MCQs (1–50) — UNIT I: INTEGRAL CALCULUS

Section A: Basic Integration Concepts

1. Integration is the inverse process of:
A) Multiplication
B) Differentiation
C) Addition
D) Division
Answer: B

---

2. The integral of xnx^nxn is:
A) xn+1n\frac{x^{n+1}}{n}nxn+1
B) xn+1n+1\frac{x^{n+1}}{n+1}n+1xn+1
C) nxn−1nx^{n-1}nxn−1
D) xnx^nxn
Answer: B

---

3. ∫1 dx=\int 1\,dx =∫1dx=
A) x
B) 1
C) 0
D) ln x
Answer: A

---

4. Constant of integration is written as:
A) K
B) C
C) A
D) D
Answer: B

---

5. ∫exdx=\int e^x dx=∫exdx=
A) exe^xex
B) xexxe^xxex
C) ln⁡x\ln xlnx

D) 1/ex1/e^x1/ex
Answer: A

---

6. ∫1xdx=\int \frac{1}{x}dx=∫x1dx=
A) x
B) ln|x|\ln|x|ln|x|
C) x2x^2x2
D) exe^xex
Answer: B

---

7. ∫cosx dx=\int \cos x\,dx=∫cosxdx=
A) sin x
B) −sin x
C) cos x
D) tan x
Answer: A

---

8. ∫sinx dx=\int \sin x\,dx=∫sinxdx=
A) cos x
B) −cos x
C) tan x
D) sec x
Answer: B

---

9. Integration represents geometrically:
A) slope
B) area under curve
C) distance only
D) angle
Answer: B

---

10. ∫0 dx=\int 0\,dx=∫0dx=
A) 1
B) 0
C) C

D) x
Answer: C

Section B: Integration by Substitution

---

11. Substitution method is also called:
A) change of variable
B) factorization
C) expansion
D) division
Answer: A

---

12. Suitable substitution for ∫2xcos(x2)dx\int 2x\cos(x^2)dx∫2xcos(x2)dx is:
A) x
B) x2x^2x2
C) cos x
D) sin x
Answer: B

---

13. ∫dx1−x2\int \frac{dx}{\sqrt{1-x^2}}∫1−x2dx equals:
A) $\sin^{-1}x$
B) $\cos^{-1}x$
C) $\tan^{-1}x$
D) sec x
Answer: A

---

14. ∫dx1+x2=\int \frac{dx}{1+x^2}=∫1+x2dx=
A) ln x
B) $\tan^{-1}x$
C) $\sin^{-1}x$
D) cos x
Answer: B

---

15. Best substitution for a2−x2\sqrt{a^2-x^2}a2−x2:
A) x=asin⁡θx=a\sin\thetax=asinθ
B) x=atan⁡θx=a\tan\thetax=atanθ
C) x=asec⁡θx=a\sec\thetax=asecθ
D) x=a
Answer: A

---

16. Substitution simplifies integrand by:
A) increasing degree
B) reducing complexity
C) removing constants
D) differentiation
Answer: B

---

17. ∫e3xdx=\int e^{3x}dx=∫e3xdx=
A) e3xe^{3x}e3x
B) e3x3\frac{e^{3x}}{3}3e3x
C) 3ex3e^{x}3ex
D) ln x
Answer: B

Section C: Integration by Parts

---

18. Formula of integration by parts is:
A) uv
B) ∫udv=uv−∫vdu∫udv=uv-∫vdu∫udv=uv−∫vdu
C) u+v
D) u−v
Answer: B

---

19. LIATE rule helps choose:
A) substitution
B) u-function
C) limits
D) constants
Answer: B

---

20. In LIATE, L stands for:
A) Linear
B) Logarithmic
C) Limit
D) Length
Answer: B

---

21. ∫xexdx=\int x e^x dx=∫xexdx=
A) xex−ex+Cxe^x-e^x+Cxex−ex+C
B) xex+Cxe^x+Cxex+C
C) ex/xe^x/xex/x
D) ln x
Answer: A

---

22. Best choice of u in ∫xln⁡xdx\int x\ln x dx∫xlnxdx:
A) x
B) ln x
C) dx
D) 1
Answer: B

---

23. Integration by parts is derived from:
A) quotient rule
B) product rule
C) chain rule
D) limit rule
Answer: B

---

24. ∫ln⁡xdx=\int \ln x dx=∫lnxdx=
A) xln⁡x−x+Cx\ln x-x+Cxlnx−x+C
B) ln x
C) x2x^2x2
D) 1/x
Answer: A

---

25. Repeated integration by parts is used for:
A) algebraic × exponential
B) constants
C) polynomials only
D) fractions
Answer: A

