Paper - 2

Board's Question Paper (Based on March - 2020)

Time : 1 Hour] **PART - A** **[Total Marks : 50**

❖ **Select the following questions with proper alternative and answer it :**

1) Area bounded by curve $y = \tan \pi x;\ x \in \left[-\frac{1}{4}, \frac{1}{4}\right]$ and X-axis is .... [March - 2020]

(A) $\frac{\log 2}{2\pi}$ (B) $\frac{\log 2}{2}$ (C) log2 (D) $\frac{\log 2}{\pi}$

2) The area of the region bounded by the curve $y = [x]$ and the lines $x = 1$ and $x = 1.9$ is ...... Sq. units. Where [·] denotes the greatest integer function.

(A) $\frac{3}{5}$ (B) $\frac{7}{10}$ (C) $\frac{4}{5}$ (D) $\frac{9}{10}$

3) Area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 4$ is .... [March - 2020]

(A) 64π (B) 32π (C) 8π (D) $\frac{\pi}{64}$

4) The order and degree of the differential equation $(y''')^3 + (y'')^4 + (y')^4 + y = 7$ are .... respectively. [March - 2020]

(A) 4 and 1 (B) 1 and 4

(C) 3 and 3 (D) 2 and 4

5) If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)\,y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(A) $\frac{1}{4}\log_e 2$ (B) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(C) $\log_e 2$ (D) $\frac{1}{2}\log_e 2$

6) Integrating factor of the differential equation $y\,dx - (x + 2y^2)dy = 0$ is ...... [March - 2020]

(A) y (B) $-y$ (C) $-\frac{1}{y}$ (D) $\frac{1}{y}$

7) Measure of the angle between the vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ is .... [March - 2020]

(A) $\sin^{-1}\frac{2\sqrt{2}}{3}$ (B) $\pi - \cos^{-1}\frac{1}{3}$

(C) $\cos^{-1}\frac{1}{\sqrt{3}}$ (D) $\sin^{-1}\frac{1}{3}$

8) If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$ then $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = $ .... [March - 2020]

(A) –2 (B) –8 (C) 8 (D) 2

9) Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$. [March - 2020]

(A) $\frac{1}{2}\sqrt{507}$ (B) $\sqrt{387}$

(C) $\sqrt{507}$ (D) 25

10) Let $|\vec{x}| = |\vec{y}| = |\vec{x} + \vec{y}| = 1$ and if measure of the angle between $\vec{x}$ and $\vec{y}$ is α, then $\sin\alpha = $ .... [March - 2020]

(A) $-\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) 1

11) $\hat{i}\cdot(\hat{k}\times\hat{j}) + \hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{k}\cdot(\hat{j}\times\hat{i}) + \hat{i}\cdot(\hat{i}\times\hat{j}) + \hat{j}\cdot(\hat{j}\times\hat{k})$ = ... [March - 2020]

(A) –1 (B) 1 (C) 3 (D) –3

12) For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$, then evaluate $2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$. [March - 2020]

(A) 100 (B) 50 (C) –25 (D) –50

13) If the lines $\frac{2x-5}{k} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other, then value of k is .... [March - 2020]

(A) –7 (B) 14 (C) 7 (D) 26

14) The angle between the lines $\vec{r} = (4, -3, 2) + k(2, 1, 2)$, $k \in R$ and $\vec{r} = (2, 0, 5) + k(6, 3, 2)$, $k \in R$ is ....

(A) $\sin^{-1}\frac{4\sqrt{5}}{21}$ (B) $\cos^{-1}\frac{4\sqrt{5}}{21}$

(C) $\cos^{-1}\frac{4\sqrt{5}}{19}$ (D) $\sin^{-1}\frac{19}{21}$

15) The equation of the line L passing through A(–2, 2, 3) and perpendicular to $\overleftrightarrow{AB}$ is .... where B = (13, –3, 13).

