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Review

Analysis of a Reactive MHD Third Grade Fluid in a Cylindrical Pipe with Radially Applied Magnetic Field, Reynold's Variable Viscosity and Joule Heating

Bankole Vincent Akinremi*

Department of Mathematics, Adeyemi Federal University of Education, Ondo State
akinremibv@gmail.com; akinremiov@afued.edu.ng

Highlights

- The study analyses how a reactive magnetohydrodynamic third grade fluid flows through a cylindrical pipe when magnetic field, variable viscosity, and Joule heating are present.
- A Reynolds-type variable viscosity model is applied and the resulting nonlinear equations are solved numerically using the Galerkin weighted residual method.
- The results show that increasing magnetic and Hall parameters significantly influences the velocity distribution of the fluid.
- Fluid temperature increases with higher Eckert and Prandtl numbers due to stronger viscous dissipation and heat transfer effects.
- Entropy analysis indicates that heat transfer irreversibility dominates the flow, with the Bejan number increasing as the magnetic parameter increases.

Abstract

This study presents an analysis of a chemically reactive magnetohydrodynamic (MHD) third-grade fluid flowing through a cylindrical pipe. The combined effects of a radially applied magnetic field, variable viscosity, and Joule heating are examined to provide valuable insights into the fluid's behaviour, with important implications for optimising industrial processes and enhancing the performance of systems that utilise MHD fluids. The coupled governing equations are formulated, and the Reynolds viscosity model is adopted and approximated using Taylor series expansion. The resulting non-linear dimensionless equations are solved numerically using the Galerkin weighted residual method. A parametric study of the relevant physical parameters is presented graphically and discussed. The results indicate that the velocity profile exhibits an inverse relationship with the magnetic parameter, while the Bejan number increases with increasing magnetic parameter and decreases with increasing Hall parameter.

Keywords: Magnetohydrodynamics, variable viscosity, Joule heating, Bejan number

***Correspondence**
Akinremi. B.V., akinremibv@gmail.com; akinremiov@afued.edu.ng
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1.0 Introduction

Magnetohydrodynamic (MHD) flows in rectangular and cylindrical systems continue to attract significant interest in the fields of engineering science and applied mathematics. This interest arises from their numerous important applications in biological and engineering industries, such as reactive polymer flows, crude oil extraction, synthetic fibre production, paper manufacturing, as well as absorption and filtration processes in chemical engineering (Chinyoka & Makinde, 2010). The dynamics of reactive fluids flowing through pipes at low Reynolds numbers has long been an important subject in environmental engineering and science.

The steady flow of a reactive variable-viscosity fluid in a cylindrical pipe with an isothermal wall was studied by Makinde (2007), who reported the dependence of steady-state thermal ignition criticality conditions on both the Frank-Kamenetskii and viscous heating parameters. Makinde et al. (2013) numerically investigated entropy generation rates in the unsteady flow of a variable-viscosity incompressible fluid through a porous pipe with uniform suction at the surface. Ajadi (2009) obtained closed-form solutions using the homotopy analysis method to study the effects of variable viscosity and viscous dissipation on the thermal stability of a one-step exothermic reactive non-Newtonian flow in a cylindrical pipe, assuming negligible reactant consumption. In 2013, Aiyesimi et al. considered a mathematical model for dusty viscoelastic fluid flow in a circular channel and observed that increasing the magnetic field strength and viscoelastic parameter reduced the horizontal velocity of both the fluid and particles, thereby decreasing the boundary-layer thickness and increasing the absolute value of the velocity gradient at the surface. Srihari and Avinash (2015) examined the effects of radiation on unsteady MHD flow of a chemically reacting fluid past a hot vertical porous plate using a finite difference approach. They reported that the temperature and velocity of the fluid increase with increasing heat generation parameter, while both temperature and velocity decrease with increasing radiation parameter, and that fluid temperature increases with increasing Eckert number.

The effects of Hall current and chemical reaction on hydromagnetic flow over a stretching vertical surface with internal heat generation or absorption were investigated by Salem and Abd El-Aziz (2008). A finite element solution for heat and mass transfer flow incorporating Hall current, heat source, and viscous dissipation was presented by Sivaiah and Raju (2013).

Shateyi and Marewo (2014) employed a computational iterative approach known as the Spectral Local Linearisation Method (SLLM), combined with the Chebyshev spectral method, to study the effects of Hall current on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and heat source or sink. Aiyesimi et al. (2015) presented a similarity solution for the hydromagnetic boundary-layer flow of a nanofluid past a stretching sheet embedded in a Darcian porous medium with radiation, using the Adomian decomposition method.

Second law thermodynamic analysis and its design-related concept of entropy generation minimisation have become cornerstones in the field of heat transfer and thermal design. Several researchers have been motivated to investigate fundamental and applied engineering problems using second law analysis, owing to entropy production resulting from the combined effects of velocity and temperature gradients. Entropy generation is closely associated with thermodynamic irreversibility, which is inherent in all heat transfer processes. Eegunjobi and Makinde (2012) investigated the combined effects of buoyancy force and Navier slip on the entropy generation rate in a vertical porous channel with wall suction or injection. Chinyoka and Makinde (2013) studied the combined effects of Navier slip, convective cooling, variable viscosity, and suction or injection on the entropy generation rate in the unsteady flow of an incompressible viscous fluid through a channel with permeable walls.

