## UNIT–1: Differential Calculus

(Successive Differentiation, Leibnitz Theorem, Taylor & Maclaurin Series, Tangent & Normal, Partial Differentiation, Euler's Theorem, Curvature, Asymptotes, Maxima & Minima)

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**MCQs (1–75)**

**Section A: Successive Differentiation & nth Order Derivatives**

**1.** The second derivative of x3x^3x3 is:
A) 3x²
B) 6x
C) 6
D) x³
**Answer:** B

---

**2.** The nth derivative of xnx^nxn is:
A) n
B) n!
C) 0
D) x
**Answer:** B

---

**3.** The nth derivative of eaxe^{ax}eax is:
A) aeaxae^{ax}aeax
B) aneaxa^n e^{ax}aneax
C) neaxne^{ax}neax
D) enxe^{nx}enx
**Answer:** B

---

**4.** The nth derivative of sin⁗ax\sin axsinax is:
A) ansin⁗(ax+nπ/2)a^n \sin(ax+n\pi/2)ansin(ax+nπ/2)
B) ancos⁗axa^n \cos axancosax
C) sin⁗nx\sin nxsinnx
D) asin⁗axa\sin axasinax
**Answer:** A

**5.** The nth derivative of a constant is:
A) 1
B) constant
C) 0
D) infinity
**Answer:** C

---

**6.** The third derivative of x4x^4x4 is:
A) 24x
B) 12x²
C) 4x³
D) 6x
**Answer:** A

---

**7.** Successive differentiation means:
A) Integration repeatedly
B) Differentiation repeatedly
C) Partial derivative
D) Limit evaluation
**Answer:** B

---

**8.** The nth derivative of ln⁡x\ln xlnx contains:
A) powers of x in denominator
B) exponential terms
C) constants only
D) trigonometric terms
**Answer:** A

---

**9.** Dn(xm)=0D^n(x^m)=0Dn(xm)=0 when:
A) m>n
B) m=n
C) m<n
D) m=0
**Answer:** C

---

**10.** The nth derivative of cos(ax)cos(ax)cos(ax) involves:
A) sine only
B) cosine only
C) alternating sine & cosine
D) polynomial
**Answer:** C

---

### Section B: Leibnitz Theorem

**11.** Leibnitz theorem is used for derivative of:
A) Quotient
B) Product
C) Sum
D) Constant
**Answer:** B

---

**12.** Leibnitz theorem gives nth derivative of:
A) uv
B) u/v
C) u+v
D) u−v
**Answer:** A

---

**13.** First term in Leibnitz expansion is:
A) unvu^n vunv
B) u(n)vu^{(n)}vu(n)v
C) uv$^n$
D) v$^n$
**Answer:** B

---

**14.** Last term of Leibnitz theorem is:
A) uv(n)uv^{(n)}uv(n)
B) u(n)vu^{(n)}vu(n)v
C) uv
D) zero
**Answer:** A

---

**15.** Leibnitz theorem includes:
A) factorial terms
B) binomial coefficients
C) logarithms
D) matrices
**Answer:** B

---

**16.** Number of terms in Leibnitz expansion is:
A) n
B) n−1
C) n+1
D) 2n
**Answer:** C

---

**17.** Leibnitz theorem is derived using:
A) Binomial theorem
B) Taylor theorem
C) Integration
D) Limits
**Answer:** A

---

**18.** Leibnitz theorem applies to:
A) composite functions
B) product of functions
C) constants
D) matrices
**Answer:** B

---

**19.** Combination term used is:
A) nCrnC_rnCr
B) n!
C) r!
D) none
**Answer:** A

---

**20.** Leibnitz theorem simplifies:
A) higher derivatives
B) integration
C) limits
D) matrices
**Answer:** A

---

**Section C: Taylor & Maclaurin Series**

**21.** Maclaurin series is Taylor series about:
A) x=1
B) x=0
C) x=a
D) infinity
**Answer:** B

---

**22.** Taylor series expands function near:
A) origin only
B) any point a
C) infinity
D) zero only
**Answer:** B

---

**23.** Maclaurin expansion of exe^xex begins with:
A) x
B) 1
C) x²
D) 0
**Answer:** B

---

**24.** Maclaurin series of sinxsin xsinx contains:
A) even powers
B) odd powers
C) constants only
D) logarithms
**Answer:** B

---

**25.** Maclaurin series of cosxcos xcosx contains:
A) odd powers
B) even powers
C) roots
D) fractions
**Answer:** B

**26.** Taylor series is useful for:
A) approximation
B) integration only
C) matrices
D) geometry
**Answer:** A

**27.** Expansion of ln(1+x) starts with:
A) x
B) 1
C) x²
D) ln x
**Answer:** A

**28.** Taylor series requires existence of:
A) derivatives
B) integrals
C) limits only
D) constants
**Answer:** A

**29.** Maclaurin series is special case of:
A) Fourier series
B) Taylor series
C) Power series
D) Binomial series
**Answer:** B

**30.** Series expansion converts function into:
A) polynomial form
B) matrix form
C) vector form
D) integral form
**Answer:** A

---

## Section D: Tangent and Normal

**31.** Slope of tangent equals:
A) dy/dx
B) dx/dy
C) y/x
D) constant
**Answer:** A

---

**32.** Slope of normal is:
A) dy/dx
B) −dx/dy
C) −1/(dy/dx)
D) dy/dx²
**Answer:** C

---

**33.** Tangent touches curve at:
A) one point
B) two points
C) infinite points
D) none
**Answer:** A

---

**34.** Normal is perpendicular to:
A) axis
B) tangent
C) curve
D) origin
**Answer:** B

**35.** Angle between curves depends on:
A) slopes
B) intercepts
C) constants
D) area
**Answer:** A

---

**36.** Angle between curves formula uses:
A) tanθ
B) sinθ
C) cosθ
D) secθ
**Answer:** A

---

**37.** Subtangent lies on:
A) x-axis
B) y-axis
C) curve
D) origin
**Answer:** A

---

**38.** Subnormal is measured along:
A) x-axis
B) y-axis
C) tangent
D) curve
**Answer:** A

---

**39.** Parametric tangent slope equals:
A) dy/dx = (dy/dt)/(dx/dt)
B) dx/dy
C) dt/dx
D) dy/dt
**Answer:** A

**40.** Length of normal depends on:
A) derivative
B) integral
C) constant
D) matrix
**Answer:** A

---

**Section E: Partial Differentiation & Euler's Theorem**

**41.** Partial differentiation applies to:
A) single variable
B) multivariable functions
C) constants
D) matrices
**Answer:** B

---

**42.** Symbol for partial derivative is:
A) d
B) $\partial$
C) $\Delta$
D) $\sum$
**Answer:** B

---

**43.** While partial differentiation, other variables are:
A) zero
B) constant
C) infinity
D) ignored
**Answer:** B

---

**44.** Euler's theorem applies to:
A) homogeneous functions
B) exponential functions
C) logarithmic functions
D) trigonometric functions
**Answer:** A

---

**45.** If function degree is n, Euler theorem gives:
A) nf
B) f/n
C) n²f
D) 0
**Answer:** A

---

**46.** Homogeneous function means:
A) same degree terms
B) constants
C) linear only
D) quadratic only
**Answer:** A

---

**47.** Euler theorem relation includes:
A) $x\partial z/\partial x + y\partial z/\partial y$
B) x+y
C) xy
D) x/y
**Answer:** A

---

**48.** Partial derivative of constant equals:
A) 1
B) constant
C) 0
D) infinity
**Answer:** C

---

**49.** Mixed partial derivatives are:
A) always unequal
B) equal (under continuity)
C) zero
D) infinite
**Answer:** B

**50.** Order of differentiation in mixed partial derivative:
A) matters always
B) does not matter (continuous case)
C) undefined
D) random
**Answer:** B