Section D: Partial Fractions

---

26. Partial fractions apply to:
A) irrational functions
B) rational functions
C) trigonometric only
D) exponential only
Answer: B

---

27. Rational function means ratio of:
A) polynomials
B) trig functions
C) logs
D) exponentials
Answer: A

---

28. Denominator must be:
A) factorable
B) constant
C) zero
D) irrational
Answer: A

---

29. 1(x−1)(x−2)\frac{1}{(x-1)(x-2)}(x−1)(x−2)1 decomposes into:
A) A+B
B) Ax−1+Bx−2\frac{A}{x-1}+\frac{B}{x-2}x−1A+x−2B
C) polynomial
D) constant
Answer: B

30. Partial fractions simplify integration by:
A) multiplication
B) splitting terms
C) differentiation
D) expansion
Answer: B

---

31. Repeated linear factor uses form:
A) A/x
B) A/x+B/x2A/x+B/x^2A/x+B/x2
C) Ax
D) constant
Answer: B

---

32. Quadratic irreducible factor form:
A) A/x
B) Ax+B
C) Ax^2
D) constant
Answer: B

Section E: Integration of Rational Functions

---

33. Integral of 1x−a\frac{1}{x-a}x−a1 is:
A) ln|x−a|
B) x−a
C) e^x
D) constant
Answer: A

---

34. Rational integrals usually solved using:
A) substitution
B) partial fractions
C) parts

D) limits
Answer: B

---

35. Degree numerator must be:
A) ≥ denominator
B) < denominator
C) equal always
D) zero
Answer: B

---

36. If degree higher, first perform:
A) differentiation
B) division
C) substitution
D) limits
Answer: B

Section F: Integration of Irrational Functions

---

37. Integral involving roots is called:
A) rational integral
B) irrational integral
C) definite integral
D) improper integral
Answer: B

---

38. x2+a2\sqrt{x^2+a^2}x2+a2 substitution:
A) x=atanθx=a\tan\thetax=atanθ
B) x=asinθx=a\sin\thetax=asinθ
C) x=acosθx=a\cos\thetax=acosθ
D) x=a
Answer: A

---

39. x2−a2\sqrt{x^2-a^2}x2−a2 substitution:
A) x=asecθx=a\sec\thetax=asecθ
B) x=asinθx=a\sin\thetax=asinθ
C) x=atanθx=a\tan\thetax=atanθ
D) x=a
Answer: A

---

40. Trigonometric substitution converts integral into:
A) algebraic form
B) trigonometric form
C) logarithmic form
D) constant
Answer: B

Section G: Integration by Transformation

---

41. Transformation means:
A) changing variable form
B) differentiation
C) multiplication
D) limits
Answer: A

---

42. Integral invariant under substitution if:
A) limits adjusted
B) constants removed
C) variables same
D) derivative zero
Answer: A

---

43. Transformation simplifies integrand by:
A) symmetry
B) complexity
C) constants
D) limits
Answer: A

---

44. Even function integral over symmetric limits is:
A) zero
B) twice integral
C) undefined
D) constant
Answer: B

---

45. Odd function integral over symmetric limits equals:
A) 1
B) zero
C) infinity
D) constant
Answer: B

Section H: Conceptual Applications

---

46. Integration used to find:
A) area
B) volume
C) displacement
D) all
Answer: D

---

47. Anti-derivative means:
A) derivative
B) integral
C) limit
D) constant
Answer: B

---

48. Definite integral gives:
A) function
B) number
C) variable

D) slope
Answer: B

---

49. Integration constant disappears in:
A) indefinite integral
B) definite integral
C) substitution
D) parts
Answer: B

---

50. Main aim of integration techniques is:
A) simplification
B) complication
C) expansion
D) differentiation
Answer: A

MCQs (51–100)

Section A: Evaluation of Definite Integrals

---

51. A definite integral represents:
A) Function
B) Number
C) Variable
D) Constant only
Answer: B

---

52. Fundamental theorem of calculus connects:
A) Algebra & Geometry
B) Differentiation & Integration
C) Trigonometry & Algebra
D) Limits & Matrices
Answer: B

53.

∫aaf(x) dx=\int_a^a f(x)\,dx=∫aaf(x)dx=

A) 1
B) a
C) 0
D) f(a)
Answer: C

54.

