MATHEMATICS

1) $\int_0^1 x e^x \, dx =$ ______ $\left[xe^x - e^x \right]_0^1$

(A) 1 (B) 0 $(1e^1 - e^1) + (+e^0)$

(C) e (D) -1 $(e^1 - e^1) + 1$

$0 + 1$

2) Area lying in the first quadrant and bounded by ellipse $9x^2 + 16y^2 = 1$ is ______

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{48}$

(C) 12π (D) 3π

3) Area of the region bounded by the curve $x^2 = 4y$, X-axis and the line $x = 3$ is ______.

(A) $\frac{9}{4}$ (B) 2

(C) $\frac{9}{3}$ (D) $\frac{9}{2}$

4) The area bounded by the curve $y = \cos x$ between $x = -\pi/2$ and $x = \pi/2$ is ______

(A) 1 (B) 4

(C) 0 (D) 2

(Space for Rough Work)

5) The order and the degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[3]{\left(\frac{dy}{dx}\right)^4 + 2}$ is respectively ________ and ________.

(A) 3, 2

(B) 2, 3

(C) 2, 8

(D) 1, 8

$\left(\frac{d^2y}{dx^2}\right)^{3} = \left(\frac{dy}{dx}\right)^{8} + L$

6) The general solution of the differential equation $\frac{x\,dy - y\,dx}{y} = 0$ is ________.

(A) $x = cy^2$

(B) $xy = c$

(C) $y = cx$

(D) $y = cx^2$

$x\,dy - y\,dx = 0$

$\frac{dy}{y} = \frac{dx}{x}$

$\log y = \log x + \log c$

7) The Integrating Factor of the differential equation

$(\tan^{-1}y - x)\,dy = (1 + y^2)\,dx$ is ________.

linear in x

$\frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2} = \frac{dx}{dy}$

(A) $\frac{1}{1+y^2}$

(B) $\tan^{-1}y$

(C) $e^{\frac{1}{1+y^2}}$

(D) $e^{\tan^{-1}y}$

8) The angle 'θ' between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ is ________.

$(1, -1, 1)$

$(1, 1, -1)$

(A) $\cos^{-1}\left(-\frac{1}{3}\right)$

(B) $\cos^{-1}\frac{1}{3}$

(C) $\sin^{-1}\frac{1}{3}$

(D) $\sin^{-1}\left(-\frac{1}{3}\right)$

$\frac{\cancel{1-1}-1}{\sqrt{3}\sqrt{3}}$

$\frac{-1}{3}$

9) The area of a parallelogram, whose adjacent sides are given by the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{j} - 2\hat{k}$, is ________.

$(2, 3, 4)$

$(0, -1, -2)$

$(-6+4, +4, -2)$

$(-2, +4, -2)$

(A) $\sqrt{6}$

(B) $2\sqrt{6}$

(C) 24

(D) $2\sqrt{3}$

$\cancel{100+16}$ $4+16+4$

24

10) The value of $\hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{i}\cdot(\hat{j}\times\hat{j}) + \hat{k}\cdot(\hat{j}\times\hat{i}) + \hat{i}\cdot(\hat{k}\times\hat{j})$ is ________.

(A) −2

(B) −1

(C) −3

(D) −4

11) The angle, between the pair of lines, given by $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z+4}{2}$ and $\frac{x-5}{3}=\frac{y+2}{2}=\frac{z}{6}$ is ________.

(1, 2, 2)
(3, 2, 6)

3 + 4 + 12

$\frac{19}{\sqrt{1+4+4}\sqrt{\ }}$

↳ 3

(A) $\cos^{-1}\left(\frac{19}{21}\right)$ (B) $\cos^{-1}\left(\frac{\sqrt{19}}{21}\right)$

(C) $\sin^{-1}\left(\frac{19}{21}\right)$ (D) $\cos^{-1}\left(-\frac{19}{21}\right)$

12) If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular, then the value of k is ________.