In this paper, the motivation arises from the desire to gain a deeper understanding of the combined effects of a radially applied magnetic field and Hall current on the flow of a chemically reactive third-grade fluid. The relevant governing equations are solved numerically using the Galerkin weighted residual method (Finlayson, 1972; Jain, 1984; Cicelia, 2014; Keskin, 2019). The effects of various governing parameters on velocity, temperature, and concentration are presented and discussed. In addition, the entropy generation rate of a laminar MHD flow of a reactive third-grade fluid is analysed in a circular pipe, which is assumed to be electrically conducting and incompressible, in the presence of an externally applied radially exponential magnetic field.

2.0 Mathematical Formulation

A steady flow of electrically conducting, incompressible, third grade fluid with variable viscosity of Reynold's type $\mu = e^{-n(T-T_0)}$ in a non-conducting circular pipe in the absence of gravitational force is considered. The z-axis is taken along the axis of flow. A radially exponential varying magnetic field $B_r = B_0 e^{\frac{r}{2R}}$ is applied (Sahadeb, 1973) with Joule heating and no electric field is applied. The flow is induced due to constant applied pressure gradient in the z-direction and electron-atom collision frequency is assumed to be relatively high compared to the collision frequency of ions. The equations which govern the MHD flow are the continuity, momentum and Maxwell equations. In fluid dynamics studies, it is assumed that the flows meet the Clausius-Duhem inequality, and the specific Helmholtz free energy of fluid has a minimum at equilibrium (Rajagopal, 1980). Using the velocity field $V = (0,\ 0,\ \mathrm{w}(r)),$ the incompressibility condition is satisfied identically and momentum and Maxwell equations after the constitutive equations

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2\left(A_2A_1 + A_1A_2\right) + \beta_3(trA_1^2)A_1 + \gamma_1 A_4 + \gamma_2(A_3A_1 + A_1A_3)$$
$$+\gamma_3 A_2^2 + \gamma_4(A_2A_1^2 + A_1^2A_2) + \gamma_5(trA_2)A_2 + \gamma_6(trA_2)A_1^2 + \left(\gamma_7 trA_3 + \gamma_8 tr(A_2A_1)\right)A_1,$$

$$A_1 = gradV + (gradV)^T$$
$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}L + L^T A_{n-1}, \quad (n>1) \tag{2.1}$$

(Ellahi & Riaz, 2010; Reddy, et al. 2013) and the stated assumptions give

$$\frac{1}{r}\frac{d}{dr}\left(r\bar{\mu}(T)\frac{d\bar{w}}{dr}\right) + \frac{2\beta_3}{r}\frac{d}{dr}\left(r\left(\frac{d\bar{w}}{dr}\right)^3\right) - \frac{\partial\hat{p}}{\partial z} - \frac{\sigma B_0^2 e^{\frac{r}{R}}\bar{w}}{1+m^2} = 0 \tag{2.2}$$

$$\bar{\mu}(T)\left(\frac{d\bar{w}}{dr}\right)^2 + 2\beta_3\left(\frac{d\bar{w}}{dr}\right)^4 + k\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right] + Q_s\left(T-T_0\right) - \frac{dq_r}{dr} + \sigma B_0^2 e^{\frac{r}{R}}\bar{w}^2$$
$$+\frac{D_m\lambda_T}{c_s}\left(\frac{d^2C}{dr^2} + \frac{1}{r}\frac{dC}{dr}\right) = 0 \tag{2.3}$$

$$-\bar{w}\frac{dC}{dr} + D_m\left(\frac{d^2C}{dr^2} + \frac{1}{r}\frac{dC}{dr}\right) + \frac{D_m\lambda_T}{T_m}\left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right) - k_c\left(C-C_0\right) = 0 \tag{2.4}$$

with the boundary conditions

$$\frac{d\bar{w}}{dr} = 0,\ T(r) = T_0,\ C(r) = C_0 \quad \text{at} \quad r = 0$$
$$\bar{w}(r) = 0,\ T(r) = T_w,\ C(r) = C_w \quad \text{at} \quad r = R \tag{2.5}$$

where $w, T, B_0, \hat{p} = -p + \alpha\left(\frac{dw}{dr}\right)^2, \sigma, m, k, q, D_m, \lambda_T, c_p, k_c, T_0, T_w, C_0, C_w$ are fluid velocity, fluid temperature, applied magnetic field strength, modified pressure, electrical conductivity, Hall parameter, thermal conductivity, thermal radiation, molecular diffusivity, thermal diffusivity, specific heat capacity, chemical reaction rate constant, reference temperature, wall temperature, reference concentration and wall concentration. For the temperature dependent viscosity, considering the Reynold's model $\mu = e^{-n(T-T_0)}$ and following Ellahi (2013) we have $\mu = e^{-B\theta}$. The Taylor series expansion is used which gives

$$\mu \cong 1 - B\theta + \frac{(B\theta)^2}{2} + O(\theta^3), \quad \frac{1}{\mu} \cong 1 + B\theta + \frac{(B\theta)^2}{2} + O(\theta^3)$$

where $B = n(T_w - T_0)$ and assuming that $|B\theta| < 1$, then higher order terms can be neglected.