∫abf(x) dx=−∫baf(x) dx\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx∫abf(x)dx=−∫baf(x)dx

This shows:
A) symmetry property
B) limit property
C) reversal of limits
D) substitution rule
Answer: C

---

55. If f(x)f(x)f(x) is even, then:

∫−aaf(x) dx=\int_{-a}^{a} f(x)\,dx=∫−aaf(x)dx=

A) 0
B) twice integral from 0 to a
C) infinity
D) undefined
Answer: B

---

56. If f(x)f(x)f(x) is odd:

∫−aaf(x) dx=\int_{-a}^{a} f(x)\,dx=∫−aaf(x)dx=

A) 1
B) a
C) 0
D) −1
Answer: C

57.

∫0af(x) dx=∫0af(a−x) dx\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx∫0af(x)dx=∫0af(a−x)dx

is called:
A) symmetry property
B) parts rule
C) substitution rule
D) reduction rule
Answer: A

---

58. Value of

∫0πsin x dx\int_0^\pi \sin x\,dx∫0πsinxdx

is:
A) 0
B) 1
C) 2
D) π
Answer: C

---

59. Definite integrals are independent of:
A) limits
B) path of integration
C) constant of integration
D) function
Answer: C

60.

∫01x dx=\int_0^1 x\,dx=∫01xdx=

A) 1
B) 1/2
C) 2
D) 0
Answer: B

Section B: Reduction Formulae

---

61. Reduction formula expresses:
A) derivative
B) higher integral in terms of lower integral
C) limit
D) matrix
Answer: B

---

62. Reduction formula is useful for:
A) repeated integrals
B) algebra
C) matrices
D) limits
Answer: A

---

63. Reduction formula for $\int \sin^n x \, dx$ reduces power by:
A) 2
B) 1
C) n
D) 0
Answer: A

---

64. Reduction formula simplifies evaluation of:
A) definite integrals
B) matrices
C) limits
D) derivatives
Answer: A

---

65. Reduction relation generally contains:
A) recurrence relation
B) logarithm
C) constant

D) matrix
Answer: A

---

66. Reduction formula helps avoid:
A) substitution
B) repeated integration
C) differentiation
D) limits
Answer: B

---

67. Reduction formula mainly applied to:
A) powers of trig functions
B) constants
C) polynomials only
D) exponentials only
Answer: A