(A) $\frac{x-2}{3}=\frac{y+2}{13}=\frac{z+3}{2}$

(B) $\frac{x+2}{3}=\frac{y-2}{13}=\frac{z-3}{2}$

(C) $\frac{x+2}{15}=\frac{y-2}{-5}=\frac{z-3}{10}$

(D) $\frac{x-2}{15}=\frac{y+2}{-5}=\frac{z+3}{10}$

16) The objective function of a linear programming problem is .... [March - 2020]

(A) a function to be optimized

(B) a quadratic equation

(C) a constant (D) an inequality

17) The vertices of the feasible region determined by some linear constraints are (0, 2), (1, 1), (3, 3), (1, 5). Let $Z = px + qy$ where $p, q > 0$. The condition on p and q so that the maximum of Z occurs at both the points (3, 3) and (1, 5) is .... [March - 2020]

(A) $p = q$ (B) $p = 2q$ (C) $q = 2p$ (D) $p = 3q$

18) $\begin{vmatrix} 1 & -a & -b \\ a & 1 & -c \\ b & c & 1 \end{vmatrix}$ = ...

(A) $1 + a^2 + b^2 + c^2$ (B) $-(1 + a^2 + b^2 + c^2)$

(C) $1 - a^2 - b^2 - c^2$ (D) $a^2 + b^2 + c^2 - 1$

19) If P(E) = 0.8, P(F) = 0.5 and P(E/F) = 0.4 then P(E/F) = ... [March - 2020]

(A) 0.80 (B) 0.32 (C) 0.64 (D) 0.98

20) The feasible region of the inequality $x + y \le 1$ and $x - y \le 1$ lies in .... quadrants.

(A) Only I and II (B) Only I and III

(C) Only II and III (D) All the these

21) Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to ....

(A) 13 (B) 6 (C) 4 (D) 3

22) $f : N \times N \to N$, $f((m, n)) = m + n$ then the function f is ......

(A) one-one and onto.

(B) one-one and not onto.

(C) not one-one but onto.

(D) neither one-one nor onto.

23) If S is defined on R by $(x, y) \in R \Leftrightarrow xy \ge 0$. Then S is ......

(A) an equivalence relation

(B) reflexive only

(C) symmetric only (D) transitive only

24) Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

25) $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ then range of $f(x)$ is ....

(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(C) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) None of these

26) The value of $\tan\left[\cos^{-1}\left(\cos\frac{50\pi}{3}\right)\right]$ is ......

(A) $-\sqrt{3}$ (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) –1

27) The value of $\cot\left[\frac{\pi}{4} - 2\cot^{-1}3\right]$ is ......

(A) 3 (B) 7 (C) 9 (D) $\frac{3}{4}$

28) $\cot\left\{\frac{2019\pi}{2} - \left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)\right\}$ = ... [March - 2020]

(A) $\frac{17}{6}$ (B) $\frac{19}{6}$ (C) $-\frac{17}{6}$ (D) $-\frac{19}{6}$

29) For 3 × 4 matrix, elements are given by $a_{ij} = |-3i + 4j|$ then $\sum_{i=1}^{3}(a_{ij})^i$ = ... [March - 2020]

(A) 2^5 (B) 4^3 (C) 3^3 (D) 6^3

30) A is 3 × 3 matrix and det(A) = 7. If B = adj A then det(AB) = ... [March - 2020]

(A) 7 (B) 7^2 (C) 7^5 (D) 7^3

31) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 5A = kI$ then k = ... [March - 2020]

(A) 5 (B) 7 (C) –7 (D) –5

32) Matrices X and Y are inverse of each other then .... [March - 2020]

(A) XY = I, YX = –I (B) XY = YX = –I

(C) XY = YX = 0 (D) $X^{-1}Y^{-1} = Y^{-1}X^{-1} = I$

33) If $\Delta = \begin{vmatrix} x+y+z^2 & x^2+y+z & x+y^2+z \\ z^2 & x^2 & y^2 \\ x+y & y+z & x+z \end{vmatrix}$,

(where $(x \neq y \neq z)$ $x, y, z \in R - \{0\}$) then Δ = ... [March - 2020]

(A) 0 (B) 1

(C) $x + y + z$ (D) $x^2 + y^2 + z^2$

34) For $\Delta = \begin{vmatrix} 2019 & 2020 & 2021 \\ 2022 & 2023 & 2024 \\ 2025 & 2026 & 2027 \end{vmatrix}$ sum of minor and cofactor of 2020 is ..... [March - 2020]

(A) 4040 (B) 0 (C) 2020 (D) –2020

35) $-1 \le x < 0$, $0 \le y < 1$, $1 \le z < 2$ and [ ] is a greatest integer function then,

$\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ = ...