Introducing the following non-dimensional quantities (Ellahi, 2013) into (2.2) to (2.5) and the boundary conditions

$$w = w_0\bar{w},\ r = R\eta,\ \ T = (T_w - T_0)\theta + T_0,\ \ C = (C_w - C_0)\chi + C_0$$

$$\Lambda = \frac{2\beta_3 w_0^2}{\mu_0 R^2},\ M = \frac{\sigma B_0{}^2 R^2}{w_0\mu_0},\ c = \frac{R^2}{\mu_0 w_0}\frac{\partial\hat{p}}{\partial z},\ P_r = \frac{\mu_0 c_p}{k},\ E_c = \frac{w_0^2}{c_p(T_w - T_0)}, Q_H = \frac{Q_s R^2}{k},$$
$$D_u = \frac{D_m\lambda_T(C_w - C_0)}{kc_s(T_w - T_0)},\ J_H = \frac{\sigma B_0^2 R^2 c_p}{k(T_w - T_0)},\ R_p = \frac{16\sigma_*\rho c_p T_0{}^4}{3\delta_* k}, R_* = \frac{1}{1+R_p}, \Phi = \frac{K_c R^2}{D_m}, \tag{2.6}$$
$$S_c = \frac{w_0 R}{D_m}, S_r = \frac{D_m\lambda_T(T_w - T_0)}{T_m R w_0(C_w - C_0)}$$

and using Rosseland's approximation

$$q_r = -\frac{4\sigma_*}{3\delta_*}\frac{\partial T^4}{\partial r} \tag{2.7}$$

$\Lambda, M, c,\ P_r, E_c, Q_H,\ \beta_*, D_u, J_H, R_p, S_c, K_R, \sigma_*, \delta_*$ denotes third grade parameter, magnetic parameter, pressure drop, Prandtl number, Eckert number, heat source/sink parameter, material constant parameter, Dufour number, radiation parameter, Schmidt number, chemical reaction parameter, Stefan-Boltzmann constant and mean absorption coefficient. For steady flow, the time dependent terms are set to zero and the following equations were obtained respectively with the boundary conditions

$$\frac{d^2w}{d\eta^2} = -\frac{1}{\eta}\frac{dw}{d\eta} - \left(B + B^2\theta + \frac{B^2\theta^2}{2}\right)\frac{d\theta}{d\eta}\frac{dw}{d\eta} - \left(2B^2 + B^3\theta + \frac{B^4\theta^2}{2}\right)\frac{d\theta}{d\eta}\frac{dw}{d\eta}$$
$$-\frac{\Lambda}{\eta}\left(1 + B\theta + \frac{B^2\theta^2}{2}\right)\left(\frac{dw}{d\eta}\right)^3 - 3\Lambda\left(1 + B\theta + \frac{B^2\theta^2}{2}\right)\left(\frac{dw}{d\eta}\right)^2\frac{d^2w}{d\eta^2} + c \tag{2.8}$$
$$+ cB\theta + \frac{cB^2\theta^2}{2} + \left(1 + B\theta + \frac{B^2\theta^2}{2}\right)\frac{Me^\eta w}{1+m^2}$$

$$\frac{d^2\theta}{d\eta^2} = -\frac{R_*}{\eta}\frac{d\theta}{d\eta} - P_rE_cR_*\left(\frac{dw}{d\eta}\right)^2 - BP_rE_cR_*\theta\left(\frac{dw}{d\eta}\right)^2 - \frac{B^2P_rE_cR_*\theta^2}{2}\left(\frac{dw}{d\eta}\right)^2$$
$$-\Lambda P_rE_cR_*\left(\frac{dw}{d\eta}\right)^4 - Q_HR_*\theta - J_HE_cR_*e^\eta w^2 - D_uR_*\frac{d^2\chi}{d\eta^2} - \frac{D_uR_*}{\eta}\frac{d\chi}{d\eta} \tag{2.9}$$

$$\frac{d^2\chi}{d\eta^2} = S_cw\frac{d\chi}{d\eta} - \frac{1}{\eta}\frac{d\chi}{d\eta} - S_cS_r\frac{d^2\theta}{d\eta^2} - \frac{S_cS_r}{\eta}\frac{d\theta}{d\eta} + S_c\Phi\chi \tag{2.10}$$

Boundary conditions

$$\frac{dw}{d\eta} = 0, \quad \theta(\eta) = 0, \quad \chi(\eta) = 0 \quad \text{at} \quad \eta = 0$$
$$w(\eta) = 0, \quad \theta(\eta) = 1, \quad \chi(\eta) = 1 \quad \text{at} \quad \eta = 1 \tag{2.11}$$

Equations (2.8), (2.9), (2.10) and (2.11) comprise the boundary value problem to now be solved.

