Paper - 5

Board's Question Paper (Based on March - 2022)

Time : 1 Hour] **PART - A** **[Total Marks : 50**

❖ **Select the following questions with proper alternative and answer it :**

1) **The corner points of feasible region determined by the system of linear constaints are (2, 72), (15, 20), (40, 15). Let Z = 6x + 3y be the objective function. Minimum of Z occurs at** [March - 2022]

(A) (40, 15) (B) (15, 20)
(C) (2, 72) (D) (0, 0)

2) **If for a linear programming problem feasible region is bounded, then the objective function has ....** [March - 2022]

(A) both maximum and minimum value
(B) only minimum value
(C) only maximum value
(D) neither maximum nor minimum value

3) **If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct ?** [March - 2022]

(A) $P(A / B) = \frac{P(B)}{P(A)}$ (B) $P(A / B) < P(A)$
(C) $P(A / B) \geq P(A)$ (D) None of these

4) **Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is**

(A) $\frac{4}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

5) **If A is square matrix of order 3 × 3 and A · *adj* A = 10I then |*adj* A| = ...**

(A) 1 (B) 10 (C) 100 (D) 101

6) **If *n*(A) = 3 and *n*(B) = 4 then the number of functions from A to B which are one-one is ......**

(A) 4^3 (B) 3^4 (C) 4 ! (D) 12

7) **Let R be the relation in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3)}. Choose the correct answer.** [March - 2022]

(A) R is reflexive and symmetric but not transitive.
(B) R is reflexive and transitive but not symmertic.
(C) R is an equivalence relation.
(D) R is symmetric and transitive but not reflexive.

8) **$f(x) = \frac{x^3}{3} + \frac{x^2}{2} + ax + b, \forall x \in R$. If $f(x)$ is one-one function then the minimum value of *a* is ....**

(A) $\frac{1}{4}$ (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{8}$

9) **If $\tan^{-1}x = y$, then ......** [March - 2022]

(A) $0 < y < \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(C) $0 \leq y \leq \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

10) **The value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ is ......** [March - 2022]

(A) $-\frac{\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $-\frac{7\pi}{6}$ (D) $\frac{\pi}{6}$

11) **The solution set of $[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \leq 0$, is ...... where [·] is the greatest integer function.**

(A) (−∞, cot3] (B) [cot3, cot2]
(C) [cot3, ∞) (D) none of these

12) **$\cos(\tan^{-1}x)$, $|x| < 1$ is equal to** [March - 2022]

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1+x^2}}$
(C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

13) **If matrix $\begin{bmatrix} x & -5 & 3 \\ y & 0 & w \\ z & 7 & 0 \end{bmatrix}$ is skew-symmetric then $x + y + z + w =$ ....**

(A) −3 (B) −5 (C) 7 (D) 1

14) **If $[x \ -5 \ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$, then value of x is ....** [March - 2022]

(A) $\pm 4\sqrt{3}$ (B) $\pm 2\sqrt{3}$
(C) 0 (D) $\pm 6\sqrt{3}$

15) If $A = \begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}$, then what is value of A^{-1} ? [March - 2022]

(A) $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{16} & \frac{2}{16} \\ \frac{4}{16} & \frac{8}{16} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{16} & -\frac{1}{8} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$ (D) Does not exist

16) If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 8A$ is equal to .... [March - 2022]

(A) I (B) I – A (C) A (D) 3A

17) Let $A = \begin{bmatrix} 1 & -\cos\theta & -1 \\ \cos\theta & 1 & -\cos\theta \\ 1 & \cos\theta & 1 \end{bmatrix}$ where $0 < \theta < \frac{\pi}{2}$, then [March - 2022]

(A) det(A) ∈ [2, 4] (B) det(A) ∈ (2, ∞)
(C) det(A) = 0 (D) det(A) ∈ (2, 4)

18) Let A be a nonsingular square matrix of order 3 × 3. Then |adj A| is equal to ...... [March - 2022]

(A) $|A|^3$ (B) $|A|^2$ (C) $|A|$ (D) $3|A|$

19) If area of triangle is 4 sq. units with vertices (– 2, 0), (0, 4) and (0, k), then k is ...... [March - 2022]

(A) 0, 8 (B) –8 (C) 0 (D) 0, –8

20) $f(x) = \dfrac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}$, $x \neq \frac{\pi}{4}$ is continuous at $x = \frac{\pi}{4}$ then $f\left(\frac{\pi}{4}\right) = ...$.