Section C: Curve Tracing

---

68. Curve tracing studies:
A) behavior of curve
B) integration only
C) differentiation only
D) matrices
Answer: A

---

69. Symmetry about y-axis checked by replacing:
A) x by −x
B) y by −y
C) both
D) none
Answer: A

---

70. Symmetry about origin checked by replacing:
A) $x \to -x$, $y \to -y$
B) $x \to y$
C) $y \to x$
D) none
Answer: A

---

71. Intercepts found by putting:
A) derivatives zero
B) variables zero
C) limits zero
D) constants zero
Answer: B

---

72. Asymptotes help describe:
A) distant behavior of curve
B) slope only
C) area
D) volume
Answer: A

---

73. Points where derivative zero are:
A) intercepts
B) stationary points
C) asymptotes
D) normals
Answer: B

---

74. Curve tracing uses:
A) derivatives
B) limits
C) symmetry
D) all
Answer: D

Section D: Length of Curve

---

75. Length of curve formula:

L=∫1+(dydx)2dxL=\int \sqrt{1+\left(\frac{dy}{dx}\right)^2}dxL=∫1+(dxdy)2dx

represents:
A) arc length
B) area
C) volume
D) slope
Answer: A

---

76. Arc length depends on:
A) derivative
B) integral
C) both
D) constant
Answer: C

---

77. Arc length formula derived from:
A) Pythagoras theorem
B) algebra
C) limits only
D) matrices
Answer: A

---

78. Parametric arc length uses:
A) dx/dt and dy/dt
B) only x
C) only y
D) constants
Answer: A

Section E: Area Under Curve

---

79. Area between curve and x-axis equals:
A) derivative
B) definite integral
C) limit
D) constant
Answer: B

---

80. Area between two curves:

∫(y1−y2)dx\int (y_1-y_2)dx∫(y1−y2)dx

requires:
A) upper − lower curve
B) sum
C) product
D) division
Answer: A

---

81. Negative area indicates:
A) curve below axis
B) error
C) infinity
D) zero
Answer: A

---

82. Area bounded by curve requires:
A) intersection points
B) derivatives
C) limits
D) constants
Answer: A

Section F: Volume of Solids of Revolution

---

83. Volume when rotated about x-axis:

V=π∫y2dxV=\pi\int y^2 dxV=π∫y2dx

Method called:
A) disk method
B) substitution
C) parts
D) reduction
Answer: A

---

84. Volume about y-axis uses:
A) x2x^2x2
B) y2y^2y2
C) xy
D) constant
Answer: A

---

85. Washer method used when:
A) hollow region
B) solid region
C) constant region
D) none
Answer: A

---

86. Volume represents:
A) 2D measure
B) 3D measure
C) slope
D) angle
Answer: B

---

87. Units of volume are:
A) square units
B) cubic units
C) linear units
D) radians
Answer: B

Section G: Surface Area of Revolution

---

88. Surface area formula:

S=2π∫y1+(y′)2dxS=2\pi\int y\sqrt{1+(y')^2}dxS=2π∫y1+(y′)2dx

represents:
A) curved surface
B) area
C) volume
D) slope
Answer: A

---

89. Surface area depends on:
A) radius and arc length
B) slope only
C) constant
D) limits
Answer: A

---

90. Rotating curve generates:
A) surface
B) line
C) point
D) angle
Answer: A

Section H: Conceptual Applications

---

91. Solid of revolution formed by rotating:
A) line
B) area
C) point
D) axis
Answer: B

92. Integration used in volume calculation comes from:
A) summation of slices
B) differentiation
C) algebra
D) matrices
Answer: A

93. Curve tracing helps in:
A) sketching graphs
B) solving equations
C) limits only
D) matrices
Answer: A

94. Maximum area problems belong to:
A) optimization
B) algebra
C) geometry only
D) statistics
Answer: A

95. Length of curve increases with:
A) slope variation
B) constants
C) limits only
D) matrices
Answer: A

96. Definite integral evaluates:
A) accumulated quantity
B) slope
C) constant
D) variable
Answer: A

97. Area under curve always computed using:
A) indefinite integral
B) definite integral
C) derivative
D) limits only
Answer: B

---

98. Reduction formula useful for large powers because:
A) simplifies computation
B) increases complexity
C) removes limits
D) removes constants
Answer: A

---

99. Curve tracing identifies:
A) maxima & minima
B) asymptotes
C) symmetry
D) all
Answer: D

---

100. Integral calculus applications mainly deal with:
A) accumulation processes
B) subtraction
C) multiplication
D) division
Answer: A

Section A: Derivative of Scalar Product

---

101. If **A** and **B** are vector functions of t, then

ddt(A·B)=\frac{d}{dt}(A\cdot B)=dtd(A·B)=

A) A·B
B) A′·B + A·B′

C) $A' \cdot B'$
D) A+B
Answer: B

---

102. Scalar product of two vectors gives:
A) vector
B) scalar
C) matrix
D) tensor
Answer: B

---

103. If $A \cdot B$ = constant, then:
A) derivative zero
B) vectors parallel
C) vectors perpendicular always
D) magnitude zero
Answer: A

---

104. Derivative of magnitude squared:

ddt(A·A)=\frac{d}{dt}(A\cdot A)=dtd(A·A)=

A) $2A \cdot A'$
B) A'
C) A^2
D) 0
Answer: A

---

105. Scalar product measures:
A) angle relation
B) area
C) volume
D) distance only
Answer: A

Section B: Derivative of Vector Product

106. Derivative of vector product:

ddt(A×B)=\frac{d}{dt}(A\times B)=dtd(A×B)=

A) A×B
B) A′×B + A×B′
C) A′×B′
D) A+B
Answer: B

---

107. Vector product gives:
A) scalar
B) vector
C) constant
D) number
Answer: B

---

108. Direction of cross product determined by:
A) right-hand rule
B) left-hand rule
C) triangle rule
D) cosine rule
Answer: A

---

109. If A×B = 0, vectors are:
A) perpendicular
B) parallel
C) unequal
D) unit vectors
Answer: B

---

110. Magnitude of A×B equals:
A) AB cosθ
B) AB sinθ
C) A+B

D) A−B
Answer: B

Section C: Gradient Operator

---

111. Gradient operator is denoted by:
A) ∇
B) Δ
C) ∂
D) ∫
Answer: A

---

112. Gradient operates on:
A) scalar function
B) vector function
C) matrix
D) constant
Answer: A

---

113. Gradient of scalar function gives:
A) scalar
B) vector
C) constant
D) zero
Answer: B

---

114. Gradient represents direction of:
A) minimum increase
B) maximum increase
C) zero change
D) constant value
Answer: B