3.0 Method of Solution (Galerkin Weighted Residual)

Suppose an approximate solution is to be determined for the differential equation of the form

$$L(\phi) + f = 0 \tag{3.1}$$

where $\phi(x)$ is an unknown dependent variable, L is a differential operator and $f(x)$ is a known function. Let $\psi(x) = \sum_{i=1}^{N} c_iu_i(x)$ be an approximate solution to (2.8). On substituting $\psi(x)$ into (2.8), it is unlikely that (2.8) is satisfied i.e. $L(\psi)+f\neq 0$ therefore

$$L(\psi)+f=R \tag{3.2}$$

where $R(x)$ is a measure of error called the Residual (Baluch, 1983, Jain, 1984). Multiplying (3.2) by an arbitrary weight function $u(x)$ and integrating over the domain to obtain

$$\int_D u(x)\left[L(\psi)+f\right]dD=\int_D u(x)R(x)dD\neq 0 \tag{3.3}$$

Galerkin Weighted Residual method ensures equation (3.3) vanishes over the solution domain and the weight function is chosen from the basic functions $u(x)=u_i(x)\quad (i=0,...,N)$ hence

$$\langle u,R\rangle=\int_D u(x)R(x)dD=\int_D u_i(x)\left[L(u_0(x)+\sum_{i=1}^{N}c_iu_i(x))+f\right]dD=0 \tag{3.4}$$

These are a set of n-order linear equations to be solved to obtain all the c_i coefficients. The trial functions can be polynomials, trigonometric functions etc. The trial functions are usually chosen in such that the assumed function $\psi(x)$ satisfies the global boundary conditions for $\phi(x)$ though this is not strictly necessary and certainly not always possible (Finlayson, 1972). To apply the method to (2.8)-(2.10), we select an approximate solutions of the form $\psi_w(\eta)=a_0+a_1\eta+a_2\eta^2$, $\psi_\theta(\eta)=b_0+b_1\eta+b_2\eta^2$, $\psi_\chi(\eta)=c_0+c_1\eta+c_2\eta^2$ for the velocity, temperature and concentration respectively, which satisfies the boundary conditions (2.11). Applying the boundary conditions on the approximate solution we obtain the following:

$w(\eta)=a_0(1-\eta^2)$, $\theta(\eta)=\eta^2+b_1(\eta-\eta^2)$, $\chi(\eta)=\eta^2+c_1(\eta-\eta^2)$ and $u_1=(1-\eta^2)$, $u_2=(\eta-\eta^2)$, $u_3=(\eta-\eta^2)$ are the weighting functions u_i, where a_1,b_1,c_1 are the coefficients to be determined.

The residue *R* for (2.7) -(2.9) respectively are given by

$$\begin{aligned}&\frac{1}{420(Bs+2)^4}(120960(-\frac{1}{36}+(b_1-\frac{31}{36})B_c)a_0Me-88\Lambda(\frac{56}{11}+(b_1+\frac{24}{11})B_c)(m^2+1)a_0^{\,3}\\&+(2(b_1^{\,2}-9b_1-48)(m^2+1)B_c^{\,2}+((14b_1-224)m^2+(-328860M+14)b_1+283080M\\&-224)B_c-1120m^2+8820M-1120)a_0+49(\frac{40}{7}+(b_1+\frac{8}{7})B_c)(m^2+1)=0\end{aligned} \tag{3.5}$$

$$\begin{aligned}&-288E_cJ_hea_0^{\,2}+\frac{1}{420}(160E_c\Pr\Lambda a_0^{\,4}+16E_c\Pr(B_cb_1+\frac{5}{2}B_c+\frac{21}{4})a_0^{\,2}+(14Q_H+70)b_1\\&-70(c_1-4))Du+21Q_H+140)R_p+783E_cJ_ha_0^{\,2}-\frac{1}{3}(b_1-1)=0\end{aligned} \tag{3.6}$$

$$\frac{1}{60}(-S_ca_0-2\Phi-10S_r-10)c_1-\frac{S_ca_0}{10}-\frac{\Phi}{20}+\frac{2S_r}{3}+\frac{2}{3}=0 \tag{3.7}$$