(A) $[x]$ (B) $[y]$

(C) $[z]$ (D) None of these

36) Let the function f be defined by

$f(x) = \begin{cases} cx + 1, & \text{if } x \le 3 \\ dx + 3, & \text{if } x > 3 \end{cases}$

If f is continuous at $x = 3$ then $d - c$ = ... [March - 2020]

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $\frac{2}{3}$

37) If $y = (x + 3)^2 \cdot (x + 4)^3 \cdot (x + 5)^4$, then first order derivative of y with respect to x is .... [March - 2020]

(A) $\frac{1}{y}\sum_{i=1}^{3}\frac{i-1}{(x+1-i)}$ (B) $\frac{x}{y}\sum_{i=1}^{3}\frac{i+1}{(x+1+i)}$

(C) $\frac{y}{x}\sum_{i=2}^{4}\frac{i}{(x+1-i)}$ (D) $y\sum_{i=2}^{4}\left(\frac{i}{(x+1)+i}\right)$

38) If $y = \log_e(\log_\pi x)$; $(x > 1)$ then $\frac{d^2y}{dx^2}$ = ... [March - 2020]

(A) $-\frac{(x \cdot \log_e x)^2}{\log_e(ex)}$ (B) $\frac{\log_e(ex)}{(x \cdot \log_e x)^2}$

(C) $-\frac{\log_e(ex)}{(x \cdot \log_e x)^2}$ (D) $\frac{\log_e\left(\frac{e}{x}\right)}{(x \cdot \log_\pi x)^2}$

39) An iron ball having 10 cm radius is covered equally with ice. Ice is melting at the rate of 50 cm^3/min. When the thickness of ice is 5 cm, rate of decrease in radius is .... cm/min.

(A) $\frac{1}{36\pi}$ (B) $\frac{1}{18\pi}$ (C) $\frac{1}{54\pi}$ (D) $\frac{5}{6\pi}$

40) $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always increasing function in ...... interval.

(A) $(0, \pi)$ (B) $\left(0, \frac{\pi}{2}\right)$

(C) $\left(0, \frac{\pi}{4}\right)$ (D) $\left(0, \frac{3\pi}{4}\right)$

41) Function $f(x) = |\sin x|$, $x \in \left(-\frac{\pi}{2}, 0\right)$ is : [March - 2020]

(A) Only an increasing

(B) Neither increasing nor decreasing

(C) Strictly increasing

(D) Strictly decreasing

42) Local maximum value of the $f(x) = x + \frac{1}{x}, (x \neq 0)$ is ...... [March - 2020]

(A) $\frac{1}{2}$ (B) –2 (C) 2 (D) $-\frac{1}{2}$

43) $\int\sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\,dx$ = ... + C.

$\left(\text{where } x \in R - \left\{\frac{k\pi}{2} / k \in Z\right\}\right)$ [March - 2020]

(A) $\frac{2}{3}\cos^{-1}\left(\sin^{\frac{3}{2}} x\right)$ (B) $\frac{2}{3}\tan^{-1}\left(\cos^{\frac{3}{2}} x\right)$

(C) $-\frac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)$ (D) $\frac{2}{3}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right)$

44) If $\int\frac{1}{e^x + 1}\,dx = px - q\log|1 + e^x| + C$ then $p + q$ = ... [March - 2020]

(A) 0 (B) 2 (C) –2 (D) 1

45) $\int e^{x^3} \cdot 5^{x^2} \cdot x \cdot [\log 25 + 3x]\, dx = ... + C.$ [March - 2020]

