Paper - 7

Board's Question Paper (Based on March - 2023)

Time : 1 Hour] **PART - A** **[Total Marks : 50**

❖ **Select the following questions with proper alternative and answer it :**

1) The value of $\hat{k}\cdot(\hat{i}\times\hat{j})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{i}\cdot(\hat{j}\times\hat{k})$ is .... [March - 2023]

(A) 3 (B) –1 (C) 1 (D) 0

2) For vectors $\vec{a},\vec{b},\vec{c}$ if $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ = ... [March - 2023]

(A) 0 (B) –19 (C) 1 (D) 38

3) Let $\vec{a}=2\hat{i}-7\hat{j}+5\hat{k}$, $\vec{b}=\hat{i}+\hat{k}$, $\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$ then $|\vec{r}|$ = ...

(A) $\frac{11}{7}$ (B) $\frac{11}{5}\sqrt{2}$ (C) $\frac{\sqrt{914}}{7}$ (D) $\frac{11}{7}\sqrt{2}$

4) For any vectors $\vec{a}$ and $\vec{b}$, we always have $|\vec{a}||\vec{b}|$ .... $|\vec{a}\cdot\vec{b}|$. [March - 2023]

(A) ≥ (B) ≤ (C) > (D) <

5) If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$ and $\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$. and $\vec{c}=\hat{i}-2\hat{j}-\hat{k}$) such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to ....

(A) 2 (B) 4 (C) 8 (D) 10

6) The direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), directed from A to B is ....

(A) $\frac{1}{3},-\frac{2}{3},\frac{2}{3}$ (B) $-\frac{1}{3},-\frac{2}{3},\frac{2}{3}$

(C) $\frac{1}{3},\frac{2}{3},-\frac{2}{3}$ (D) $\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$

7) The equation of line passing through (2, –3, 5) and makes congruent angles with axes is ......

(A) $\vec{r}=(2,-3,5)+k(1,1,1),\ k\in R$

(B) $\vec{r}=(2,-3,5)+k\left(\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right),\ k\in R$

(C) $\vec{r}=(2,-3,5)+k\left(\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right),\ k\in R$

(D) $\vec{r}=(2,-3,5)+k(-1,-1,1),\ k\in R$

8) The corner points of the feasible region determined by the following system of linear inequalities : $2x+y\le 10$, $x+3y\le 15$, $x, y\ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z=px+qy$, where $p, q>0$, condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is .... [March - 2023]

(A) $p=q$ (B) $p=3q$ (C) $p=2q$ (D) $q=3p$

9) For linear programming problem the objective function $Z=10500x+9000y$, if the corner points of the bounded feasible region are (0, 0), (40, 0), (30, 20) and (0, 50), then the maximum value of Z is .... [March - 2023]

(A) 5,95,000 (B) 4,95,000

(C) 6,20,000 (D) 4,50,000

10) If $\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then value of Δ is given by ....

(A) $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}$

(B) $a_{11}A_{11}+a_{12}A_{21}+a_{13}A_{31}$

(C) $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$

(D) $a_{11}A_{11}+a_{21}A_{21}+a_{31}A_{31}$

11) Two events A and B will be independent, if .... [March - 2023]

(A) A and B are mutually exclusive

(B) P(A) = P(B)

(C) P(A' ∩ B') = [1 – P(A)] [1 – P(B)]

(D) P(A) + P(B) = 1

12) If A and B are two events such that P(A) ≠ 0 and P(B/A) = 1, then .... [March - 2023]

(A) A ⊂ B (B) B = ϕ (C) B ⊂ A (D) A = ϕ

13) Which of the following is a homogeneous differential equation ?

(A) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(B) $(xy)dx - (x^3 + y^3)dy = 0$

(C) $(x^3 + 2y^2)dx + 2xy\ dy = 0$

(D) $y^2\ dx + (x^2 - xy - y^2)dy = 0$

14) W is the set of all words of English alphabet. The relation R on W is defined as follow : R = {(x, y) ∈ W × W, x and y has at least one letter common}. Then R is,

(A) reflexive and symmetric but not transitive.

(B) reflexive, symmetric and transitive.

(C) reflexive and transitive but not symmetric.

(D) transitive and symmetric but not reflexive.

15) $f: R \to R$, $f(x) = 3^x + 3^{|x|}$ is ......

(A) one-one and onto.

(B) one-one but not onto.

(C) many-one and onto.

(D) many-one and not onto.

16) Let A = {2, 3, 4, 5, ... , 30} and '≃' be an equivalence relation on A × A, defined by (a, b) '≃' (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(A) 5 (B) 6 (C) 8 (D) 7

17) $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$ = ... where x > 1. [March - 2023]

(A) $\sec^{-1}x$ (B) $\sin^{-1}x$

(C) $\text{cosec}^{-1}x$ (D) $\cos^{-1}x$

18) $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ = ...[March - 2023]

(A) π (B) 0 (C) $-\frac{\pi}{2}$ (D) $2\sqrt{3}$

19) $\cos^{-1}(\cos 6)$ = ...