Considering the orthogonality of the residues above, we have

$$\langle u_1, R_a\rangle = \int_0^1 \begin{bmatrix} (1-\eta^2)(\frac{1}{420(Bs+2)^4}(120960(-\frac{1}{36}+(b_1-\frac{31}{36})B_c)a_0 Me - 88\Lambda(\frac{56}{11}+(b_1+\frac{24}{11})B_c) \\ (m^2+1)a_0^{\ 3} + (2(b_1^{\ 2}-9b_1-48)(m^2+1)B_c^{\ 2} + ((14b_1-224)m^2 + (-328860M + \\ 14)b_1 + 283080M - 224)B_c - 1120m^2 + 8820M - 1120)a_0 + 49(\frac{40}{7}+(b_1+\frac{8}{7})B_c) \\ (m^2+1)) \end{bmatrix} d\eta = 0$$

$$\langle u_2, R_b\rangle = \int_0^1 \begin{bmatrix} (\eta-\eta^2)(-288E_c J_h e a_0^{\ 2} + \frac{1}{420}(160E_c \Pr \Lambda a_0^{\ 4} + 16E_c \Pr(B_c b_1 + \frac{5}{2}B_c + \frac{21}{4})a_0^{\ 2} \\ +(14Q_H+70)b_1 - 70(c_1-4))Du + 21Q_H + 140)R_p + 783E_c J_h a_0^{\ 2} - \frac{1}{3}(b_1-1)) \end{bmatrix} d\eta = 0$$

$$\langle u_3, R_c\rangle = \int_0^1 \left[(\eta-\eta^2)(\frac{1}{60}(-S_c a_0 - 2\Phi - 10S_r - 10)c_1 - \frac{S_c a_0}{10} - \frac{\Phi}{20} + \frac{2S_r}{3} + \frac{2}{3}) \right] d\eta = 0$$

The symbolic calculation software MAPLE 2022 is used to compute the values of a_1, b_1, c_1 and the approximate solutions.

4.0 Irreversibility Ratio

Inherent irreversibility in a pipe flow occurs owing to exchange of energy and momentum within the fluid and the solid boundaries. The entropy generation is owed to heat transfer and the effects of fluid friction. The equation for rate of entropy generation per unit volume (Makinde *et al.*, 2013; Chinyoka & Makinde, 2013) is given

$$S^m = \frac{k}{T_w^2}\left(\frac{dT}{dr}\right)^2 + \frac{\mu}{T_w}\left(\frac{dw}{dr}\right)^2 + \frac{2\beta_3}{T_w}\left(\frac{dw}{dr}\right)^4 \tag{4.1}$$

where the first term in (4.1) is the irreversibility due to heat transfer, the second and third terms are entropy generation due to viscous dissipation. Introducing the dimensionless quantities in (2.6) to (4.1), we have

$$N_s = \frac{r^2 S^m}{k} = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2 + \frac{B_R\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_*\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \tag{4.2}$$

where $\Omega = \frac{T_w}{T_w - T_0}, B_R = \frac{\mu w_0^{\ 2}}{k(T_w - T_0)}, \beta_* = \frac{\beta_3 w_0^{\ 4}}{kR^2(T_w - T_0)}$ are temperature difference parameter, Brickman number and third grade parameter and

$$N_{Ihf} = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2, N_{Svd} = \frac{B_R\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_*\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \tag{4.3}$$

where N_{Ihf} is irreversibility due to heat transfer and N_{Svd} gives entropy generation due to viscous dissipation. The Bejan number is defined as

$$B_e = \frac{N_{Ihf}}{N_{Svd}} \tag{4.4}$$

such that $0 \le B_e \le 1$ denoting $B_e = 1$ is the limit at which heat transfer irreversibility dominates , $B_e = 0$ is the limit at which fluid friction irreversibility dominates, and $B_e = \frac{1}{2}$ connotes equal contribution.(Bejan, 1996)

5.0 Results

In this investigation, the application of a radially exponential magnetic field to reactive MHD third grade fluid flow in the presence of Joule heating and variable viscosity using the Galerkin weighted residual method was considered. The velocity, temperature and chemical functions are analysed by computing their numerical values and plotting their respective graphs against various thermophysical variables of interest.

