Paper - 9

Board's Question Paper (March - 2024)

Time : 1 Hour] **PART - A** **[Total Marks : 50**

❖ **Select the following questions with proper alternative and answer it :**

1) The projection of the vector $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ on the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is ....

(A) $\frac{\sqrt{10}}{6}$ (B) $\frac{10}{\sqrt{6}}$ (C) $\frac{\sqrt{10}}{17}$ (D) $\frac{10}{\sqrt{17}}$

2) The area of a parallelogram whose adjacent sides are $\vec{a} = \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j}$ is ....

(A) $\sqrt{42}$ (B) $2\sqrt{21}$ (C) $\sqrt{21}$ (D) $\frac{1}{2}\sqrt{21}$

3) For the vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}| = \frac{2}{3}$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is ....

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

4) Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$, then the vector equation of the line is ....

(A) $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$

(B) $\vec{r} = 3\hat{i} + 7\hat{j} - 2\hat{k} + \lambda(5\hat{i} - 4\hat{j} + 6\hat{k})$

(C) $\vec{r} = 3\hat{i} + 7\hat{j} + 2\hat{k} + \lambda(5\hat{i} - 4\hat{j} + 6\hat{k})$

(D) $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

5) The angle between the pair of lines

$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ and

$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$ is ....

(A) $\cos^{-1}\left(\frac{17}{21}\right)$ (B) $\sin^{-1}\left(\frac{17}{21}\right)$

(C) $\sin^{-1}\left(\frac{19}{21}\right)$ (D) $\cos^{-1}\left(\frac{19}{21}\right)$

6) If the lines $\frac{1-x}{3} = \frac{y-2}{1} = \frac{z-1}{2}$ and $\frac{x-2}{p} = \frac{y-1}{2} = \frac{z-2}{1}$ are perpendicular to each other, then $p =$ ....

(A) 0 (B) $-\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{4}{3}$

7) For linear programming problem, the objective function is $Z = 3x + 2y$. If the corner points of the bounded feasible region are (12, 0), (4, 2), (1, 5) and (0, 10), then the maximum value of Z is ....

(A) 46 (B) 36 (C) 13 (D) 56

8) Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum value of Z occurs at (3, 0) and (1, 1) is ....

(A) $p = \frac{q}{2}$ (B) $p = 2q$

(C) $p = 3q$ (D) $p = q$

9) The probability of obtaining even number on each die, when a pair of dice is rolled is ....

(A) $\frac{1}{2}$ (B) $\frac{1}{9}$ (C) $\frac{1}{4}$ (D) $\frac{1}{36}$

10) Given that events A and B are such that $P(A) = 0.5$, $P(A \cup B) = 0.6$, $P(B) = K$. If A and B are mutually exclusive events then K = ...

(A) 0.2 (B) 0.1 (C) 0.11 (D) 0

11) The relation $R = \{(a, b), (b, a)\}$ is defined on the set $\{a, b, c\}$, then R is ....

(A) Symmetric, but not reflexive and transitive

(B) Reflexive, but not symmetric and transitive

(C) Transitive, but not reflexive and symmetric

(D) An equivalence relation

12) Function defined as $f : N \to N$, $f(x) = x^6$, then ....

(A) f is one-one and onto

(B) f is many-one and onto

(C) f is one-one, but not onto

(D) f is neither one-one nor onto

13) Let A = {1, 2, 3}, then number of equivalence relations containing (1, 2) is ....
(A) 3 (B) 4 (C) 2 (D) 1

14) $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = ...
(A) π (B) $\frac{\pi}{2}$ (C) $\frac{5\pi}{6}$ (D) 0

15) $\tan^{-1}\left(\tan\frac{31\pi}{6}\right)$ = ...
(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{31\pi}{6}$ (D) $-\frac{\pi}{6}$

16) If $\cos^{-1}x = y$, then ....
(A) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (B) $-\frac{\pi}{2} < y < \frac{\pi}{2}$
(C) $0 < y < \pi$ (D) $0 \le y \le \pi$

17) $\cos(\tan^{-1}x)$ = ... ($|x| < 1$)
(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{x}{\sqrt{1-x^2}}$
(C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

18) The number of all possible matrices of order 3 × 2 with each entry 1 or 2 is ....
(A) 64 (B) 512 (C) 32 (D) 128