(A) $-\frac{7}{10}$ (B) $\frac{7}{10}$

(C) $\frac{10}{7}$ (D) $-\frac{10}{7}$

(-3, 2K, 2)
(3K, 1, -5)
-9K + 2K - 10
-7K = 10

13) The Cartesian equation of the line which passes through the point (1, -3, 5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is :

(A) $\frac{x-1}{3}=\frac{y+3}{5}=\frac{z-5}{6}$ (B) $\frac{x+3}{1}=\frac{y-4}{-3}=\frac{z+8}{5}$

(C) $\frac{x+3}{-3}=\frac{y-4}{4}=\frac{z+8}{-8}$ (D) $\frac{x-1}{-3}=\frac{y+3}{4}=\frac{z-5}{-8}$

14) The coordinates of the corner points of the bounded feasible region are $(0, 6)$, $(3, 3)$, $(9, 9)$, $(0, 12)$. The maximum of the objective function $z = 6x + 12y$ is : 162 144

(A) 152

(B) 162

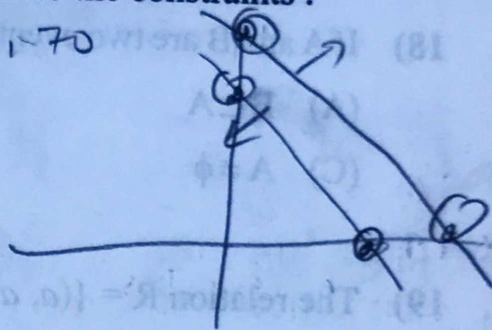
(C) 144

(D) 166

15) Minimise objective function $z = 7x + 3y$ subject to the constraints :

15 (0,5), (5,0) 35 30, 70

$x + y \leq 5, x + y \geq 10, x \geq 0, y \geq 0$ is :



(A) 15

(B) 35

(C) 70

(D) No feasible region and hence no feasible solution

16) If, for independent events A and B, $P(A) = p$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$ are given then, the value of p is ________.

$\frac{6}{10} - \frac{5}{10} = \frac{2p}{5}$

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

$\frac{1}{10} = \frac{2p}{5}$

(C) $\frac{3}{5}$

(D) $\frac{1}{3}$

$\frac{3}{5} = p + \frac{1}{2} - \frac{p}{2}$

17) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is :

(A) $\frac{1}{3}$ (B) 0

(C) $\frac{1}{12}$ (D) $\frac{1}{36}$

18) If A and B are two events such that $P(B) \neq 0$ and $P(A|B) = 1$, then _______.

(A) $B \subset A$ (B) $A \subset B$ $P(A \cap B) = P(B)$

(C) $A \neq \phi$ (D) $B \neq \phi$

19) The relation $R = \{(a, a), (b, b), (c, c), (a, c)\}$, is defined on the set $\{a, b, c\}$, is _______.

(A) Reflexive and transitive but not symmetric

(B) Reflexive and symmetric but not transitive

(C) Transitive and symmetric but not reflexive

(D) An equivalence relation

20) $f: Z \rightarrow Z, f(x) = x^3 + 2$ is defined then function f is _______.

(A) One - one but not onto

(B) One - one and onto

(C) Not one - one but onto $x^3 + 2 = y^3 + 2$

(D) Neither one - one nor onto

21) If $y = \tan^{-1}x$ then ________.

(A) $0 \le y \le \pi$

(B) $0 < y < \pi$

(C) $-\pi/2 < y < \pi/2$

(D) $-\pi/2 \le y \le \pi/2$

22) The value of $\tan^{-1}(-1) + \sec^{-1}(-2) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is ________.

$-\frac{\pi}{4} + \pi - \frac{\pi}{3} + \cancel{\pi/4}$

(A) $-\pi/6$

(B) $-\pi/3$

(C) π

(D) $2\pi/3$

$I^3 - 3A^2 - 3A - \cancel{I^3} - 3A$

$-3A - A^3$

$-3A - 3A - 3A - 3A$

23) $\sin^{-1}\left(\sin\frac{23\pi}{6}\right) =$ ________.