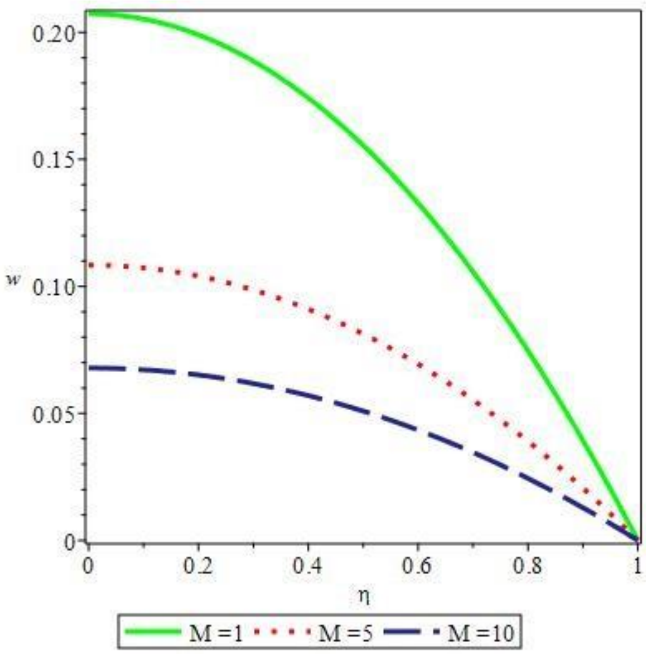


Figure 1 Velocity Profile for various valves of Magnetic parameter

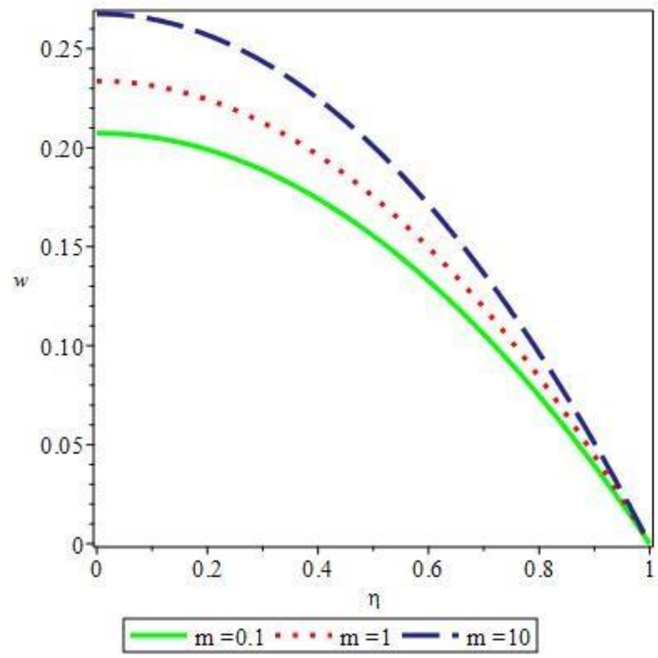


Figure 2 Velocity Profile for various valves of Hall parameter

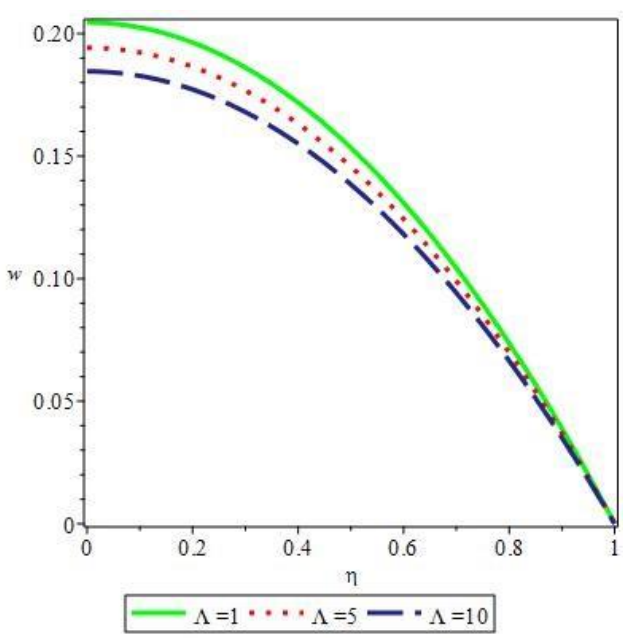


Figure 3 Velocity profile for various values of Third grade parameter

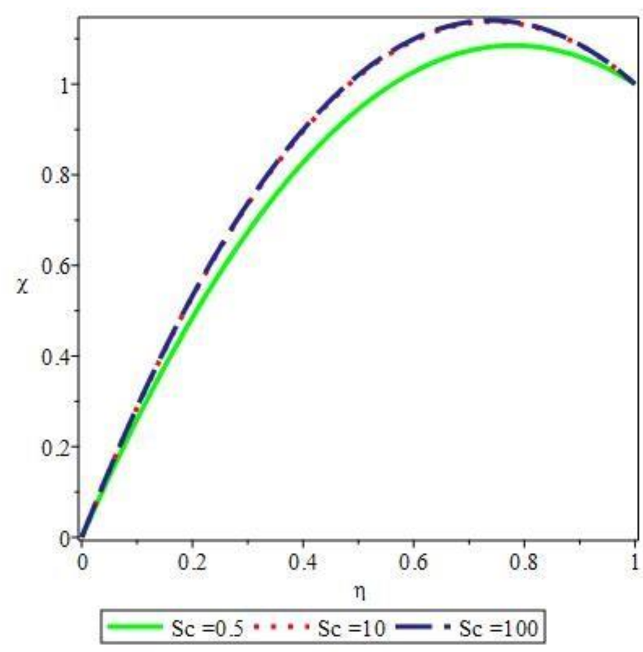
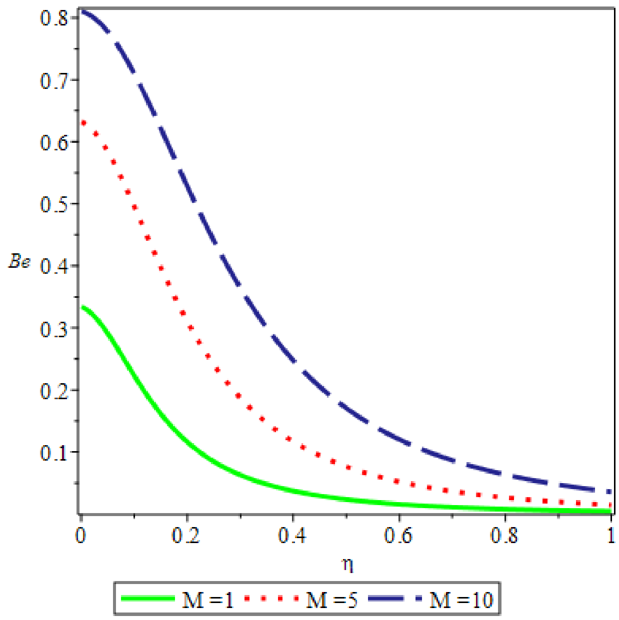
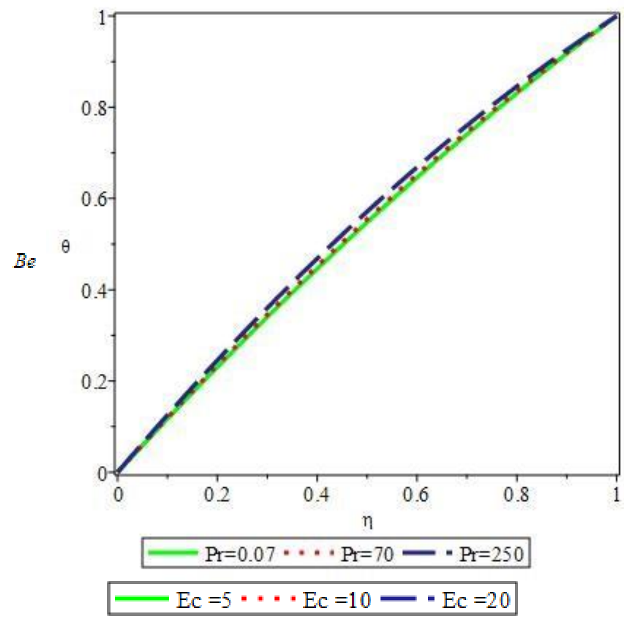