---

### Section F: Curvature & Asymptotes

**51.** Curvature measures:
A) slope
B) bending of curve
C) area
D) volume
**Answer:** B

---

**52.** Radius of curvature is reciprocal of:
A) slope
B) curvature
C) area
D) tangent
**Answer:** B

---

**53.** Higher curvature means:
A) flatter curve
B) sharper bend
C) straight line
D) constant slope
**Answer:** B

---

**54.** Asymptote is a line which:
A) cuts curve always
B) approaches curve at infinity
C) tangent always
D) normal always
**Answer:** B

---

**55.** Horizontal asymptote equation is:
A) x = constant
B) y = constant
C) xy = constant
D) slope zero
**Answer:** B

---

**56.** Vertical asymptote equation is:
A) x = constant
B) y = constant
C) x+y=0
D) slope infinity
**Answer:** A

---

**57.** Oblique asymptote occurs when degree difference is:
A) 0
B) 1
C) 2
D) 3
**Answer:** B

---

**58.** Straight line has curvature:
A) 1
B) 0
C) infinity
D) constant
**Answer:** B

---

**59.** Radius of curvature of straight line is:
A) 1
B) 0
C) infinity
D) constant
**Answer:** C

---

**60.** Curvature depends on:
A) second derivative
B) first derivative only
C) constant
D) integral
**Answer:** A

---

## Section G: Maxima & Minima (Two Variables)

**61.** Critical point occurs when:
A) derivatives zero
B) derivatives infinite
C) constant value
D) integral zero
**Answer:** A

---

**62.** For maxima/minima in two variables we use:
A) partial derivatives
B) integration
C) limits
D) matrices
**Answer:** A

---

**63.** Second derivative test uses:
A) Hessian determinant
B) integration
C) limits
D) matrices
**Answer:** A

---

**64.** If $D>0$ and $f_{xx}>0$, point is:
A) maximum
B) minimum
C) saddle
D) none
**Answer:** B

---

**65.** If $D>0$ and $f_{xx}<0$, point is:
A) maximum
B) minimum
C) saddle
D) none
**Answer:** A

---

**66.** If $D<0$, point is:
A) maximum
B) minimum
C) saddle point
D) zero
**Answer:** C

---

**67.** Function of two variables written as:
A) $f(x)$
B) $f(x,y)$
C) $f(x^2)$
D) $f(y)$
**Answer:** B

---

**68.** Necessary condition for extrema is:
A) first derivatives zero
B) second derivatives zero
C) integral zero
D) constant
**Answer:** A

---

**69.** Maxima means function value is:
A) smallest nearby
B) largest nearby
C) zero
D) infinite
**Answer:** B

**70.** Minima means function value is:
A) largest nearby
B) smallest nearby
C) zero
D) constant
**Answer:** B

**71.** Saddle point means:
A) max only
B) min only
C) neither max nor min
D) constant
**Answer:** C

**72.** Optimization problems use:
A) maxima & minima
B) integration
C) limits
D) algebra
**Answer:** A

**73.** Second order partial derivatives help in:
A) classification of extrema
B) integration
C) limits
D) geometry only
**Answer:** A

**74.** Local maximum is also called:
A) relative maximum
B) absolute minimum
C) saddle
D) asymptote
**Answer:** A

**75.** Maximum and minimum problems belong to:
A) optimization theory
B) matrices
C) statistics
D) algebra only
**Answer:** A

## UNIT–II : COORDINATE GEOMETRY (2D)

*(Change of Rectangular Axes, Rotation & Shifting of Origin, Second Degree Equation, Conics, Tangent & Normal using Calculus, Polar Equation)*

**(Question Number starts from 76 as requested)**

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**MCQs (76–150)**

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**Section A: Change of Rectangular Axes**

**76.** Change of axes is used to:
A) simplify equation
B) increase degree
C) integrate function
D) differentiate function
**Answer:** A

---

**77.** Translation of axes means:
A) rotation only
B) shifting origin
C) reflection
D) scaling
**Answer:** B

---

**78.** Rotation of axes changes:
A) intercepts
B) orientation of axes
C) degree of curve
D) constants only
**Answer:** B

**79.** In rotation through angle θ:
A) axes remain fixed
B) axes rotate
C) curve rotates only
D) origin shifts
**Answer:** B

**80.** New coordinates after shifting origin are expressed using:
A) linear relations
B) quadratic relations
C) logarithmic relations
D) exponential relations
**Answer:** A

**81.** Equation form remains same after transformation because:
A) degree unchanged
B) constants vanish
C) variables vanish
D) axes fixed
**Answer:** A

**82.** Rotation eliminates which term in second degree equation?
A) x²
B) y²
C) xy
D) constant
**Answer:** C

**83.** Angle of rotation is obtained using:
A) tanθ = B/A
B) tan2θ = B/(A−C)
C) sinθ = A/B
D) cosθ = B/C
**Answer:** B

**84.** Shifting origin replaces:
A) x → X+h
B) x → X−h
C) x → hX
D) x → X²
**Answer:** B

---

**85.** Purpose of transformation is mainly:
A) simplification
B) complication
C) integration
D) differentiation
**Answer:** A

---

## Section B: Rotation and Shifting of Origin

**86.** Rotation keeps origin:
A) fixed
B) shifted
C) removed
D) undefined
**Answer:** A

---

**87.** Shifting origin changes:
A) axes direction
B) origin position
C) slope
D) angle only
**Answer:** B

---

**88.** Combined transformation includes:
A) rotation & translation
B) differentiation
C) integration
D) reflection only
**Answer:** A

**89.** After shifting origin, constants may:
A) disappear
B) increase degree
C) become zero sometimes
D) become infinite
**Answer:** C

---

**90.** Transformation helps identify:
A) nature of conic
B) derivative
C) integral
D) limit
**Answer:** A

---

**91.** Coordinate transformation preserves:
A) degree of equation
B) constants only
C) axes length
D) slope always
**Answer:** A

---

**92.** Rotation formulas involve:
A) $\sin\theta$ and $\cos\theta$
B) tan only
C) logarithm
D) exponentials
**Answer:** A

---

**93.** New axes after rotation are:
A) parallel
B) inclined
C) perpendicular always
D) coincident
**Answer:** B

---

**94.** Transformation removes linear terms by:
A) rotation
B) shifting origin
C) integration
D) differentiation
**Answer:** B

---

**95.** Canonical form means:
A) simplified standard form
B) expanded form
C) polynomial form
D) integral form
**Answer:** A

---

**Section C: General Equation of Second Degree**

General form:
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

---

**96.** Degree of general conic equation is:
A) 1
B) 2
C) 3
D) 4
**Answer:** B

---

**97.** Term responsible for rotation is:
A) $Ax^2$
B) $Cy^2$
C) $Bxy$
D) $Dx$
**Answer:** C

---

**98.** Nature of conic depends on:
A) coefficients

B) constants only
C) variables only
D) slope
**Answer:** A

---

**99.** Discriminant used is:
A) $B^2-4AC$
B) $A^2+B^2$
C) AC
D) A−C
**Answer:** A

---

**100.** If B2−4AC=0B^2−4AC=0B2−4AC=0, conic is:
A) ellipse
B) parabola
C) hyperbola
D) circle
**Answer:** B

---

**101.** If B2−4AC<0B^2−4AC<0B2−4AC<0, conic is:
A) ellipse
B) hyperbola
C) parabola
D) pair of lines
**Answer:** A

---

**102.** If B2−4AC>0B^2−4AC>0B2−4AC>0, conic is:
A) ellipse
B) parabola
C) hyperbola
D) circle
**Answer:** C

---

**103.** Circle is special case of:
A) ellipse
B) parabola

C) hyperbola
D) line
**Answer:** A

---

**104.** Equal coefficients of $x^2$ and $y^2$ indicate:
A) circle
B) parabola
C) hyperbola
D) line
**Answer:** A

---

**105.** Conic classification depends mainly on:
A) A, B, C
B) D only
C) E only
D) F only
**Answer:** A

---

**Section D: Tangent and Normal using Calculus**

**106.** Equation of tangent uses:
A) dy/dx
B) integral
C) limit only
D) matrix
**Answer:** A

---

**107.** Slope of tangent at point equals:
A) derivative value
B) function value
C) integral value
D) constant
**Answer:** A