$\pi + \frac{5\pi}{6}$

(A) $-\pi/6$

(B) $\pi/6$

(C) $23\pi/6$

(D) $-5\pi/6$

$I^3 - 3A^2 - 3A - \cancel{I^2} - 2AI$

A^2

$-3A - 3A - 2A$

$-A$

24) If A is square matrix such that $A^2 = A$, then $(I - A)^3 - (I + A)^2 =$ ________.

(A) $2(I - A)$

(B) $I - A$

(C) I

(D) 0

$I^3 - 3A^2I - 3AI^2 - (I^3 + 3A^2I + 3AI^2 + 2A^3$

$-3A - 3A + 3A + 3A + 2A^2 \cdot A$

25) If $A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$ and $A + A' = I$, then the value of $\cos\alpha$ is ________.

$\begin{pmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix} + \begin{bmatrix} \sin\alpha & +\cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) -1

(D) 0

$-\cos\alpha +$

26) If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ then $I + A^2 =$ ________.

$0 \; 0 \; -1(0-1)$
$0 \; -1(0-1)$
$-1(0)$

$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

(A) 0

(B) I+A

(C) A

(D) 2I

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

27) If area of ΔPQR is 3 sq. units with vertices P (k, 1), Q (2, 4) and R (1, 1). Then value of k is ________.

(A) $-3, 1$

(B) $0, 2$

(C) $-1, 3$

(D) $1, 3$

28) If $\begin{vmatrix} 2017 & 2018 \\ 2019 & 2020 \end{vmatrix} + \begin{vmatrix} 2021 & 2022 \\ 2023 & 2024 \end{vmatrix} = 2k$, then $k^3 =$ ________.

(A) -8

(B) 8

(4038)(4044)

(C) 0

(D) -64

29) If $A = \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$ then $A^{-1} =$ __________.

(A) $\frac{1}{24}\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(B) $\frac{1}{24}\begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{24}\begin{bmatrix} -6 & 4 \\ 3 & -2 \end{bmatrix}$

(D) Does not exist

$\frac{-2k\sin x}{-2}$

$k\sin x = 2024$

30) If function f is continuous at point $x = \pi/2$ and $f(x) = \begin{cases} \frac{2k\cos x}{\pi - 2x} & , x \neq \pi/2 \\ 2024 & , x = \pi/2 \end{cases}$; then the value of k is __________.

$\frac{2k\cos x}{x - 2x}$

(A) ~~1012~~

(B) 506

(C) 2024

(D) 4048

$\frac{(x-2)(2k\cos x) -}{(x-2x)^2}$

$-2K = 2024$

$\frac{(-1)(2k(0)) - (\pi - 2x)(-2k\sin x)}{(\pi - 2x)^2}$

$(x-2x)(-2k\sin x)$

$-2k\sin x = 0$

$k\sin x = 0$

31) $\frac{d}{dx}\left(e^{x\log x} + e^3\right) =$ __________.

(A) $(1 + \log x)$

(B) $x^x(1 + \log x)$

(C) $x^x \log x$

(D) $x^x(1 + \log x) + e^3$

32) If $x = a(1 - \cos\theta)$, $y = a(\theta + \sin\theta)$ then $\frac{dy}{dx}$ = ________.

$2\sin^2 \theta/2 - 1$

$\frac{1+\cos\theta}{1+\sin\theta}$ $\quad$ $2\sin\theta/2 \cos\theta/2$

(A) $\cot \theta/2$ (B) $\tan \theta/2$

$\frac{1-2\cos^2\theta/2}{2\sin\theta/2 \cos\theta/2}$

(C) $-\cot \theta/2$ (D) $-\tan \theta/2$

$\frac{1+\cos\theta}{1+\sin\theta}$ $\quad 2 \quad$ $\frac{1}{2s \operatorname{cosec}\theta}$ -

33) If $\frac{d^2y}{dx^2} - my = 0$ satisfies for $y = 7\sin x + 5\cos x$ then the value of m is ________.