Figure 4 Velocity profile for various values of radiation parameter

Figure 6 Temperature profile for various values of Prandtl number





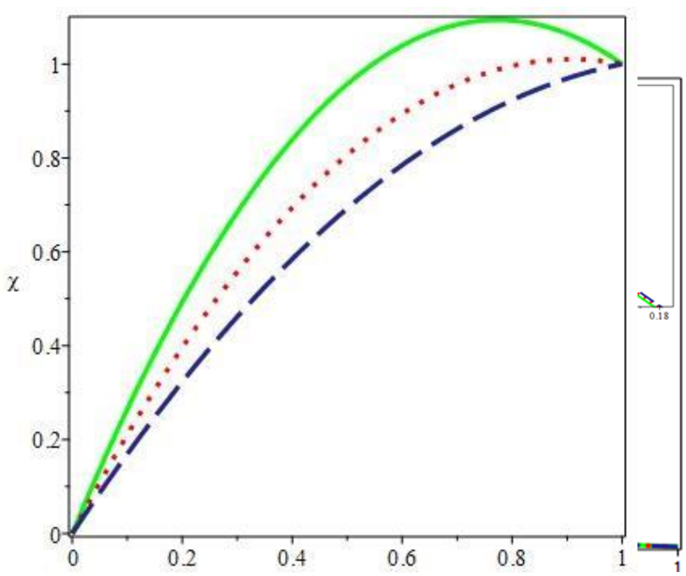
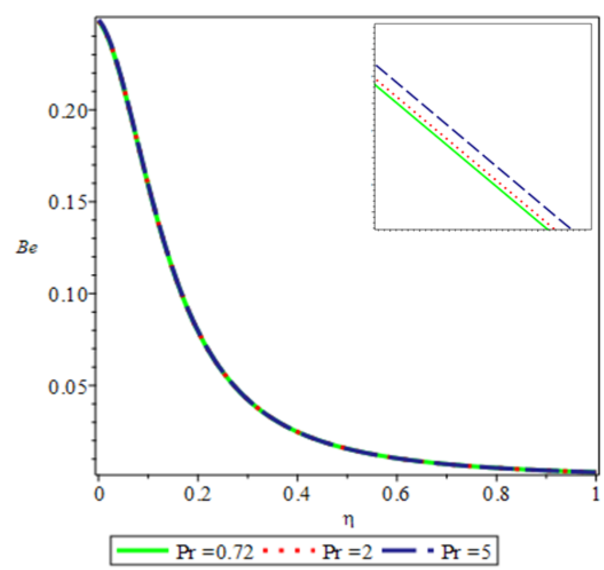