---

**108.** Normal slope equals:
A) dy/dx

B) −1/(dy/dx)
C) dx/dy
D) zero
**Answer:** B

---

**109.** Tangent touches curve at:
A) one point locally
B) two points always
C) infinite points
D) none
**Answer:** A

---

**110.** Tangent equation form:
A) $y - y_1 = m(x - x_1)$
B) y=mx
C) x+y=0
D) ax+by+c=0 only
**Answer:** A

---

**111.** Normal passes through:
A) slope zero
B) perpendicular direction
C) origin only
D) axis only
**Answer:** B

---

**112.** Tangent to circle is perpendicular to:
A) radius
B) diameter
C) chord
D) axis
**Answer:** A

---

**113.** Calculus method uses:
A) differentiation
B) integration

C) matrices
D) vectors
**Answer:** A

---

**114.** Tangent slope at turning point is:
A) zero
B) infinite
C) constant
D) undefined always
**Answer:** A

---

**115.** Vertical tangent occurs when:
A) dx/dy = 0
B) dy/dx = 0
C) slope finite
D) constant
**Answer:** A

---

## Section E: Polar Coordinates

**116.** Polar coordinates represented by:
A) (x,y)
B) (r,$\theta$)
C) ($\theta$,x)
D) (r,x)
**Answer:** B

---

**117.** Relation between Cartesian and Polar is:
A) x = r cos$\theta$
B) y = r sin$\theta$
C) both A & B
D) none
**Answer:** C

---

**118.** Polar equation represents:
A) distance & angle

B) slope only
C) intercept only
D) derivative
**Answer:** A

---

**119.** Pole means:
A) origin
B) axis
C) tangent
D) normal
**Answer:** A

---

**120.** Initial line is:
A) polar axis
B) normal
C) tangent
D) diameter
**Answer:** A

---

**121.** r represents:
A) radius vector
B) slope
C) derivative
D) intercept
**Answer:** A

---

**122.** $\theta$ represents:
A) distance
B) angle
C) slope
D) constant
**Answer:** B

---

**123.** Circle in polar form may be:
A) $r = a \cos\theta$
B) $r = a \sin\theta$

C) both
D) none
**Answer:** C

---

**124.** Cardioid is polar curve when equation contains:
A) a(1+cosθ)
B) ax²
C) y²
D) x+y
**Answer:** A

---

**125.** Polar coordinates useful for:
A) symmetric curves
B) algebra only
C) matrices
D) limits
**Answer:** A

---

**Section F: Additional Conceptual MCQs**

**126.** Transformation simplifies identification of:
A) conics
B) derivatives
C) integrals
D) limits
**Answer:** A

---

**127.** Equation without xy-term indicates axes are:
A) rotated
B) rectangular
C) parallel
D) shifted
**Answer:** B

---

**128.** Rotation angle zero implies:
A) no rotation

B) shift only
C) reflection
D) scaling
**Answer:** A

---

**129.** Conic symmetry depends on:
A) coefficients
B) constants only
C) derivative
D) integral
**Answer:** A

---

**130.** Tangent line approximates curve:
A) locally
B) globally
C) randomly
D) infinitely
**Answer:** A

---

**131.** Normal gives direction of:
A) perpendicular slope
B) parallel slope
C) zero slope
D) infinite slope
**Answer:** A

---

**132.** Polar axis corresponds to:
A) positive x-axis
B) y-axis
C) tangent
D) normal
**Answer:** A

---

**133.** Negative r means point lies:
A) opposite direction
B) same direction

C) origin only
D) infinity
**Answer:** A

---

**134.** Polar curve symmetry checked by replacing:
A) θ by −θ
B) r by −r
C) both
D) none
**Answer:** C

---

**135.** Transformation does NOT change:
A) geometric nature
B) algebraic degree
C) both A & B
D) constants
**Answer:** C

---

**Section G: Advanced Concept MCQs**

**136.** Conic degenerates when equation represents:
A) pair of lines
B) circle
C) parabola
D) hyperbola
**Answer:** A

---

**137.** Tangent at vertex of parabola is:
A) axis parallel
B) perpendicular to axis
C) random
D) undefined
**Answer:** B

---

**138.** Normal at vertex of parabola coincides with:
A) axis

B) tangent
C) chord
D) asymptote
**Answer:** A

---

**139.** Hyperbola has how many asymptotes?
A) 1
B) 2
C) 3
D) none
**Answer:** B

---

**140.** Ellipse has eccentricity:
A) >1
B) =1
C) <1
D) 0 only
**Answer:** C

---

**141.** Parabola eccentricity equals:
A) 0
B) 1
C) 2
D) infinity
**Answer:** B

---

**142.** Hyperbola eccentricity is:
A) <1
B) =1
C) >1
D) 0
**Answer:** C

---

**143.** Focus-directrix definition applies to:
A) conic sections
B) lines

C) circles only
D) matrices
**Answer:** A

---

**144.** Polar equation of straight line is:
A) r cos(θ−α)=p
B) x+y=0
C) y=mx
D) ax²
**Answer:** A

---

**145.** Angle between radius vector and tangent involves:
A) dr/dθ
B) dy/dx
C) integral
D) constant
**Answer:** A

---

**146.** Pedal equation relates:
A) radius vector & perpendicular distance
B) slope & intercept
C) derivative only
D) integral only
**Answer:** A

---

**147.** Symmetry about pole checked by replacing:
A) r → −r
B) θ → π−θ
C) θ → −θ
D) all
**Answer:** D

---

**148.** Rectangular axes always meet at:
A) right angle
B) acute angle
C) obtuse angle

D) variable angle
**Answer:** A

---

**149.** Coordinate geometry connects:
A) algebra & geometry
B) algebra & calculus only
C) geometry only
D) arithmetic
**Answer:** A

---

**150.** Main aim of coordinate geometry is:
A) geometric study using algebra
B) differentiation
C) integration
D) statistics
**Answer:** A

**UNIT–1 Differential Calculus**

**Step-by-Step Solutions (Q1–Q30)**

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**☑ SECTION A — Successive Differentiation & nth Derivative**

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**Q1. Find nth derivative of y=xneaxy=x^n e^{ax}y=xneax**

Step 1: Use Leibnitz Rule
Dn(uv)=∑r=0n(nr)u(n−r)v(r)D^n(uv)=\sum_{r=0}^{n} {n \choose r} u^{(n-r)}v^{(r)}Dn(uv)=r=0∑n (rn)u(n−r)v(r)

Let

u=xn,v=eaxu=x^n,\quad v=e^{ax}u=xn,v=eax

---

## Step 2: Derivatives

$$\frac{d^k}{dx^k}(x^n)=\frac{n!}{(n-k)!}x^{n-k}$$

$$\frac{d^r}{dx^r}(e^{ax})=a^r e^{ax}$$

---

## Step 3: Substitute

$$y^{(n)}=e^{ax}\sum_{r=0}^{n}\binom{n}{r}\frac{n!}{r!}x^{r}a^{\,n-r}$$

---

## ✅ Final Answer

$$y^{(n)}=e^{ax}\sum_{r=0}^{n}\frac{(n!)^2}{(r!)^2(n-r)!}a^{n-r}x^{r}$$

---

## Q2. nth derivative of $y=e^{ax}\sin bx$

### Step 1: Write in complex form

$$\sin bx=\Im(e^{ibx})$$

$$y=\Im(e^{(a+ib)x})$$

---

### Step 2: Differentiate n times

$$D^n(e^{(a+ib)x})=(a+ib)^n e^{(a+ib)x}$$

---

### Step 3: Convert to polar form

Let

$$r=\sqrt{a^2+b^2},\quad \theta=\tan^{-1}(b/a)$$

$$(a+ib)^n=r^n(\cos n\theta+i\sin n\theta)$$

---

## ✅ Final Answer

$$y^{(n)}=r^n e^{ax}\sin(bx+n\theta)$$

---

## Q3. nth derivative of $y=\ln x$

### Stepwise Differentiation

$y'=\frac1x$ y'=\frac1xy'=x1 $y''=-\frac{1}{x^2}$ y''=−1x2y''=-\frac1{x^2}y''=−x21 $y'''=\frac{2!}{x^3}$ y'''=2!x3y'''=\frac{2!}{x^3}y'''=x32!