(A) $e^{x^3} \cdot 5^{x^2} \cdot x$ (B) $\frac{1}{6} \cdot e^{x^3} \cdot 5^{x^2}$

(C) $\frac{1}{6} \cdot e^{x^3} \cdot 5^{x^2} \cdot x$ (D) $e^{x^3} \cdot 5^{x^2}$

46) $\int \frac{dx}{\sqrt{2x - x^2}} = ... + C.$ [March - 2020]

(A) $2 \sin^{-1}(x - 1)$ (B) $\frac{1}{2} \sin^{-1}(x - 1)$

(C) $\sin^{-1}(x - 1)$ (D) $\log\left|(x-1)+\sqrt{2x-x^2}\right|$

47) $\int_{-1}^{\sqrt{3}} \frac{dx}{1 + x^2} = ...$ [March - 2020]

(A) $\frac{7\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

48) $\int_0^{\pi} \cos^3 x \cdot \sin^4 x\, dx = ...$ [March - 2020]

(A) $-\pi$ (B) 0 (C) π (D) 2π

49) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^5 x \cos^2 x\, dx = ...$ [March - 2020]

(A) $\frac{1}{\sqrt{2}} - 1$ (B) 0

(C) $\left(\frac{\pi}{6}\right)^5 - \left(\frac{\pi}{6}\right)^2$ (D) $\left(\frac{\pi}{6}\right)^2 - \left(\frac{\pi}{6}\right)^5$

50) $\int_0^2 f(x)dx = ...;$ where $f(x) = \max\{x, x^2\}$. [March - 2020]

(A) $\frac{8}{3}$ (B) $\frac{13}{6}$ (C) $\frac{17}{6}$ (D) $\frac{19}{6}$

Time : 2 Hours] **PART - B** **[Total Marks : 50**

Section - A

❖ **Answer any 8 of the following given question 1 to 12 : (Each carries 2 marks)** **[16]**

1) Show that,

$\sin^{-1}\left(2x\sqrt{1 - x^2}\right) = 2\sin^{-1} x,\ -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

2) $2 \tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x),\ x \in \left(0, \frac{\pi}{2}\right)$

3) Find $\frac{dy}{dx}$ in the following :

$\sin^2 y + \cos xy = k$

4) Evaluate the integrals using substitution :

$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}\, dx$

5) Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

6) Find the area under the given curves and given lines :

$y = x^4;\ x = 1,\ x = 5$ and X-axis

7) For the differential equation find the general solution :

$(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$

8) Show that each of the given three vectors is a unit vector :

$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right),\ \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right),$

$\frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right).$

Also, show that they are mutually perpendicular to each other.

9) Find the angle between the following pairs of lines :

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

10) Find the distance of a point (2, 4, –1) from the line $\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9}$

11) A dice marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent ?

12) In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Section - B

❖ Answer any 6 of the following given questions 13 to 21 : (Each carries 3 marks) [18]

13) Let A = R − {3} and B = R − {1}. Consider the function $f : A \to B$ defined by,

$f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one-one and onto ? Justify your answer.

14) Find the values of x, y, z if the matrix

$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation A'A = I.

15) Solve system of linear equations, using matrix method, in

$2x - y = -2,\ 3x + 4y = 3$

16) Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t. x.

17) Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [−3, −1].

18) If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

19) Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

20) Determine graphically the minimum value of the objective function $Z = -50x + 20y$ ...(1)

subject to the constraints :

$2x - y \geq -5$ ...(2)

$3x + y \geq 3$ ...(3)

$2x - 3y \leq 12$ ...(4)

$x \geq 0,\ y \geq 0$ ...(5)

21) A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train ?

Section - C

❖ Answer any 4 of the following given questions from 22 to 27 : (Each carries 4 marks) [16]

22) If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

23) Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

24) Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$.

25) A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

26) Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin(\pi x)|\, dx$.

27) Find the general solution of the given differential equation :

$\frac{dy}{dx} + 2y = \sin x$

❖ ❖ ❖