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{3}$ (D) 3

21) What is differentiation of $\cos^{-1}(\sin x)$ with respect to x ? [March - 2022]

(A) $\frac{\pi}{2} - 1$ (B) –1 (C) 1 (D) $\frac{\pi}{2}$

22) If $e^y(x + 1) = 1$, then what is the value of $\frac{d^2y}{dx^2} = ...$ [March - 2022]

(A) $-\left(\frac{dy}{dx}\right)^2$ (B) $\left(\frac{dy}{dx}\right)$

(C) $-\left(\frac{dy}{dx}\right)$ (D) $\left(\frac{dy}{dx}\right)^2$

23) The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is .... [March - 2022]

(A) $(-2\sqrt{2}, 4)$ (B) $(2\sqrt{2}, 0)$
(C) $(2\sqrt{2}, 4)$ (D) $(-2\sqrt{2}, 0)$

24) Which of the following function is decreasing on $\left(0, \frac{\pi}{8}\right)$?

(A) $\cos 4x$ (B) $\sin x$
(C) $-\cos x$ (D) $\tan 4x$

25) $f(x) = x^2 + 4x + 5$ has minimum value .... $(x \in R)$

(A) 4 (B) 2 (C) 1 (D) – 1

26) The interval in which $y = x^2 e^{-x}$ is increasing is [March - 2022]

(A) (2, ∞) (B) (–2, 0) (C) (–∞, ∞) (D) (0, 2)

27) $\int_0^{\pi} \sin x \, dx = ...$ [March - 2022]

(A) 0 (B) 2 (C) 1 (D) –2

28) $\int \frac{1}{\sqrt{(x-1)(x-2)}} dx = ... + C.$ [March - 2022]

(A) $\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right|$

(B) $\log\left|x + \sqrt{x^2 - 3x + 2}\right|$

(C) $\log\left|\left(x - \frac{3}{2}\right) - \sqrt{x^2 - 3x + 2}\right|$

(D) $\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right|$

29) $\int e^{3x} \cdot \sin(4x - 5) dx = ... + C.$ [March - 2022]

(A) $\frac{e^{3x}}{25}[3\sin(4x - 5) - 4\cos(4x - 5)]$

(B) $\frac{e^{3x}}{25}[3\cos(4x - 5) - 4\sin(4x - 5)]$

(C) $\frac{e^{3x}}{25}[3\sin(4x - 5) + 4\cos(4x - 5)]$

(D) $\frac{e^{3x}}{25}[4\sin(4x - 5) - 3\cos(4x - 5)]$

30) $\int \sqrt{x^2+4x+1}\,dx = ... + C.$ [March - 2022]

(A) $\frac{x+2}{2}\sqrt{x^2+4x+1} - 9\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(B) $\frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(C) $\frac{x+2}{2}\sqrt{x^2+4x+1} + \frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(D) $\frac{x+2}{2}\sqrt{x^2+4x+1} + 9\log\left|x+2+\sqrt{x^2+4x+1}\right|$

31) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 - \cos x + \tan^5 x)$ is .... [March - 2022]

(A) π (B) 2 (C) 0 (D) 1

32) $\int_{-1}^{1} \sin^5 x \cos^4 x\, dx = ...$ [March - 2022]

(A) –2 (B) 2 (C) 0 (D) 3

33) The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$ is ...... [March - 2022]

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) 0 (D) $\frac{\pi}{12}$

34) $\int x^2 e^{x^3}\,dx$ equals [March - 2022]

(A) $\frac{1}{2}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^3} + C$

(C) $\frac{1}{3}e^{x^2} + C$ (D) $\frac{1}{2}e^{x^2} + C$

35) The area of the parabola $y^2 = 12x$ bounded by its latus rectum is .... [March - 2022]

(A) 18 (B) 12 (C) 24 (D) 30

36) Smaller area enclosed by the circle $x^2 + y^2 = 16$ and the line $x + y = 4$ is .... [March - 2022]

(A) $4(\pi - 4)$ (B) $8\pi - 4$

(C) $4(\pi - 1)$ (D) $4(\pi - 2)$

37) The area of the region bounded by the ellipse $9x^2 + 4y^2 = 36$ is .... [March - 2022]

(A) 6π (B) 36π (C) 12π (D) 72π

38) The order and degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = \sqrt{\frac{d^2y}{dx^2}}$ respectively are .... [March - 2022]

(A) 2, 1 (B) 2, 2 (C) 1, 2 (D) 4, 2

39) The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is .... [March - 2022]

(A) e^x (B) x^2 (C) e^{2x} (D) x

40) The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is .... [March - 2022]

(A) $\log|y^2 + 1| = \log|1 + x^2| + C$

(B) $\sin^{-1}y = \sin^{-1}x + C$

(C) $\tan^{-1}y = \tan^{-1}x + C$

(D) $\cos^{-1}y = \cos^{-1}x + C$

41) The vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is .... [March - 2022]