Pattern:

$y^{(n)}=(-1)^{n-1}\frac{(n-1)!}{x^n}$ y(n)=(−1)n−1(n−1)!xny^{(n)}=(-1)^{n-1}\frac{(n-1)!}{x^n}y(n)=(−1)n−1xn(n−1)!

---

y(n)=(−1)n−1(n−1)!xn$\boxed{y^{(n)}=(-1)^{n-1}\frac{(n-1)!}{x^n}}$y(n)=(−1)n−1xn(n−1)!

---

## Q4. nth derivative of $y=\frac1{ax+b}$ y=1ax+by=\frac1{ax+b}y=ax+b1

Write:

$y=(ax+b)^{-1}$ y=(ax+b)−1y=(ax+b)^{-1}y=(ax+b)−1

Using chain rule repeatedly:

$y^{(n)}=(-1)^n n!\, a^n (ax+b)^{-(n+1)}$ y(n)=(−1)nn!an(ax+b)−(n+1)y^{(n)}=(-1)^n n! a^n (ax+b)^{-(n+1)}y(n)=(−1)nn!an(ax+b)−(n+1)

---

y(n)=(−1)nn!an(ax+b)n+1$\boxed{y^{(n)}=\frac{(-1)^n n! a^n}{(ax+b)^{n+1}}}$y(n)=(ax+b)n+1(−1)nn!an

---

## Q5. nth derivative of $y=x^2\cos ax$ y=x2cos⁡axy=x^2\cos axy=x2cosax

Apply Leibnitz theorem.

Let:

$u=x^2,\quad v=\cos ax$ u=x2,v=cos⁡axu=x^2,\quad v=\cos axu=x2,v=cosax

Derivatives:

$u'=2x,\; u''=2,\; u'''=0$ u'=2x, u''=2, u'''=0u'=2x,\; u''=2,\; u'''=0u'=2x,u''=2,u'''=0

Hence only first three terms survive.

$$y^{(n)}=x^2 D^n(\cos ax)+2nx D^{n-1}(\cos ax)+n(n-1)D^{n-2}(\cos ax)$$

---

## Final Result

Substitute:

$$D^n(\cos ax)=a^n\cos(ax+n\pi/2)$$

---

## Q6. Show amplitude-phase form

From Q2:

$$D^n[e^{ax}\cos bx]=r^n e^{ax}\cos(bx+n\theta)$$

where

$$r=\sqrt{a^2+b^2}$$

✓ proved.

---

## Q7. nth derivative of $y=\tan^{-1}x$

$$y'=\frac{1}{1+x^2}$$

Repeated differentiation gives:

$$y^{(n)}=\frac{(-1)^{n-1}(n-1)!P_{n-1}(x)}{(1+x^2)^n}$$

where $P_{n-1}(x)$ is polynomial.

---

## Q8. nth derivative of

$$y=\frac{x}{x^2+a^2}$$

Write using partial fractions:

$$\frac{x}{x^2+a^2}=\frac12\left(\frac1{x-ia}+\frac1{x+ia}\right)$$

Differentiate using result of Q4.

---

**Q9. nth derivative of $y=\sin(ax+b)$**

Successive differentiation pattern:

$$\sin \to \cos \to -\sin \to -\cos$$

Hence,

$$\boxed{y^{(n)}=a^n\sin(ax+b+n\pi/2)}$$

---

**Q10. Higher order derivatives (Theory)**

Higher derivatives measure:

- acceleration (physics)
- curvature
- error estimation
- oscillation behavior

Applications:

- mechanics
- signal processing
- numerical approximation

---

## ✅ SECTION B — LEIBNITZ THEOREM

---

**Q11. Statement & Proof**

If $y = uv$,

$$D^n(uv) = \sum_{r=0}^{n} \binom{n}{r} u^{(n-r)} v^{(r)}$$

---

1. Verify for n=1:

$$(uv)' = u'v + uv'$$

2. Assume true for n=k.
3. Differentiate both sides.
4. Use binomial identity:

$$\binom{k}{r} + \binom{k}{r-1} = \binom{k+1}{r}$$

Hence proved by induction.

---

**Q12. $y = x^n e^{ax}$**

Apply Leibnitz:

$$y^{(n)} = \sum_{r=0}^{n} \binom{n}{r} \frac{n!}{r!} x^r a^{n-r} e^{ax}$$

---

**Q13. $y = x^2 \sin x$**

Only first three derivatives of $x^2$ exist:

$$y^{(n)} = x^2 D^n(\sin x) + 2nx D^{n-1}(\sin x) + n(n-1) D^{n-2}(\sin x)$$

---

**Q14. $y = x^3 \cos x$**

Same process; terms up to third derivative remain.

### Q15. y=xnln⬚xy=x^n\ln xy=xnlnx

Use Leibnitz rule:

y(n)=∑r=0n(nr)Dn−r(xn)Dr(ln⬚x)y^{(n)}=\sum_{r=0}^{n}{n\choose r} D^{n-r}(x^n)D^r(\ln x)y(n)=r=0∑n(rn)Dn−r(xn)Dr(lnx)

Substitute known formulas.

---

### Q16. Significance

Leibnitz theorem helps:

- higher derivatives quickly
- series expansions
- differential equations

---

### Q17. y=eax(x2+1)y=e^{ax}(x^2+1)y=eax(x2+1)

y(n)=eax[an(x2+1)+2nan−1x+n(n−1)an−2]y^{(n)}=e^{ax}[a^n(x^2+1)+2na^{n-1}x+n(n-1)a^{n-2}]y(n)=eax[an(x2+1)+2nan−1x+n(n−1)an−2]

---

### Q18. Case n=1

D(uv)=u′v+uv′D(uv)=u'v+uv'D(uv)=u′v+uv′

Hence Leibnitz reduces to product rule.

---

### ✅ SECTION C — TAYLOR & MACLAURIN SERIES

---

### Q19. Taylor's Theorem

$$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f''(a)+\cdots$$

Proof uses repeated integration and mean value theorem.

---

### Q20. Maclaurin Series

Put $a=0$:

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\cdots$$

---

### Q21. Maclaurin for $e^x$

All derivatives $= e^x$

At $x=0 \to 1$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}$$

---

### Q22. Maclaurin for sin x

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$$

---

### Q23. ln(1+x)

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$

Valid for $|x|<1$.

---

### Q24. Taylor expansion of cos x about $x=\pi/4$

Use formula:

$$f(x)=f(a)+(x-a)f'(a)+\cdots$$

Compute derivatives at $a=\pi/4$.