(A) 6 (B) $2\pi - 6$

(C) $\pi - 6$ (D) None of these

20) $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x = ... [March - 2023]

(A) $0, \frac{1}{2}$ (B) 0 (C) $1, \frac{1}{2}$ (D) $\frac{1}{2}$

21) Total number of possible matrices of order 3 × 3 with entry 2 or 9 is ....[March - 2023]

(A) 27 (B) 81 (C) 18 (D) 512

22) If A and B are symmetric matrices of same order, then AB – BA is .... [March - 2023]

(A) a skew symmetric matrix

(B) a zero matrix

(C) a symmetric matrix

(D) an identity matrix

23) If α is a root of the equation $x^2 + x + 1 = 0$ and $A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then A^{31} = ...

(A) A (B) A^2 (C) A^3 (D) I^3

24) For square matrix A if $A = B + \frac{C}{2}$, where B is skew symmetric matrix and C is symmetric matrix, then C = ... [March - 2023]

(A) A + A' (B) $\frac{A + A'}{2}$ (C) A – A' (D) $\frac{A - A'}{2}$

25) For determinant A, if $A = \begin{vmatrix}1 & 2 & 13\\ 3 & 0 & 5\\ 6 & 7 & 11\end{vmatrix}$ and p, q, r are cofactors of 13, 5 and 11 respectively, then p + 3q + 6r = ... [March - 2023]

(A) 232 (B) 241 (C) 0 (D) 243

26) If $\Delta = \begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$, then it's maximum value is .... (Here θ ∈ R)

(A) $\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\sqrt{2}$ (D) $\frac{2\sqrt{3}}{4}$

27) If $\begin{vmatrix}x^3+4x & x+3 & x-2\\ x-2 & 5x & x-1\\ x-3 & x+2 & 4x\end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, then value of f is ....

(A) 0 (B) 15 (C) 17 (D) 1

28) If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ and $\frac{dy}{dx} = \frac{2^{x+1}\log 2}{f(x)}$ then f(0) = ...

(A) 2 (B) –2 (C) 0 (D) 2 log 2

29) If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ = ... [March - 2023]

(A) 0 (B) y (C) –y (D) $-\frac{dy}{dx}$

30) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, then $\frac{dy}{dx}$ = ... [March - 2023]

(A) $\cot\frac{\theta}{2}$ (B) $\tan\frac{\theta}{2}$ (C) $\frac{1}{2}\cot\frac{\theta}{2}$ (D) $\frac{1}{2}\tan\theta$

31) The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is .... [March - 2023]

(A) 10π (B) 8π (C) 12π (D) 11π

32) The local minimum value of $x^2 + \frac{16}{x}$ $(x \neq 0)$ is ....

(A) 12 (B) 22 (C) – 12 (D) 2

33) A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of ....

(A) 1 m/h (B) 0.1 m/h
(C) 1.1 m/h (D) 0.5 m/h

34) The maximum value of $f(x) = [x(x-1) + 1]^{\frac{1}{3}}$, $0 \le x \le 1$ is .... [March - 2023]

(A) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$ (B) 1 (C) $\frac{1}{2}$ (D) 0

35) $\int \frac{1}{x + x\log x} dx$ = ... + C. [March - 2023]

(A) $\frac{-1}{(1+\log x)^2}$ (B) $1 + \log x$
(C) $\log|\log(ex)|$ (D) $\frac{\log x}{x}$

36) $\int \frac{dx}{\sqrt{2x - x^2}}$ = ... + C. [March - 2023]

(A) $\log\left|x - 1 + \sqrt{2x - x^2}\right|$
(B) $\sin^{-1}(x-1)$
(C) $\log\left|\frac{x}{2-x}\right|$ (D) $\cos^{-1}(x-1)$

37) $\int \frac{(x-3)e^x}{(x-1)^3} dx$ = ... + C. [March - 2023]

(A) $\frac{e^x}{(x-1)^3}$ (B) $\frac{e^x}{(x-3)^3}$
(C) $\frac{e^x}{(x-3)^2}$ (D) $\frac{e^x}{(x-1)^2}$

38) $\int \sqrt{x^2 - 8x + 7}\,dx$ = ... + C. [March - 2023]

(A) $\frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} + 9\log\left|x - 4 + \sqrt{x^2 - 8x + 7}\right|$

(B) $\frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} - 3\sqrt{2}\log\left|x - 4 + \sqrt{x^2 - 8x + 7}\right|$

(C) $\frac{1}{2}(x+4)\sqrt{x^2 - 8x + 7} + 9\log\left|x + 4 + \sqrt{x^2 - 8x + 7}\right|$

(D) $\frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|x - 4 + \sqrt{x^2 - 8x + 7}\right|$