19) If $A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}4 & 0\\1 & -2\\0 & 3\end{bmatrix}$, then AB = ...
(A) $\begin{bmatrix}4 & 0\\1 & -2\\0 & 3\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
(C) $\begin{bmatrix}4 & 0\\1 & -2\end{bmatrix}$ (D) not defined

20) If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ = ...
(A) I – A (B) A (C) I (D) 3A

21) If $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then A^{10} = ...
(A) 512 A (B) 1024 A
(C) 10 A (D) A

22) If $f(\theta) = \begin{vmatrix}\cos\theta & -\sin\theta\\ \sin\theta & -\cos\theta\end{vmatrix}$, then $f\left(\frac{\pi}{6}\right)$ = ...
(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

23) If $A = \begin{bmatrix}1 & 0 & 0\\0 & 2 & 3\\1 & 0 & 1\end{bmatrix}$, then A |adj A| = ...
(A) 4 (B) 2 (C) 8 (D) 6

24) If $A = \begin{bmatrix}5 & -2\\4 & 3\end{bmatrix}$, then A (adj A) = ...
(A) A (B) I (C) 23 I (D) 23 A

25) If area of a triangle is 3 sq. units with the vertices (3, 5), (2, 2) and (k, 2), then k = ...
(A) 0, –4 (B) 0, 4 (C) 3, 1 (D) –3, 1

26) If $f(x) = \begin{cases} kx + 1, & x \le \frac{\pi}{2} \\ \sin x, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k = ...
(A) $\frac{2}{\pi}$ (B) $-\frac{2}{\pi}$ (C) 1 (D) 0

27) If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$, then $\frac{dy}{dx}$ = ...
(A) y (B) y – 1
(C) 0 (D) does not exit

28) Differentiation of $\sin^2x$ w.r.t. $\cos^2x$ is ....
(A) $-\tan^2x$ (B) $\tan^2x$ (C) –1 (D) 1

29) The rate of change of the volume of a sphere with respect to radius r, at r = 3 cm is .... cm^3/s.
(A) 36π (B) 12π (C) 24π (D) 81π

30) The total revenue in Rupees received from the sale of x units of a product is given by, $R(x) = x^2 + 6x + 5$. The marginal revenue, when x = 20 is ₹ ....
(A) 126 (B) 525 (C) 46 (D) 96

31) The function given by $f(x) = x^2 - 6x + 10$ is an increasing in .... interval.
(A) $(-\infty, 3)$ (B) $(3, \infty)$
(C) (–3, 3) (D) (0, 6)

32) The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is ....
(A) (0, 0) (B) (2, 2)
(C) $(2\sqrt{2}, 0)$ (D) $(2\sqrt{2}, 4)$

33) $\int \frac{\text{cosec}^2 x}{\sec^2 x}dx$ = ... + c
(A) $-\cot x - x$ (B) $\tan x - x$
(C) $\cot x - x$ (D) $-\cot x + x$

34) $\int \frac{1}{x + x\log x} dx$ = ... + c

(A) $\log|\log x|$ (B) $1 + \log x$

(C) $\log|\log ex|$ (D) $\frac{(1+\log x)^2}{2}$

35) $\int \frac{1}{e^x + 1} dx$ = ... + c

(A) $\log\left|\frac{e^x+1}{e^x}\right|$ (B) $\log\left|\frac{e^x}{e^x+1}\right|$

(C) $\log\left|\frac{1}{e^x+1}\right|$ (D) $\log\left|\frac{e^x-1}{e^x+1}\right|$

36) $\int \frac{dx}{x^2 + 2x + 5}$ = ... + c

(A) $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$ (B) $\tan^{-1}\left(\frac{x+1}{2}\right)$

(C) $\tan^{-1}(x+1)$ (D) $\frac{1}{2}\tan^{-1}(x+1)$

37) $\int_{-1}^{1} \sin^7 x \cdot \cos^6 x \, dx$ = ...

(A) 2 (B) –1 (C) 0 (D) 1

38) $\int e^x \tan x(1 + \tan x) \, dx$ = ... + c

(A) $e^x \tan x$ (B) $e^x (\tan x - 1)$

(C) $e^x \sec x$ (D) $e^x (\tan x + 1)$

39) $\int_0^{2\pi} \sin^3 x \cos^2 x \, dx$ = ...

(A) –1 (B) 2π (C) 1 (D) 0

40) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos^{\frac{3}{2}} x}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x} dx$ = ...

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{2}$

41) The area bounded by the curve $y = \cos x$ between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ is ....