---

## Q25. Taylor expansion of $e^x$ about x=1

$$e^x=e\left[1+(x-1)+\frac{(x-1)^2}{2!}+\cdots\right]$$

---

## Q26. Maclaurin for $\tan^{-1}x$

$$\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots$$

---

## ✅ SECTION D — APPLICATIONS

---

## Q27. Approximate $e^{0.1}$

$$e^{0.1}=1+0.1+\frac{0.01}{2}+\frac{0.001}{6}=1.10517$$

---

## Q28. Limit

Use series:

$$\sin x=x-\frac{x^3}{6}+\ldots \quad \frac{\sin x-x}{x^3}=-\frac{1}{6}$$

---

## Q29. Approximate $\sqrt{1.02}$

$$(1+x)^{1/2}=1+\frac{x}{2}-\frac{x^2}{8}$$

Put $x=0.02$:

=1.00995=1.00995=1.00995

---

## Q30. Approximate $\ln(1.1)$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}$$

Put $x=0.1$:

$$=0.1-0.005+0.00033 =0.09533$$

## Q31.

Expand $\sqrt{1+x}$ using **Maclaurin's series** up to the term containing $x^4$.

Solution

Maclaurin series:

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\cdots$$

Let:

$$f(x)=(1+x)^{1/2}$$

---

Step 1: Derivatives

$$f'(x)=\frac12(1+x)^{-1/2}$$
$$f''(x)=-\frac14(1+x)^{-3/2}$$
$$f'''(x)=\frac{3}{8}(1+x)^{-5/2}$$
$$f^{(4)}(x)=-\frac{15}{16}(1+x)^{-7/2}$$

---

Step 2: Values at $x=0$

$$f(0)=1$$
$$f'(0)=\frac12$$
$$f''(0)=-\frac14$$
$$f'''(0)=\frac38$$
$$f^{(4)}(0)=-\frac{15}{16}$$

---

Step 3: Substitute

$$\sqrt{1+x}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}$$

✓ Final Answer

$$\boxed{1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}}$$

---

## Q32.

Expand $\ln(1-x)$ using Maclaurin's series up to $x^4$.

### Solution

Known expansion:

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$

Replace $x$ by $-x$:

$$\ln(1-x)=-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}\right)$$

---

✓ Final Answer

$$\boxed{\ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}}$$

---

## Q33.

Expand $\tan^{-1}x$ by Maclaurin's series and hence evaluate approximately $\tan^{-1}(0.2)$.

### Solution

Maclaurin expansion:

$$\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots$$

---

## Step 1: Substitute $x=0.2$

$$=0.2-\frac{(0.2)^3}{3}+\frac{(0.2)^5}{5}$$

---

## Step 2: Calculate

$$0.2^3=0.008 \qquad 0.2^5=0.00032$$

$$=0.2-\frac{0.008}{3}+\frac{0.00032}{5}$$

$$=0.2-0.00267+0.000064$$

---

## ✓ Final Answer

$$\boxed{\tan^{-1}(0.2)\approx 0.1974}$$

---

## Q34.

Use Taylor's series to expand $\ln x$ about $x=1$ up to $(x-1)^3$.

### Solution

Taylor expansion about $a=1$:

$$f(x)=f(1)+(x-1)f'(1)+\frac{(x-1)^2}{2!}f''(1)+\cdots$$

Let $f(x)=\ln x$

---

### Step 1: Derivatives

$$f'(x)=\frac{1}{x} \qquad f''(x)=-\frac{1}{x^2} \qquad f'''(x)=\frac{2}{x^3}$$

---

### Step 2: Values at $x=1$

$$f(1)=0,\quad f'(1)=1$$
$$f''(1)=-1,\quad f'''(1)=2$$

---

### Step 3: Substitute

$$\ln x=(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}$$

✅ **Final Answer**

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$$

---

## Q35.

Using Maclaurin's series, evaluate the limit:

$$\lim_{x\to0}\frac{e^x-1-x}{x^2}$$

**Solution**

Maclaurin expansion:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

---

**Step 1: Substitute**

$$e^x - 1 - x = \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

---

**Step 2: Divide by $x^2$**

$$= \frac{1}{2} + \frac{x}{6} + \cdots$$

---

**Step 3: Take limit**

$$x\to0 \quad \boxed{\frac{1}{2}}$$

## SECTION A — Tangent & Normal (Cartesian Form)

## Q36.

Find equations of tangent and normal to the curve

$$y = x^2 + 3x$$

at $x=1$.

## Solution

Given:

$y = x^2 + 3x$

### Step 1: Differentiate

$\frac{dy}{dx} = 2x + 3$

At $x = 1$:

$m = 2(1) + 3 = 5$

Point:

$y = 1 + 3 = 4 \Rightarrow (1,4)$

---

### Step 2: Tangent

$y - 4 = 5(x-1)$

$\boxed{y = 5x - 1}$

---

### Step 3: Normal slope

$m_n = -\frac{1}{5}$

Normal:

$y - 4 = -\frac{1}{5}(x-1)$

---

## Q37.

Find tangent and normal to $y = \ln x$ at $x = e$.

### Solution

$\frac{dy}{dx} = \frac{1}{x}$

At $x = e$:

$m = \frac{1}{e}$

Point $(e, 1)$

Tangent:

$$y-1=\frac{1}{e}(x-e)$$

Normal slope:

$$=-e$$

Normal:

$$y-1=-e(x-e)$$

---

## Q38.

Find tangent and normal to $y=x^3$ at (1,1).

$$\frac{dy}{dx}=3x^2$$

At $x=1$: slope = 3

Tangent:

$$y-1=3(x-1)$$

Normal slope:

$$-\frac{1}{3}$$

---

## Q39.

Find tangent to curve $x^2+y^2=25$ at (3,4).

Differentiate implicitly:

$$2x+2y\frac{dy}{dx}=0 \quad \frac{dy}{dx}=-\frac{x}{y}$$

At (3,4):

m=−34m=-\frac34m=−43

Tangent:

y−4=−34(x−3)y-4=-\frac34(x-3)y−4=−43(x−3)

---

## Q40.

Prove tangent to circle is perpendicular to radius.

Slope of radius:

mr=yxm_r=\frac{y}{x}mr=xy

Slope of tangent:

mt=−xym_t=-\frac{x}{y}mt=−yx mrmt=−1m_r m_t=-1mrmt=−1

Hence perpendicular ✓

---

## SECTION B — Parametric Form

---

## Q41.

Find tangent to:

x=t2, y=t3x=t^2,\; y=t^3x=t2,y=t3

at t=1t=1t=1.

Solution
dydx=dy/dtdx/dt=3t22t=3t2\frac{dy}{dx}=\frac{dy/dt}{dx/dt} =\frac{3t^2}{2t}=\frac{3t}{2}dxdy =dx/dtdy/dt=2t3t2=23t

At t=1t=1t=1: slope = 3/2

Point (1,1)

Tangent:

$$y-1=\frac{3}{2}(x-1)$$

---

## Q42.

Find normal to parametric curve of Q41.

Normal slope:

$$-\frac{2}{3}$$

$$y-1=-\frac{2}{3}(x-1)$$

---

## Q43.

Find tangent to cycloid

$$x=a(t-\sin t),\ y=a(1-\cos t)$$

### Solution

$$\frac{dy}{dx}=\frac{a\sin t}{a(1-\cos t)}=\frac{\sin t}{1-\cos t}$$

---

## Q44.

Explain geometrical meaning of parametric derivative.

### Solution (Theory)

Slope represents rate of change of y w.r.t x along parameter motion.

---

## SECTION C — Angle Between Two Curves

---

## Q45.

Find angle between curves:

y=x2,y=2xy=x^2,\quad y=2xy=x2,y=2x

at origin.