$y_1 = 7\cos x - 5\sin x$

$y_2 = -7\sin x - 5\cos x$

(A) 1 (B) 0

(C) −1 (D) −2

$-7\sin x - 5\cos x - m(7\sin x ...) = 0$

$-y_2 - m(y) = 0 \quad -my = y_2$

$m = -1$

34) The rate of change of the surface area of a sphere with respect to its radius r, when $r = 6$ cm, is ________ cm^2/s.

(A) 24π (B) 12π

(C) 48π (D) 144π

$2\pi r + r + h$

(Space for Rough Work)

$2\pi \frac{dr}{dt} + \frac{dr}{dt} \pi r^2 + \pi r h$

$\pi r(r + h)$

$2\pi r \frac{dr}{dt} + \pi \frac{dh}{dt}$

35) For function $f(x) = \sin 3x$; $x \in \left[0, \frac{\pi}{2}\right]$, f is ________.

(A) Increasing in $\left[0, \frac{\pi}{2}\right]$ $3\sin x - 4\sin^3 x$

$3\cos x - 12 \sin^2 x \cos x$

(B) Decreasing in $\left[0, \frac{\pi}{2}\right]$

(C) Decreasing in $\left[0, \pi/6\right)$ and increasing in $\left(\pi/6, \pi/2\right)$

(D) Increasing in $\left[0, \pi/6\right)$ and decreasing in $\left(\pi/6, \pi/2\right)$

36) The absolute maximum value of the function $f(x) = \sin x + \cos x$, $x \in [0, \pi]$ is ________.

$r = \sqrt{a^2 + b^2}$

$\sqrt{1+1}$

(A) 0 (B) $\frac{1}{\sqrt{2}}$

(C) 1 (D) $\sqrt{2}$

$e^{2x} - 1 + 1 \cdot \frac{e^{2x}}{e^{2x}+1} - 1$

37) $\int \frac{e^{2x}-1}{e^{2x}+1}\, dx =$ ________ $+ C$ $\frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}\, dx - \int \frac{1}{e^{2x}+1}\, dx$

$\log(e^{2x}+1)$ $\frac{1}{2}\log(e^{2x}+1)$

(A) $\log(e^{2x} - 1) + x$ (B) $\log(e^{2x} + 1) - x$

(C) $\log(e^{2x} + 1) + x$ (D) $\log(e^{2x} - 1) - x$ $\frac{1}{-(e^{2x}-1)}$ $-e^{2x}+1$

$e^{2x} + 1 - 1 + 1$

(Space for Rough Work)

$\frac{1}{\quad} + \frac{1}{2x}$ $\frac{1 + e^{2x} + 1}{e^{2x}+1}$ $-e^{2x} - 1$

$\frac{1}{2}\sqrt{x - \frac{x}{4}}$

38) $\int \frac{1}{\sqrt{4x - x^2}}\, dx =$ ____________ + C

$\frac{1}{2l}$ $\sqrt{x\left(1 - \frac{x}{4}\right)}$

(A) $\sin^{-1}\left(\frac{x-2}{2}\right)$

(B) $\frac{1}{2}\tan^{-1}\left(\frac{x-2}{2}\right)$

(C) $\log\left|(x-2) + \sqrt{4x - x^2}\right|$

(D) $\frac{1}{4}\log\left|\frac{x}{x-4}\right|$

$\frac{1}{\sqrt{(x-2)^2 - (2)^2}}$

$4 - (x - 2)^2$

39) $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right) dx =$ ____________ +C $= e^x$

$2\sin^2 \frac{x}{2}$ $\frac{-1}{2\sin^2 \frac{x}{2}} + \frac{\sin x}{2\sin^2 \frac{x}{2}}$

(A) $e^x \tan\frac{x}{2}$

(B) $e^x \tan x$

(C) $e^x \cot\frac{x}{2}$

(D) $e^x \cot x$

$\frac{1+\sin x}{1 + (2\sin^2 \frac{x}{2} - 1)}$

40) $\int_{-\pi/2}^{\pi/2} \left(x^5 - x^3\cos x + \sin^3 x - 3\right) dx =$ ________

(A) 3π

(B) $-\pi$

(C) -3π

(D) 0

$1 + \sin x$