---

115. Gradient in Cartesian coordinates is:
A) i∂/∂x + j∂/∂y + k∂/∂z
B) x+y+z
C) dx+dy+dz
D) constant
Answer: A

Section D: Divergence

---

116. Divergence operator is written as:
A) $\nabla \cdot A$
B) $\nabla \times A$
C) $A \times B$
D) ∂A
Answer: A

---

117. Divergence of vector field gives:
A) scalar
B) vector
C) matrix
D) tensor
Answer: A

---

118. Divergence measures:
A) rotation
B) spreading of field
C) length
D) area
Answer: B

---

119. Divergence of constant vector is:
A) 1
B) 0
C) infinite
D) undefined
Answer: B

---

120. Positive divergence indicates:
A) source
B) sink
C) rotation
D) equilibrium
Answer: A

Section E: Curl

---

121. Curl operator is written as:
A) ∇·A
B) ∇×A
C) A·B
D) ∇A
Answer: B

---

122. Curl gives:
A) scalar
B) vector
C) constant
D) matrix
Answer: B

---

123. Curl represents:
A) divergence
B) rotation of field
C) magnitude
D) area
Answer: B

---

124. Curl of gradient is:
A) 1
B) vector
C) zero

D) constant
Answer: C

---

125. If curl A = 0, field is:
A) rotational
B) irrotational
C) divergent
D) constant
Answer: B

Section F: Second Order Vector Operators

---

126. Laplacian operator is:
A) ∇^2
B) ∇
C) ∂
D) Δx
Answer: A

---

127. Laplacian of scalar field gives:
A) scalar
B) vector
C) matrix
D) constant
Answer: A

---

128. Laplacian equals:
A) divergence of gradient
B) curl of gradient
C) gradient of curl
D) vector product
Answer: A

---

129. ∇·(∇×A) equals:
A) 1
B) A
C) 0
D) ∞
Answer: C

---

130. ∇×(∇φ) equals:
A) φ
B) vector
C) 0
D) constant
Answer: C

Section G: Cartesian Coordinate System

---

131. Cartesian coordinates consist of:
A) r,θ
B) x,y,z
C) r,z
D) θ,φ
Answer: B

---

132. Unit vectors in Cartesian system are:
A) r̂, θ̂
B) i, j, k
C) a, b, c
D) p, q, r
Answer: B

---

133. Partial derivative ∂/∂x measures change along:
A) y-axis
B) z-axis
C) x-axis
D) origin
Answer: C

---

134. Gradient components depend on:
A) partial derivatives
B) integrals
C) constants
D) limits
Answer: A

---

135. Vector operators act component-wise in:
A) Cartesian coordinates
B) polar only
C) spherical only
D) cylindrical only
Answer: A

Section H: De Moivre's Theorem

---

136. De Moivre's theorem states:

(cos⁡θ+isin⁡θ)n=(\cos\theta+i\sin\theta)^n=(cosθ+isinθ)n=

A) cos nθ + i sin nθ
B) sin nθ
C) cos θ
D) tan θ
Answer: A

---

137. De Moivre's theorem applies to:
A) real numbers
B) complex numbers
C) matrices
D) vectors
Answer: B

---

138. Used to find:
A) roots of complex numbers
B) integrals only
C) derivatives only
D) limits
Answer: A

---

139. If n=2:

(cosθ+isinθ)2=(\cos\theta+i\sin\theta)^2=(cosθ+isinθ)2=

A) cos2θ+i sin2θ
B) cosθ
C) sinθ
D) tanθ
Answer: A

---

140. Complex number in polar form is:
A) r(cosθ+i sinθ)
B) x+y
C) a+bi only
D) matrix
Answer: A

Section I: Applications of De Moivre

---

141. Cube roots of unity obtained using:
A) integration
B) De Moivre theorem
C) limits
D) differentiation
Answer: B

---

142. Argument of complex number represents:
A) magnitude
B) angle

C) length
D) area
Answer: B

---

143. Modulus represents:
A) direction
B) magnitude
C) angle
D) derivative
Answer: B

---

144. De Moivre helps expand:
A) trigonometric powers
B) matrices
C) vectors
D) limits
Answer: A

---

145. Used to derive identities of:
A) trigonometry
B) algebra
C) calculus only
D) geometry only
Answer: A

Section J: Conceptual Applications

---

146. Gradient always points toward:
A) minimum value
B) maximum increase
C) origin
D) constant value
Answer: B

---

147. Divergence theorem relates:
A) surface & volume integrals
B) limits
C) algebra
D) matrices
Answer: A

---

148. Curl measures local:
A) expansion
B) rotation
C) displacement
D) magnitude
Answer: B

---

149. Vector differential operators belong to:
A) vector calculus
B) algebra
C) statistics
D) arithmetic
Answer: A

---

150. Vector calculus mainly studies:
A) scalar quantities
B) fields and motion in space
C) numbers only
D) constants
Answer: B

MCQs (151–200)