### Solution

Slopes:

m1=0,m2=2m_1=0,\quad m_2=2m1=0,m2=2

Angle formula:

tan⁡θ=|m1−m21+m1m2|\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|tanθ=1+m1m2 m1−m2 =21=2=\frac{2}{1}=2=12=2 θ=tan⁡−1(2)\theta=\tan^{-1}(2)θ=tan−1(2)

---

## Q46.

State formula for angle between curves.

tan⁡θ=|m1−m21+m1m2|\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|tanθ=1+m1m2 m1−m2

---

## Q47.

Find angle between curves y=x2y=x^2y=x2 and x=y2x=y^2x=y2 at intersection.

Solve intersection → (0,0),(1,1)

Compute slopes and substitute in formula.

---

## SECTION D — Subtangent & Subnormal

---

## Q48.

Show subtangent length:

=ydy/dx=\frac{y}{dy/dx}=dy/dxy

### Proof

From tangent geometry using similar triangles.

---

## Q49.

Find subtangent of $y = x^2$.

$$\frac{dy}{dx} = 2x$$

$$\text{subtangent} = \frac{x^2}{2x} = \frac{x}{2}$$

---

## Q50.

Find subnormal for $y = x^2$.

Formula:

$$y\frac{dy}{dx} = x^2(2x) = 2x^3$$

---

## Q51.

Define geometrical meaning of subnormal.

Projection of normal on x-axis.

---

## SECTION E — Partial Differentiation

---

## Q52.

Find:

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$$

for $z = x^2y + y^3$.

Solution

∂z∂x=2xy\frac{\partial z}{\partial x}=2xy∂x∂z=2xy ∂z∂y=x2+3y2\frac{\partial z}{\partial y}=x^2+3y^2∂y∂z=x2+3y2

---

### Q53.

Find second order partial derivatives of z=x2y3z=x^2y^3z=x2y3.

zx=2xy3z_x=2xy^3zx=2xy3 zxx=2y3z_{xx}=2y^3zxx=2y3 zy=3x2y2z_y=3x^2y^2zy=3x2y2

---

### Q54.

Show mixed derivatives equality.

∂2z∂x∂y=∂2z∂y∂x\frac{\partial^2z}{\partial x\partial y} =\frac{\partial^2z}{\partial y\partial x}∂x∂y∂2z=∂y∂x∂2z

(Clairaut theorem)

---

### SECTION F — Euler's Theorem

---

### Q55.

State Euler's theorem.

If zzz homogeneous degree n:

xzx+yzy=nzx z_x+y z_y=nzxzx+yzy=nz

---

### Q56.

Verify Euler theorem for

z=x2+y2z=x^2+y^2z=x2+y2 zx=2x, zy=2yz_x=2x,\;z_y=2yzx=2x,zy=2y
xzx+yzy=2x2+2y2=2zxz_x+yz_y=2x^2+2y^2=2zxzx+yzy=2x2+2y2=2z

Verified ✓

---

**Q57.**

Verify Euler theorem for $z=x^3y^2$ $z=x^3y^2$ $z=x3y2$.

Degree = 5

LHS:

$x(3x^2y^2)+y(2x^3y)=5x^3y^2=5z$ $x(3x^2y^2)+y(2x^3y)=5x^3y^2=5z$ $x(3x2y2)+y(2x3y)=5x3y2=5z$

---

**SECTION G — Curvature**

---

**Q58.**

Find curvature of $y=x^2$ $y=x^2$ $y=x2$.

Formula:

$K=\frac{|y''|}{(1+y'^2)^{3/2}}$ $K=\frac{|y''|}{(1+y'^2)^{3/2}}$ $K=(1+y'2)3/2|y''|$ $y'=2x,\quad y''=2$ $y'=2x,\quad y''=2$ $y'=2x,y''=2$ $K=\frac{2}{(1+4x^2)^{3/2}}$ $K=\frac{2}{(1+4x^2)^{3/2}}$ $K=(1+4x2)3/22$

---

**Q59.**

Find radius of curvature.

$\rho=\frac{1}{K}=\frac{(1+4x^2)^{3/2}}{2}$ $\rho=\frac{1}{K}=\frac{(1+4x^2)^{3/2}}{2}$ $\rho=K1=2(1+4x2)3/2$

---

**Q60.**

Show curvature of straight line = 0.

Since $y''=0\Rightarrow K=0$ $y''=0\Rightarrow K=0$ $y''=0⇒K=0$.

---

## SECTION H — Asymptotes

---

## Q61.

Find asymptotes of:

$y = \frac{2x^2+3}{x}$

Divide:

$y = 2x + \frac{3}{x}$

As $x \to \infty$

$y = 2x$

Oblique asymptote ✓

---

## Q62.

Find asymptotes of hyperbola

$xy = 1$

As $x \to \infty$, $y \to 0$

Hence axes are asymptotes.

---

## Q63.

Define horizontal & vertical asymptotes.

- $y = a$ horizontal
- $x = b$ vertical

---

## SECTION I — Maxima & Minima (Two Variables)

**Q64.**

Find extrema of:

$f = x^2 + y^2$

Solution

$f_x = 2x = 0 \Rightarrow x = 0$

$f_y = 2y = 0 \Rightarrow y = 0$

Second derivatives:

$D = 4 > 0, \; f_{xx} > 0$

Minimum at (0,0).

---

**Q65.**

Find extrema of $f = x^2 - y^2$.

$D = -4 < 0$

Saddle point.

---

**Q66.**

State second derivative test.

$D = f_{xx}f_{yy} - (f_{xy})^2$

Conditions classify extrema.

---

**Q67.**

Find extrema of $f = xy$.

Critical point (0,0)

$D = -1 < 0$

Saddle point.

---

## Q68.

Explain geometrical meaning of saddle point.

Surface rises one direction and falls another.

---

## Q69.

Optimization application example (Theory).

Used in economics, engineering design, machine learning.

---

## Q70.

Explain steps to find maxima & minima of two variables.

1. Find partial derivatives
2. Equate to zero
3. Find critical points
4. Compute second derivatives
5. Apply determinant test

## SECTION A — Change of Rectangular Axes

---

## Q71.

Explain change of rectangular axes and derive transformation equations when origin is shifted.

Solution

Suppose old origin $= O(0,0)$
New origin $= O'(h,k)$

Let new coordinates be $(X,Y)$.

Then,

$$x=X+h,\qquad y=Y+k$$

These are **translation equations**.

- Axes remain parallel.
- Only origin changes.

$$\boxed{x=X+h,\; y=Y+k}$$

---

**Q72.**

Shift origin to $(2, -3)$ in equation

$$x^2+y^2-4x+6y+9=0$$

Solution

Use:

$$x=X+2,\quad y=Y-3$$

Substitute:

$$(X+2)^2+(Y-3)^2-4(X+2)+6(Y-3)+9=0$$

Expand:

$$X^2+4X+4+Y^2-6Y+9-4X-8+6Y-18+9=0$$

Simplify:

$$X^2+Y^2-4=0$$

Final Answer
$$\boxed{X^2+Y^2=4}$$

**Q73.**

Show shifting origin removes linear terms.

Solution

General equation:

$$Ax^2 + By^2 + Dx + Ey + F = 0$$

Put:

$$x = X + h, \; y = Y + k$$

Choose $h, k$ such that coefficients of X and Y vanish.

Thus linear terms eliminated ✓.

**Q74.**

Find new equation after shifting origin to center of circle:

$$x^2 + y^2 - 6x + 8y + 9 = 0$$

Solution

Center:

$$(3, -4)$$

Use:

$$x = X + 3, \; y = Y - 4$$

Substitute → simplify:

$$X^2 + Y^2 = 16$$

**Q75.**

Explain geometrical meaning of translation of axes.

Solution (Theory)

- Figure unchanged.
- Only reference frame moves.
- Distances and angles preserved.

---

## SECTION B — Rotation of Axes

---

### Q76.

Derive rotation transformation formulas.

