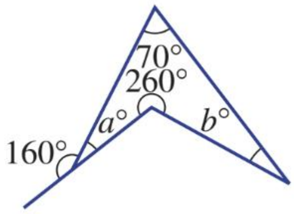


3. Find the values of a and b in each of the following diagrams.

a.



b.



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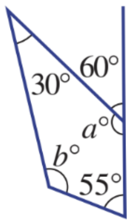
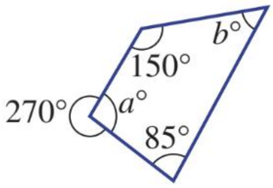
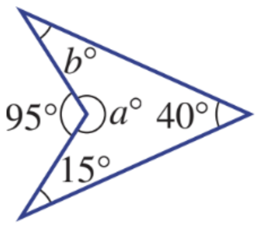


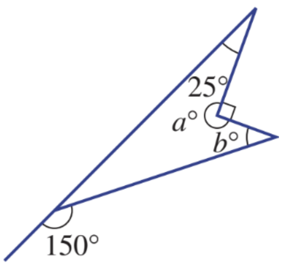
d.



e.

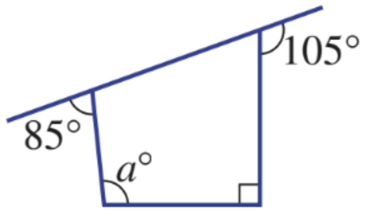


f.

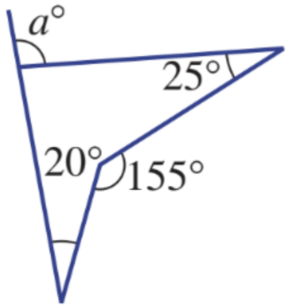


4. For each of these diagrams, find the value of a. You may need to find some other angles first.

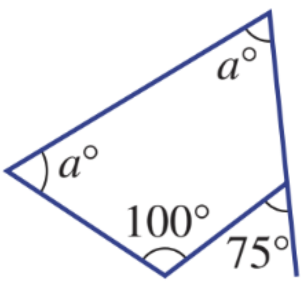
a.



b.

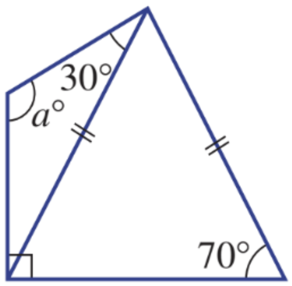


c.

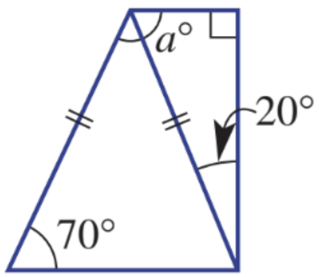


5. These diagrams include both triangles and quadrilaterals. Find the value of a.

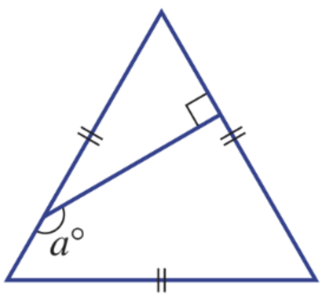
a.



b.



c.



Chapter 9.4 Symmetry

Summary

- Symmetry refers to a figure being identical on both sides when divided by a line (line symmetry) or rotated (rotational symmetry).
- A shape has line symmetry if it can be folded along a line so that both halves match.
- Rotational symmetry exists when a shape can be rotated (less than $360°$) and still look the same.
- Shapes can have one or more lines of symmetry and different orders of rotational symmetry.
- Symmetry is often found in nature, art, architecture, and geometric shapes.

Examples

- A square has 4 lines of symmetry and rotational symmetry of order 4.
- An equilateral triangle has 3 lines of symmetry and rotational symmetry of order 3.
- The letter ‘A’ has 1 vertical line of symmetry.
- A circle has infinite lines of symmetry and infinite rotational symmetry.

Word Problems

Intermediate Questions

1. What is line symmetry?

2. Name a shape with exactly 1 line of symmetry.

3. What is rotational symmetry?

4. How many lines of symmetry does a rectangle have?

5. Does a circle have rotational symmetry?

6. Which letters in the alphabet have line symmetry?

7. Which letters have rotational symmetry?

8. Draw a shape with 2 lines of symmetry.

9. Find the order of rotational symmetry for a square.

10. Does a scalene triangle have symmetry?

Hard Questions

1. Explain the difference between line and rotational symmetry.

2. Describe a shape with no symmetry at all.

3. Draw and label a hexagon showing its symmetry.

4. Which shapes have more than 2 types of symmetry?

5. Describe a real-life object that has symmetry.

6. Explain how symmetry is used in art and design.

7. Construct a pattern with rotational symmetry of order 3.

8. How many symmetries does a regular pentagon have?

9. Compare the symmetry of a rhombus and a kite.

10. What happens to symmetry when a shape is distorted?

Chapter 9.5 Reflection, Rotation and Translation

Summary

- Transformations are ways of moving or changing shapes in geometry.
- Reflection is flipping a shape over a line (mirror line) to create a mirror image.
- Rotation is turning a shape around a fixed point (centre of rotation) by a certain angle and direction.
- Translation is sliding a shape from one position to another without rotating or flipping it.
- Each transformation maintains the shape’s size and angles (congruence).

Examples

- Reflecting a triangle over the y-axis creates a mirror image.
- Rotating a square $90°$ clockwise around its centre keeps its size and shape.
- Translating a shape 4 units right and 2 units down moves it without changing its appearance.
- A letter like 'Z' has different reflection and rotation effects depending on the axis or centre.

Word Problems

Intermediate Questions

1. What is a transformation in geometry?

2. Define reflection in your own words.

3. What is the line of reflection?

4. Describe what rotation means.

5. What is the centre of rotation?

6. Explain what a translation does to a shape.

7. Does a shape change size during reflection?

8. What does it mean to translate a shape?

9. Draw a shape and reflect it over a vertical line.

10. Rotate a triangle $90°$ clockwise around a point.

Hard Questions

1. Compare reflection, rotation, and translation.

2. Describe how each transformation affects the shape’s orientation.

3. Explain the properties preserved during transformation.

4. Perform a translation and then a reflection on a square.

5. Describe the effect of rotating a shape $180°$.

6. Construct a reflection of a pentagon using a mirror line.

7. Create a sequence of transformations for moving a shape from point A to point B.

8. Describe a real-world example of each transformation.

9. Explain how transformations are used in computer graphics.

10. Compare congruent shapes before and after a transformation.