Figure 9 Variation of Bejan number for various values of Magnetic parameter



6.0 Discussion

Figures 1 to 4 illustrates the effect of magnetic, Hall, third grade and radiation parameters on velocity profiles for steady MHD fluid flow with radially applied magnetic field and variable viscosity. In Figure 1, magnetic parameter was varied between $M = 1$ and $M = 10$, while other parameters were held constant that is $\Lambda = 0.1, B_c = 1, m = 1, E_c = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ R_p = 0.1$, result showed that magnetic parameter enhances the velocity as it reduces and converging to zero at $\eta = 1$. In Figure 2, Hall parameter was varied between $m = 0.1$ and $m = 10$, while other parameters were held constant that is $\Lambda = 0.1, B_c = 1, M = 0.1, E_c = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ R_p = 0.1$, result showed that the Hall parameter enhances the flow field as it increases. Figure 3 shows that decreasing the thirdgrade parameter enhances the velocity profile, when varying the third grade parameter between $\Lambda = 1$ and $\Lambda = 10$, while other parameters were held constant that is $\text{M} = 1, B_c = 1, m = 1, E_c = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ R_p = 0.1$. In Figure 4, radiation parameter was varied between $R_p = 0.1$ and $R_p = 0.4$, while other parameters were held constant that is $\Lambda = 0.1, B_c = 1, m = 1, E_c = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ M = 1$, result showed that radiation has the tendency of increasing the flow profile as it increases.

Figures 5 and 6 show the effect of Eckert and Prandtl number on temperature profile. It is observed that increase in Eckert and Prandtl number enhances the temperature profile when varied between $E_c = 0.1$ and $E_c = 100$, and $P_r = 0.07\ and\ P_r = 250$ respectively. Figure 7 depicts chemical reaction parameter impeding the concentration profile with higher magnitude of influence when $K_r = 1$ when varied between $K_r = 1$ and $K_r = 4$. Schmidt number enhances the concentration profile with higher magnitude of influence at $S_c = 100$ when varied from $S_c = 0.5$ and $S_c = 100$, while keeping other parameters is $\Lambda = 0.1, B_c = 1, m = 1, E_c = 0.1, R_p = 0.4, K_r = 1,\ J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ M = 1$ constant.

Figures 9 to 12 indicate that the irreversibility due to heat transfer dominates from the centreline of the pipe for Magnetic parameter, Eckert number, Dufour number and Prandtl number respectively. Figure 9 shows that the Bejan number increases with increase in magnetic parameter varied between $M = 1\ and\ M = 10$ for constant parameters $\Lambda = 0.1, B_c = 1, m = 0.1, E_c = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ R_p = 0.1$, while figure 10 describes increasing the Eckert number varied between $E_c = 5\ and\ E_c = 20$ for constant parameters $\Lambda = 0.1, B_c = 1, m = 0.1, M = 1, D_{uf} = 0.1, J_H = 0.71, Q_H = 0.1, P_r = 0.72\ and\ R_p = 0.1$ increases the Bejan number. The Dufour and Prandtl numbers were varied between 0.1 to 0.5, and 0.72 to 5 respectively against the Bejan number, increase in both Dufour and Prandtl numbers showed appreciable increase in Bejan number which is maximum at at the pipe centre line and decreases close to the pipe wall.

η	$w(\eta)$ RK4	$w(\eta)$ GM	$\theta(\eta)$ RK4	$\theta(\eta)$ GM	$\chi(\eta)$ RK4	$\chi(\eta)$ GM
0	0.2426873919	0.242897239	0	0	0	0
0.2	0.2329296447	0.233181350	0.212145367	0.228824682	0.274100248	0.485904134
0.4	0.1974790573	0.184256705	0.418372663	0.443237038	0.497472225	0.828856201
0.6	0.1483545360	0.140386061	0.618504523	0.643237038	0.685937704	1.028856201
0.8	0.0825577950	0.078967159	0.814154594	0.828824692	0.850546605	1.085904134
1.0	0	0	1.0	1.0	1.0	1.0

Table 6: *Comparison of GM Result for Steady Flow with Radial Magnetic Field and Variable Viscosity with RK4*

Table 6 shows the computational result comparison between the Galerkin Weighted Residual method solution and Runge Kutta method solution for the velocity, temperature and concentration profiles respectively for

$\Lambda = M = m = Q_H = R_H = E_c = D_u = Bi = 0.1, \vartheta a = \text{Re} = 0.2, S_r = 0.5, S_c = 0.6, \text{Pr} = J_H = 0.72, \Phi = 1$. The two solutions can be seen to have good agreement.

7.0 Conclusion

In this numerical investigation, the irreversibility ratio of a steady reactive magnetohydrodynamic third-grade fluid flow in a circular pipe is presented using the Galerkin method. Numerical expressions for the velocity, temperature and concentration were obtained which were used to compute the entropy generation number. Special emphasis has been focused here to the variations of pertinent parameters of physical significance on the Bejan number. The main findings of the present analysis are:

- The velocity is enhanced for increasing values of m, R_p and inhibited for M, Λ
- The temperature is enhanced for values of Ec, Pr and inhibited for Pr, Re and Du
- The concentration is enhanced for values of S_c and inhibited for K_r
- M, E_c, D_u and Pr enhances the Bejan number showing the dominance of irreversibility due to heat transfer over entropy generation due to viscous dissipation in the flow regime.

8.0 Ethical Considerations

Not applicable, as the research does not involve living subjects or sensitive data.

9.0 Limitations

Not applicable.

10.0 References

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