Solution

If axes rotated by angle θ:

$x = X\cos\theta - Y\sin\theta$
$y = X\sin\theta + Y\cos\theta$

Derived using trigonometric projection.

---

### Q77.

Show rotation preserves distance.

Solution

Distance:

$$x^2 + y^2$$

Substitute rotation formulas → simplifies to:

$$X^2 + Y^2$$

Hence invariant ✓.

## Q78.

Find angle of rotation removing xy-term from:

$3x^2+4xy+2y^2=0$

Solution

Formula:

$$\tan 2\theta = \frac{B}{A-C}$$

Here:
A=3, B=4, C=2

$$\tan 2\theta = \frac{4}{1} = 4$$

$$2\theta = \tan^{-1}(4)$$

$$\boxed{2\theta = \tan^{-1}(4)}$$

## Q79.

Why rotation removes xy-term?

Solution

Cross-product term arises due to tilted axes.
Proper rotation aligns axes with principal directions.

## Q80.

Transform equation after rotation through θ.

Solution

Substitute rotation relations into original equation and simplify.

**SECTION C — Combined Transformation**

## Q81.

Explain combined rotation and translation.

### Solution

Steps:

1. Remove linear terms (shift origin)
2. Remove xy-term (rotate axes)

Result → canonical form.

---

## Q82.

Reduce equation:

$$x^2+y^2+4x-6y+9=0$$

to standard form.

### Solution

Complete squares:

$$(x+2)^2+(y-3)^2=4$$

Circle with center (-2,3).

---

## Q83.

Find transformation reducing equation into principal axes.

### Solution

Use:

$$\tan 2\theta=\frac{B}{A-C}$$

Then substitute rotation equations.

---

## Q84.

Explain invariant quantities under transformation.

Remain unchanged:

- Degree
- Nature of conic
- Discriminant B2−4ACB^2-4ACB2−4AC

---

## Q85.

Show degree of equation remains unchanged after transformation.

Transformation equations are linear $\rightarrow$ highest power preserved.

---

## SECTION D — General Equation of Second Degree

---

General form:

Ax2+Bxy+Cy2+Dx+Ey+F=0Ax^2+Bxy+Cy^2+Dx+Ey+F=0Ax2+Bxy+Cy2+Dx+Ey+F=0

---

## Q86.

Classify conic using discriminant.

Δ=B2−4AC\Delta=B^2-4ACΔ=B2−4AC

- =0=0=0 → Parabola
- <0<0<0 → Ellipse
- >0>0>0 → Hyperbola

---

## Q87.

Determine nature of:

$2x^2+3xy+y^2=0$  $B^2-4AC=9-8=1>0$

Hyperbola.

---

## Q88.

Find nature of:

$x^2+4y^2=1$

Since A,C same sign → ellipse.

---

## Q89.

Show circle is special case of ellipse.

Solution

Ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

If $a=b$:

$$x^2+y^2=a^2$$

Circle ✓.

---

**Q90.**

Find center of general conic.

Solution

Solve:

$$\frac{\partial}{\partial x}=0,\quad \frac{\partial}{\partial y}=0$$

(simultaneous linear equations).

---

**SECTION E — Reduction to Canonical Form**

---

**Q91.**

Reduce:

$$x^2-y^2+4x-6y=0$$

to standard form.

Solution

Complete squares:

$$(x+2)^2-(y+3)^2=5$$

Hyperbola.

---

**Q92.**

Explain principal axes.

Solution

Axes obtained after removing xy-term; symmetry axes of conic.

---

**Q93.**

Show transformation preserves conic type.

Because discriminant invariant.

---

**Q94.**

Explain geometrical interpretation of second-degree equation.

Solution

Represents:

- ellipse
- parabola
- hyperbola
- pair of lines

depending on coefficients.

---

**Q95.**

Outline steps to reduce general second-degree equation.

Solution (Algorithm)

1. Write general equation.
2. Shift origin → remove linear terms.
3. Rotate axes → remove xy-term.
4. Obtain canonical form.
5. Identify conic.

**UNIT–II: Coordinate Geometry (2D)**

**Conditions for General Equation of Second Degree to Represent Parabola, Ellipse and Hyperbola**

General Equation:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

These questions are designed for **B.Sc / BCA / Engineering Semester Examination (15 Marks Level)**.

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## ✅ LONG QUESTIONS WITH SOLUTIONS (96–120)

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### SECTION A — Fundamental Theory & Conditions

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### Q96.

State the condition under which the general second-degree equation represents a parabola.

Solution

Given:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Discriminant:

$$\Delta = B^2 - 4AC$$

Condition:

$$\boxed{B^2 - 4AC = 0}$$

Hence equation represents a **parabola**.

---

### Q97.

State condition for ellipse.

Solution

If

$$B^2 - 4AC < 0$$

then equation represents an **ellipse**.

Special case:
If $A=C$ and $B=0 \rightarrow$ circle.

---

**Q98.**

State condition for hyperbola.

If

$$\boxed{B^2-4AC>0}$$

the equation represents a **hyperbola**.

---

**Q99.**

Explain geometrical meaning of discriminant $B^2-4AC$.

It determines curvature orientation:

- Negative $\rightarrow$ closed curve (ellipse)
- Zero $\rightarrow$ transition case (parabola)
- Positive $\rightarrow$ open curve (hyperbola)

---

**Q100.**

Show circle is special case of ellipse.

Ellipse:

$$Ax^2+Cy^2+Dx+Ey+F=0$$

If:

$A=C,\; B=0$

then:

$$x^2+y^2+Dx+Ey+F=0$$

Circle ✓.

---

## SECTION B — Identification Problems

---

### Q101.

Identify nature of:

$$3x^2+2xy+y^2=0$$

Solution

$A=3,\; B=2,\; C=1$

$$B^2-4AC=4-12=-8<0$$

Hence **ellipse**.

---

### Q102.

Identify conic:

$$x^2+4xy+4y^2=0 \qquad B^2-4AC=16-16=0$$

Parabola.

---

### Q103.

Identify nature:

$2x^2 - 3xy - y^2 = 0$ $9 + 8 = 17 > 0$

Hyperbola.

---

## Q104.

Show equation represents circle:

$x^2 + y^2 - 6x + 4y + 9 = 0$

Solution

A=C=1, B=0 → circle.

---

## Q105.

Determine nature:

$4x^2 + 9y^2 = 36$

Since same sign and B=0 → ellipse.

---

## SECTION C — Degenerate Cases

---

## Q106.

Explain degenerate conic.

Solution

When conic reduces to:

- pair of lines
- coincident lines
- point

Condition:
Determinant

$$\begin{vmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{vmatrix}=0$$

---

## Q107.

Show equation represents pair of lines:

$$x^2-y^2=0$$

Factor:

$$(x-y)(x+y)=0$$

Pair of straight lines.

---

## Q108.

Explain condition for real ellipse.

Solution

Need:

$$A,C>0 \quad \text{and} \quad B^2-4AC<0$$

---

## Q109.

Explain imaginary ellipse.

Solution

If ellipse equation gives negative RHS after reduction → no real points.

---

## Q110.

Condition for rectangular hyperbola.

Solution

If:

$$A + C = 0$$

hyperbola is rectangular.

---

## SECTION D — Reduction & Classification

---

### Q111.

Reduce equation and identify conic:

$$x^2 + y^2 + 4x - 2y - 4 = 0$$

Solution

Complete squares:

$$(x+2)^2 + (y-1)^2 = 9$$

Circle.

---

### Q112.

Identify:

$$9x^2 + 16y^2 = 144$$

Divide:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Ellipse.

---

## Q113.

Identify:

$y^2 = 8x$

Compare with $y^2 = 4ax$

Parabola.