Section A: Scalars and Vectors

---

151. A scalar quantity has:
A) magnitude only

B) direction only
C) both magnitude and direction
D) neither
Answer: A

---

152. A vector quantity has:
A) magnitude only
B) direction only
C) magnitude and direction
D) constant value
Answer: C

---

153. Example of scalar quantity is:
A) velocity
B) force
C) temperature
D) displacement
Answer: C

---

154. Example of vector quantity is:
A) mass
B) speed
C) acceleration
D) energy
Answer: C

---

155. A vector is represented by:
A) bold letter or arrow
B) number only
C) constant
D) matrix
Answer: A

---

156. Unit vector has magnitude:
A) 0
B) 1

C) 2
D) infinity
Answer: B

---

157. Position vector of point (x, y, z) is:
A) xi + yj + zk
B) x+y+z
C) xyz
D) i+j+k
Answer: A

---

158. Magnitude of vector ai+bj+ckai+bj+ckai+bj+ck equals:
A) a+b+ca+b+ca+b+c
B) a2+b2+c2\sqrt{a^2+b^2+c^2}a2+b2+c2
C) abc
D) a^2+b^2
Answer: B

---

159. Zero vector has magnitude:
A) 1
B) −1
C) 0
D) infinite
Answer: C

---

160. Two vectors are equal if they have:
A) same magnitude only
B) same direction only
C) same magnitude and direction
D) same position
Answer: C

Section B: Vector Point Functions

---

161. Vector point function assigns:
A) scalar to vector
B) vector to each point
C) number only
D) constant
Answer: B

---

162. Vector function of scalar variable is written as:
A) f(x)
B) r(t)
C) x+y
D) ax
Answer: B

---

163. Example of vector function:
A) t2t^2t2
B) ti+t2jti+t^2jti+t2j
C) ln t
D) sin t
Answer: B

---

164. Vector function represents:
A) curve in space
B) line only
C) point only
D) constant
Answer: A

---

165. Parameter in vector function is usually:
A) scalar
B) vector
C) matrix
D) constant
Answer: A

---

166. Position vector describes:
A) location of particle
B) velocity
C) force
D) acceleration only
Answer: A

---

167. Vector valued function has components:
A) scalar functions
B) matrices
C) constants
D) limits
Answer: A

Section C: Continuity of Vector Function

---

168. Vector function is continuous if:
A) each component continuous
B) magnitude constant
C) direction fixed
D) derivative zero
Answer: A

---

169. Continuity depends on:
A) component functions
B) constants
C) matrices
D) limits only
Answer: A

---

170. Limit of vector function exists if:
A) component limits exist
B) magnitude zero
C) direction constant
D) infinite value
Answer: A

171. Continuous vector function has:
A) no breaks
B) constant slope
C) zero value
D) fixed direction
Answer: A

172. Discontinuity occurs when:
A) any component discontinuous
B) magnitude constant
C) derivative exists
D) vector zero
Answer: A

Section D: Differentiation of Vector Function

173. Derivative of vector function represents:
A) velocity
B) position
C) displacement
D) mass
Answer: A

174. If r(t)=xi+yj+zkr(t)=xi+yj+zkr(t)=xi+yj+zk, then

drdt=\frac{dr}{dt}=dtdr=

A) derivative of each component
B) zero
C) constant
D) integral
Answer: A

175. Differentiation of vector is performed:
A) component-wise
B) randomly
C) algebraically only
D) geometrically only
Answer: A

---

176. Second derivative of position vector gives:
A) velocity
B) acceleration
C) displacement
D) force
Answer: B

---

177. If velocity constant, acceleration is:
A) zero
B) infinity
C) constant
D) undefined
Answer: A

---

178. Derivative of constant vector equals:
A) same vector
B) zero vector
C) unit vector
D) infinity
Answer: B

Section E: Derivative Rules

---

179. Derivative of sum of vectors equals:
A) sum of derivatives
B) product
C) difference
D) zero
Answer: A

180.

ddt(A+B)=\frac{d}{dt}(A+B)=dtd(A+B)=

A) A+B
B) A′+B′
C) AB
D) 0
Answer: B

---

181. Derivative of scalar multiple:

ddt[cA]=\frac{d}{dt}[cA]=dtd[cA]=

A) cA′
B) A′
C) c+A
D) 0
Answer: A

---

182. Product rule for scalar and vector:
A) (uv)'=u'v+uv'
B) u+v
C) uv
D) constant
Answer: A

---

183. If vector magnitude constant then derivative is:
A) parallel
B) perpendicular
C) zero always
D) infinite
Answer: B

---

184. Tangent vector to curve equals:
A) derivative of position vector

B) position vector
C) acceleration
D) scalar
Answer: A

Section F: Unit Tangent and Motion

---

185. Unit tangent vector is:
A) normalized velocity vector
B) position vector
C) acceleration vector
D) zero vector
Answer: A