(A) $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

(B) $40\hat{i} - 8\hat{j} + 16\hat{k}$

(C) $\frac{4}{3}\hat{i} - \frac{8}{30}\hat{j} + \frac{16}{30}\hat{k}$

(D) None

42) If $\vec{a}$ is unit vector and $(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = 8$, then $|\vec{x}|$ is .... [March - 2022]

(A) 4 (B) 3 (C) 7 (D) None

43) If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 12$, then value of $|\vec{a}\times\vec{b}|$ is .... [March - 2022]

(A) 16 (B) 10 (C) 5 (D) 14

44) The area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is .... [March - 2022]

(A) $\sqrt{42}$ (B) 42 (C) $\sqrt{21}$ (D) 21

45) The angle 'θ' between the vectores $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is .... [March - 2022]

(A) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (B) $\cos^{-1}\left(-\frac{1}{3}\right)$

(C) $-\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (D) None of the above

46) The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is ..... [March - 2022]

(A) 1 (B) –1 (C) 0 (D) 3

47) The coordinates of a point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) nearer the origin are ....

(A) (9, –13, 4) (B) $(8\sqrt{14}, -12, -1)$

(C) $(-8\sqrt{14}, 12, 1)$ (D) (–7, –11, 4)

48) The equation of a line parallel to Y- axis and passing through the point (2, 3, 4) is .... [March - 2022]

(A) $\frac{x+2}{1}=\frac{y+3}{0}=\frac{z+4}{1}$

(B) $\frac{x-2}{0}=\frac{y+3}{1}=\frac{z-4}{0}$

(C) $\frac{x-2}{1}=\frac{y-3}{0}=\frac{z-4}{1}$

(D) $\frac{x-2}{0}=\frac{y-3}{1}=\frac{z-4}{0}$

49) If cos α, cos β, cos γ are direction cosines then cos 2α + cos 2β + cos 2γ = ...

(A) –1 (B) 0 (C) 4 (D) 3

50) The general solution of a differential equation of the type $\frac{dx}{dy}+P_1x=Q_1$ is ....

(A) $y\cdot e^{\int P_1\,dy}=\int(Q_1e^{\int P_1dy})\,dy+C$

(B) $y\cdot e^{\int P_1\,dx}=\int(Q_1e^{\int P_1dx})\,dx+C$

(C) $x\cdot e^{\int P_1\,dy}=\int(Q_1e^{\int P_1dy})\,dy+C$

(D) $x\cdot e^{\int P_1\,dx}=\int(Q_1e^{\int P_1dx})\,dx+C$

Time : 2 Hours] PART - B [Total Marks : 50

Section - A

❖ Answer any 8 of the following given question 1 to 12 : (Each carries 2 marks) [16]

1) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2}<x<\frac{\pi}{2}$ in the simplest form. [March - 2022]

2) Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$. [March - 2022]

3) If $y=e^{x+e^{x+e^{x+....\infty}}}$, then find $\frac{dy}{dx}$.

4) Evaluate $\int\log x\,dx$

5) Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

6) Find the area of the region bounded by the curve $y = 2x - x^2$ and X-axis.

7) Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is $\sin x\cos y\,dx+\cos x\sin y\,dy=0$.

8) Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ from the vertices of a right angled triangle. [March - 2022]

9) Find the values of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. [March - 2022]

10) Find the direction cosines of the line passing through the two points (–2, 4, –5) and (1, 2, 3).

11) Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that one of them is black and other is red. [March - 2022]

12) Events A and B are such that $P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B are independent ?

Section - B

❖ Answer any 6 of the following given questions 13 to 21 : (Each carries 3 marks) [18]

13) Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5 ?

14) Express the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **[March - 2022]**

15) Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

16) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\dfrac{d^2y}{dx^2}$. **[March - 2022]**

17) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

18) Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$. **[March - 2022]**

19) Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

20) Solve the following problem graphically :
Minimise and Maximise $Z = 3x + 9y$
Subject to the constraints :
$x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x \geq 0$, $y \geq 0$. **[March - 2022]**

21) A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B ? **[March - 2022]**

Section - C

❖ **Answer any 4 of the following given questions from 22 to 27 : (Each carries 4 marks) [16]**

22) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$. **[March - 2022]**

23) Solve the following system of equations by matrix method.

$3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$

24) If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$. **[March - 2022]**

25) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$. Also find the maximum volume. **[March - 2022]**

26) Find $\displaystyle\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}}\, dx$. **[March - 2022]**

27) Find the particular solution of the differential equation : $(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$; $y = 1$ when $x = 0$. **[March - 2022]**

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