---

## Q114.

Identify:

$x^2 - 9y^2 = 1$

Opposite signs → hyperbola.

---

## Q115.

Explain why parabola has single squared variable after rotation.

Solution

Rotation aligns axis of symmetry removing second squared term.

---

**SECTION E — Conceptual & Proof Based**

---

## Q116.

Prove discriminant invariant under rotation.

Solution

Rotation substitutes linear combinations of x,y only → coefficients change but expression $B^2 - 4AC$ remains constant.

## Q117.

Explain why ellipse is closed curve.

Both squared terms same sign → bounded distance.

## Q118.

Explain why hyperbola has two branches.

Opposite signs allow solutions in two separate regions.

## Q119.

Derive condition for parabola from quadratic form.

Quadratic form matrix determinant zero $\Rightarrow$ one eigenvalue zero $\Rightarrow$ parabola.

## Q120.

Summarize steps to classify any second-degree equation.

1. Write coefficients A,B,C.
2. Compute $B2-4ACB^2-4ACB2-4AC$.
3. Apply condition:

| Value | Conic |
| --- | --- |
| =0 | Parabola |
| <0 | Ellipse |
| >0 | Hyperbola |

4. Check degeneracy.
5. Reduce to standard form.

## LONG QUESTIONS WITH SOLUTIONS (121–150)

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### SECTION A — Tangent & Normal (Using Calculus)

---

### Q121.

Find the equations of tangent and normal to the curve

$y=x^2+2x+1$

at $x=1$.

**Solution**

$$\frac{dy}{dx}=2x+2$$

At $x=1$:

$m=4$

Point:

$$y=(1+1)^2=4 \Rightarrow (1,4)$$

**Tangent**

$$y-4=4(x-1) \quad \boxed{y=4x}$$

**Normal slope**

$$=-\frac{1}{4}$$

Normal:

$y-4=-14(x-1)$ $y-4=-\frac{1}{4}(x-1)$ $y-4=-41(x-1)$

---

## Q122.

Find tangent and normal to $y=x^3-3x$ $y=x^3-3x$ $y=x^3-3x$ at $x=1$ $x=1$ $x=1$.

$\frac{dy}{dx}=3x^2-3$ $\frac{dy}{dx}=3x^2-3$ $\frac{dy}{dx}=3x^2-3$

At $x=1$ $x=1$ $x=1$: slope = 0

Point $(1, -2)$

Tangent:

$y=-2$ $y=-2$ $y=-2$

Normal:

$x=1$ $x=1$ $x=1$

---

## Q123.

Find tangent and normal to $y=x$ $y=\sqrt{x}$ $y=x$ at $x=4$ $x=4$ $x=4$.

$\frac{dy}{dx}=12x$ $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$ $\frac{dy}{dx}=2x1$

At $x=4$ $x=4$ $x=4$:

$m=14$ $m=\frac{1}{4}$ $m=41$

Point $(4,2)$

Tangent:

$y-2=14(x-4)$ $y-2=\frac{1}{4}(x-4)$ $y-2=41(x-4)$

Normal slope = $-4$.

## Q124.

Find tangent to curve $x^2+y^2=13$ at (2,3).

Differentiate implicitly:

$$2x+2y\frac{dy}{dx}=0 \quad \frac{dy}{dx}=-\frac{x}{y}$$

At (2,3):

$$m=-\frac{2}{3}$$

Tangent:

$$y-3=-\frac{2}{3}(x-2)$$

## Q125.

Prove tangent to circle is perpendicular to radius using calculus.

Slope of radius:

$$m_r=\frac{y}{x}$$

Slope tangent:

$$-\frac{x}{y} \quad m_r m_t=-1$$

Hence perpendicular.

## Q126.

Find tangent and normal to $y=\ln x$ at $x=1$.

$$y'=1/x$$

Slope $=1$

Point $(1,0)$

Tangent:

$y = x - 1$

Normal:

$y = -x + 1$

---

## Q127.

Find tangent to $y = e^x$ at $x = 0$.

$y' = e^x$

Slope $= 1$

Point $(0, 1)$

Tangent:

$y = x + 1$

---

## Q128.

Find normal to $y = x^2$ at $x = 2$.

$y' = 4$

Normal slope:

$-\frac{1}{4}$

Point $(2, 4)$

Normal:

$y - 4 = -\frac{1}{4}(x - 2)$

---

## Q129.

Find tangent where curve $y=x^3$ is parallel to line $y=12x+1$.

Slope required $=12$.

$$3x^2=12 \Rightarrow x=2$$

Point $(2,8)$

Tangent:

$$y-8=12(x-2)$$

---

## Q130.

Find tangent where slope is zero for $y=x^3-6x^2+9x$.

$$y'=3x^2-12x+9$$

Set zero:

$$x=1,3$$

Horizontal tangents at those points.

---

## SECTION B — Tangent & Normal (Parametric Form)

---

## Q131.

Find tangent to:

$$x=t^2,\ y=t^3$$

at $t=2$.

$$\frac{dy}{dx}=\frac{3t^2}{2t}=\frac{3t}{2}$$

Slope$=3$

Point $(4,8)$

Tangent:

$$y - 8 = 3(x - 4)$$

---

### Q132.

Find normal to Q131.

Normal slope:

$$-\frac{1}{3}$$

$$y - 8 = -\frac{1}{3}(x - 4)$$

---

### Q133.

Find tangent to parametric curve

$$x = a\cos t,\ y = a\sin t$$

$$\frac{dy}{dx} = -\cot t$$

---

### Q134.

Show tangent to circle in parametric form.

Point:

$$(a\cos t, a\sin t)$$

Tangent:

$$x\cos t + y\sin t = a$$

---

### Q135.

Explain geometrical meaning of parametric differentiation.

Slope obtained from motion along parameter.

---

## SECTION C — Polar Equations

---

### Q136.

Define polar coordinates and derive relations.

$$x = r\cos\theta, \quad y = r\sin\theta$$

---

### Q137.

Convert $r = 2\cos\theta$ into Cartesian form.

$$r^2 = 2r\cos\theta \implies x^2 + y^2 = 2x$$

Circle.

---

### Q138.

Convert $r = a$ into Cartesian.

$$x^2 + y^2 = a^2$$

Circle centered at origin.

---

### Q139.

Find slope of polar curve.

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

---

### Q140.

Find tangent to $r = a(1 + \cos\theta)$.

Differentiate r and substitute into slope formula.

## SECTION D — Polar Tangent & Normal

### Q141.

Angle between radius vector and tangent.

$\tan\phi=\frac{r}{dr/d\theta}$

### Q142.

Find tangent at pole for $r=a\theta$.

At $r=0\Rightarrow\theta=0$

Tangent = initial line.

### Q143.

Find pedal equation relation.

Pedal length $p$:

$p=r\sin\phi$

### Q144.

Show circle symmetric about polar axis.

Replace $\theta\to-\theta$.

Equation unchanged.

### Q145.

Test symmetry about pole.

Replace r→−rr\to -rr→−r.

## SECTION E — Applications & Theory

### Q146.

Explain advantages of polar coordinates.

- symmetry handling
- circular motion
- spiral curves

### Q147.

Compare Cartesian and polar tangents.

Cartesian → dy/dx
Polar → dr/dθ relation.

### Q148.

Show cardioid equation represents heart-shaped curve.

r=a(1+cos θ)r=a(1+\cos\theta)r=a(1+cosθ)

Symmetry and cusp at pole.

### Q149.

Find tangent at θ = π/2 for r=2sin θr=2\sin\thetar=2sinθ.

Substitute into slope formula → vertical tangent.

**Q150.**

Outline steps to find tangent of polar curve.

1. Write r=f($\theta$)
2. Compute dr/d$\theta$
3. Use slope formula
4. Find point coordinates
5. Apply point–slope form.