---

186. Speed equals:
A) magnitude of velocity
B) displacement
C) acceleration
D) direction
Answer: A

---

187. Velocity direction is along:
A) tangent to path
B) normal always
C) axis
D) origin
Answer: A

---

188. Acceleration depends on:
A) change in velocity
B) position only
C) constant value
D) mass
Answer: A

Section G: Conceptual Vector Calculus

---

189. Vector differentiation useful in:
A) particle motion
B) algebra
C) matrices
D) limits
Answer: A

---

190. Curve described by vector function lies in:
A) space
B) number line
C) matrix
D) constant plane
Answer: A

---

191. If all components differentiable, vector function is:
A) differentiable
B) discontinuous
C) constant
D) undefined
Answer: A

---

192. Vector derivative gives instantaneous:
A) rate of change
B) position
C) constant
D) magnitude only
Answer: A

---

193. Differentiation preserves:
A) vector nature
B) scalar nature
C) constant value
D) magnitude only
Answer: A

Section H: Trigonometric Vector Functions

---

194. Derivative of sin t i\sin t\,isinti equals:
A) cos t i
B) sin t i
C) −sin t i
D) tan t i
Answer: A

---

195. Derivative of cos t j\cos t\,jcostj equals:
A) sin t j
B) −sin t j
C) cos t j
D) tan t j
Answer: B

---

196. Vector function (cos t)i+(sin t)j(\cos t)i+(\sin t)j(cost)i+(sint)j represents:
A) circle
B) line
C) parabola
D) hyperbola
Answer: A

---

197. Speed of circular motion remains:
A) constant
B) zero
C) infinite
D) variable always
Answer: A

---

198. Acceleration in circular motion directed toward:
A) center
B) tangent
C) outward

D) axis
Answer: A

Section I: Advanced Conceptual

---

199. Differentiation of vector function mainly studies:
A) motion in space
B) algebraic equations
C) matrices
D) limits
Answer: A

---

200. Vector calculus combines:
A) algebra & geometry
B) calculus & vectors
C) matrices & limits
D) statistics & algebra
Answer: B

LONG QUESTIONS WITH SOLUTIONS (Q1–Q30)

SECTION A — Integration by Substitution

Q1. Evaluate

$$\int 2x\cos(x^2)\,dx$$

Solution

Let

$$u=x^2 \Rightarrow du=2x\,dx$$

Integral becomes:

∫cosu du=sinu+C\int \cos u\,du=\sin u+C∫cosudu=sinu+C

Substitute back:

sin(x2)+C\boxed{\sin(x^2)+C}sin(x2)+C

Q2. Evaluate

∫xx2+1 dx\int \frac{x}{x^2+1}\,dx∫x2+1xdx

Let u=x2+1u=x^2+1u=x2+1

du=2x dxdu=2x\,dxdu=2xdx =12∫duu=\frac12\int\frac{du}{u}=21∫udu
=12ln|x2+1|+C=\frac12\ln|x^2+1|+C=21ln|x2+1|+C

Q3. Evaluate

∫dx1−x2\int \frac{dx}{\sqrt{1-x^2}}∫1−x2dx

Using standard substitution x=sinθx=\sin\thetax=sinθ:

dx=cosθdθdx=\cos\theta d\thetadx=cosθdθ =∫dθ=θ+C=\int d\theta=\theta+C=∫dθ=θ+C
sin−1x+C\boxed{\sin^{-1}x+C}sin−1x+C

Q4. Evaluate

∫e3x dx\int e^{3x}\,dx∫e3xdx

Let u=3xu=3xu=3x

du=3dxdu=3dxdu=3dx =13e3x+C=\frac13 e^{3x}+C=31e3x+C

Q5. Evaluate

∫dxxlnx\int \frac{dx}{x\ln x}∫xlnxdx

Let u=lnxu=\ln xu=lnx

du=dxxdu=\frac{dx}{x}du=xdx =∫duu=ln|lnx|+C=\int\frac{du}{u}=\ln|\ln x|+C=∫udu=ln|lnx|+C

SECTION B — Integration by Parts

Q6. Evaluate

∫xexdx\int x e^x dx∫xexdx

Formula:

∫udv=uv−∫vdu\int udv=uv-\int vdu∫udv=uv−∫vdu

Take:

u=x,dv=exdxu=x,\quad dv=e^x dxu=x,dv=exdx du=dx,v=exdu=dx,\quad v=e^xdu=dx,v=ex
=xex−∫exdx=xe^x-\int e^x dx=xex−∫exdx ex(x−1)+C\boxed{e^x(x-1)+C}ex(x−1)+C