39) $\int \frac{dx}{e^x + e^{-x}}$ = ... + C. [March - 2023]

(A) $\tan^{-1}(e^x)$ (B) $\log(e^x - e^{-x})$
(C) $\tan^{-1}(e^{-x})$ (D) $\log(e^x + e^{-x})$

40) If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)dx$ = ... [March - 2023]

(A) $\frac{a+b}{2}\int_a^b f(x)dx$ (B) $\frac{b-a}{2}\int_a^b f(x)dx$

(C) $\frac{a+b}{2}\int_a^b f(b+x)dx$ (D) $\frac{a+b}{2}\int_a^b f(b-x)dx$

41) $\int_0^1 x(1-x)^n\,dx$ = ... [March - 2023]

(A) $\frac{1}{n^2 - 3n + 2}$ (B) $\frac{1}{n^2 - 3n - 2}$

(C) $\frac{1}{n^2 + 3n + 2}$ (D) $\frac{1}{n^2 + 3n - 2}$

42) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x + 1)dx$ = ... [March - 2023]

(A) π (B) 1 (C) 0 (D) 2

43) If area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{\pi}{6}$, then equation of ellipse is ....

(A) $4x^2 + 9y^2 = 1$ (B) $\frac{x^2}{36} + y^2 = 1$

(C) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (D) $x^2 + y^2 = 36$

44) Find area of the region bounded by the line $y = x$, Y-axis and the lines $y = 1$, $y = 5$.

(A) 12 (B) 6 (C) 24 (D) 14

45) The area bounded by the $y = \cos x$ between $x = 0$ and $x = \frac{3\pi}{2}$ is .... [March - 2023]

(A) 1 (B) 3 (C) 2 (D) 4

46) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ are .... and .... respectively. [March - 2023]

(A) 2, 3 (B) 2, not defined

(C) 3, 2 (D) not defined, 2

47) The number of arbitrary constants in the particular solution of a differential equation of third order are .... [March - 2023]

(A) 3 (B) 1 (C) 2 (D) 0

48) The Integrating Factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay$ $(-1 < y < 1)$ is ....

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$

(C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$

49) The co-ordinates of a point on the line passing through the points (1, – 1, 2) and (3, 1, 1) at a distance $3\sqrt{11}$ units from the point $\hat{i} - \hat{j} + 2\hat{k}$ is ....

(A) (10, 2, – 5) (B) (– 8, – 4, – 1)

(C) (8, 4, 1) (D) (– 10, – 2, – 5)

50) A line makes an angle of measure α with X-axis and Y-axis $\cot \alpha \in$ ......

(A) (0, 1) (B) (– 1, 1) (C) [– 1, 1] (D) [0, 1]

Time : 2 Hours] PART - B [Total Marks : 50

Section - A

❖ Answer any 8 of the following given question 1 to 12 : (Each carries 2 marks) [16]

1) Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

2) Prove that : $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

3) If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$, for $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1 + x)^2}$. [March - 2023]

4) Find : $\int \frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} dx$

5) Using integration, find the area of the region bounded by the line $2y = 5x + 7$, X-axis and the lines $x = 2$ and $x = 8$.

6) Find area of the region bounded by $y = (x - 1)(x - 2)$ and X-axis.

7) If on any point (x, y) on the curve slope of tangent is equal to sum of coordinate of that point then find equation of the curve if it passes through origin.

8) If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

9) Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (–1, –2, 1), (1, 2, 5).

10) Find the cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x + 3}{3} = \frac{y - 4}{5} = \frac{z + 8}{6}$.

11) An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.

12) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$ then find $P(A' \mid B)$.

Section - B

❖ **Answer any 6 of the following given questions 13 to 21 : (Each carries 3 marks)** **[18]**

13) Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a − b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

14) If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$, find A(BC), (AB)C and show that (AB)C = A(BC).

15) If $A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then prove that $A^{-1} = A^2$.

16) If $2x = y^{\frac{1}{m}} + y^{\frac{-1}{m}}$ $(n \geq 1)$, then prove that $(x^2 - 1)\, y_2 + xy_1 = m^2 y$.

17) If critical points of $y = a \log x + bx^2 + x$ are $x = -1$ and $x = 2$, then find a and b.

18) Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$.

19) Find the foot of perpendicular from the point (2, 3, −8) to the line $\frac{4 - x}{2} = \frac{y}{6} = \frac{1 - z}{3}$. Also, find the perpendicular distance from the given point to the line.

20) Maximise $Z = -x + 2y$, subject to the constraints : $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

21) Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

Section - C

❖ **Answer any 4 of the following given questions from 22 to 27 : (Each carries 4 marks)** **[16]**

22) If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

23) Solve the following system of linear equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

24) Find derivative of given function w.r.t. x

$(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

25) A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$.

26) Evaluate : $\int \sqrt{\tan x}\, dx$

27) In a bank, principal increases continuously at the rate of 5% per year. In how many years ₹ 1000 double itself ?

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