(A) 2 (B) 1 (C) 3 (D) 4

42) Area of the region bounded by the curve $y^2 = 4x$, Y-axis and the line $y = 3$ is ....

(A) 3 (B) $\frac{9}{2}$ (C) $\frac{9}{4}$ (D) 2

43) The area bounded by the curve $y = x|x|$, X-axis and the ordinates $x = 0$ and $x = 1$ is given by ....

(A) $\frac{1}{3}$ (B) 0 (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

44) The order of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is ....}$$

(A) 3 (B) 4

(C) 2 (D) undefined

45) The number of arbitrary constants in the particular solution of a differential equation of fourth order are ....

(A) 4 (B) 0 (C) 3 (D) 2

46) The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ is ....

(A) $\log x$ (B) $2\log x$ (C) $\frac{2}{x}$ (D) x^2

47) The general solution of a differential equation $\frac{y\,dx - x\,dy}{y} = 0$ is ....

(A) $y = Cx$ (B) $y = Cx^2$

(C) $x = Cy^2$ (D) $xy = C$

48) The vector in the direction of a vector $\vec{a} = 4\hat{i} + 3\hat{j} - 2\hat{k}$, that has magnitude $2\sqrt{29}$ is ....

(A) $4\hat{i} + 3\hat{j} - 2\hat{k}$

(B) $\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

(C) $8\hat{i} + 6\hat{j} - 4\hat{k}$

(D) $\frac{2}{\sqrt{29}}\hat{i} + \frac{3}{2\sqrt{29}}\hat{j} - \frac{1}{\sqrt{29}}\hat{k}$

49) The direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), directed from A to B is ....

(A) $\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$ (B) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

(C) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ (D) $\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

50) The angle between the vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ is ....

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) 0

Time : 2 Hours] **PART - B** **[Total Marks : 50**

Section - A

❖ **Answer any 8 of the following given question 1 to 12 : (Each carries 2 marks)** **[16]**

1) Prove that : $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.

2) Prove that : $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$, where $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.

3) If $x^y + y^x = 1$, then find $\frac{dy}{dx}$.

4) Obtain $\int \frac{dx}{\sqrt{8+3x-x^2}}$

5) Find the area of the region bounded by the ellipse $9x^2 + 16y^2 = 144$.

6) Find the area of the region bounded by the line $y = 3x + 2$, the X-axis and the ordinates $x = -1$ and $x = 1$.

7) Find the general solution of the differential equation : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$.

8) If vertices of triangle are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$, determine the type of triangle they form.

9) Find the Cartesian equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

10) Find vector and the Cartesian equations of the line through the point (5, 2, –4) and which is parallel to the vector $3\hat{i} - 2\hat{j} + 8\hat{k}$.

11) Given that A and B are events such that P(A) = 0.6, P(B) = 0.3, P(A ∩ B) = 0.2. Find P(A|B) and P(B|A).

12) An unbiased die is thrown twice. Let the event E be 'odd number on the first throw' and F the event 'odd number on the second throw'. Check the independence of events E and F.

Section - B

❖ **Answer any 6 of the following given questions 13 to 21 : (Each carries 3 marks)** **[18]**

13) Check whether the relation R in R defined by $R = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

14) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 = 6A^2 - 7A - 2I$.

15) Using the cofactor of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

16) If $y = (\tan^{-1}x)^2$, show that

$$(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} - 2 = 0$$

17) Find the maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval $x \in [1, 3]$. Find the maximum value of the same function in $x \in [-3, -1]$.

18) The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

19) Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and}$$

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}.$$

20) Solve the following linear programming problem graphically :
For $Z = -3x + 4y$
Subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$.
Find minimum and maximum value of Z.

21) In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolts is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B ?

Section - C

❖ **Answer any 4 of the following given questions from 22 to 27 : (Each carries 4 marks)** **[16]**

22) Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

23) Using matrix method, solve the system of equations :

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 6$, $\frac{1}{y} + \frac{3}{z} = 11$, $\frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 0$

24) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\left(\frac{d^2y}{dx^2}\right)_{t = \frac{\pi}{4}}$.

25) Show that the semi-vertical angle of the cone of the maximum volume and of the given slant height is $\tan^{-1}\sqrt{2}$.

26) Evaluate : $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$

27) Find the particular solution of the differential equation $2ye^{\frac{x}{y}}\, dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$, given that $x = 0$ when $y = 1$.

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