Q7. Evaluate

∫xsinx dx\int x\sin x\,dx∫xsinxdx

Let:

u=x, dv=sinxdxu=x,\ dv=\sin xdxu=x, dv=sinxdx du=dx, v=−cosxdu=dx,\ v=-\cos
xdu=dx, v=−cosx =−xcosx+sinx+C=-x\cos x+\sin x+C=−xcosx+sinx+C

Q8. Evaluate

∫lnx dx\int \ln x\,dx∫lnxdx

Write:

∫lnxdx=∫1·lnxdx\int \ln x dx=\int 1\cdot \ln x dx∫lnxdx=∫1·lnxdx

Take:

u=lnx, dv=dxu=\ln x,\ dv=dxu=lnx, dv=dx =xlnx−x+C=x\ln x-x+C=xlnx−x+C

Q9. Evaluate

∫x2exdx\int x^2 e^x dx∫x2exdx

Apply parts repeatedly:

Result:

ex(x2−2x+2)+C\boxed{e^x(x^2-2x+2)+C}ex(x2−2x+2)+C

Q10. Prove integration by parts formula.

From product rule:

ddx(uv)=udvdx+vdudx\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}dxd(uv)=udxdv+vdxdu

Integrating:

∫udv=uv−∫vdu\int udv=uv-\int vdu∫udv=uv−∫vdu

✓ proved.

SECTION C — Partial Fractions

Q11. Evaluate

∫dx(x−1)(x−2)\int \frac{dx}{(x-1)(x-2)}∫(x−1)(x−2)dx

Decompose:

1(x−1)(x−2)=Ax−1+Bx−2\frac1{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}(x−1)(x−2)1=x−1A+x−2B

Solve:

A=−1, B=1A=-1,\ B=1A=−1, B=1

Integral:

ln|x−2x−1|+C\ln\left|\frac{x-2}{x-1}\right|+Clnx−1x−2+C

Q12. Evaluate

∫xx2−1dx\int \frac{x}{x^2-1}dx∫x2−1xdx

Factor denominator:

=12∫2xx2−1dx=\frac12\int\frac{2x}{x^2-1}dx=21∫x2−12xdx =12ln|x2−1|+C=\frac12\ln|x^2-1|+C=21ln|x2−1|+C

Q13. Evaluate

∫dxx2−4\int \frac{dx}{x^2-4}∫x2−4dx

Factor:

(x−2)(x+2)(x-2)(x+2)(x−2)(x+2)

Result:

14ln|x−2x+2|+C\frac14\ln\left|\frac{x-2}{x+2}\right|+C41lnx+2x−2+C

Q14. Explain steps of partial fraction decomposition.

1. Factor denominator
2. Assume constants
3. Equate coefficients
4. Integrate each term

Q15. Evaluate

∫3x+5x2+2xdx\int \frac{3x+5}{x^2+2x}dx∫x2+2x3x+5dx

Split:

3x+5x(x+2)=Ax+Bx+2\frac{3x+5}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}x(x+2)3x+5=xA+x+2B

Solve:
A=5/2, B=1/2

Integrate → logarithmic result.

SECTION D — Rational & Irrational Functions

Q16. Evaluate

∫x dx\int \sqrt{x}\,dx∫xdx =∫x1/2dx=23x3/2+C=\int x^{1/2}dx =\frac{2}{3}x^{3/2}+C=∫x1/2dx=32x3/2+C

Q17. Evaluate

∫dxx\int \frac{dx}{\sqrt{x}}∫xdx =2x+C=2\sqrt{x}+C=2x+C

Q18. Evaluate

∫a2−x2 dx\int \sqrt{a^2-x^2}\,dx∫a2−x2dx

Substitute:

x=asinθx=a\sin\thetax=asinθ

Result:

x2a2−x2+a22sin−1xa+C\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C2xa2−x2+2a2sin−1ax+C

Q19. Evaluate

∫dxx2+a2\int \frac{dx}{\sqrt{x^2+a^2}}∫x2+a2dx

Substitute x=atanθx=a\tan\thetax=atanθ

Result:

ln|x+x2+a2|+C\ln|x+\sqrt{x^2+a^2}|+Cln|x+x2+a2|+C

Q20. Explain trigonometric substitution method.

Used when integrand contains:

- a2−x2\sqrt{a^2-x^2}a2−x2
- x2+a2\sqrt{x^2+a^2}x2+a2
- x2−a2\sqrt{x^2-a^2}x2−a2

Convert algebraic form → trigonometric form.

SECTION E — Integration by Transformation

Q21. Evaluate

∫0af(x)dx=∫0af(a−x)dx\int_0^a f(x)dx=\int_0^a f(a-x)dx∫0af(x)dx=∫0af(a−x)dx

Proof

Put:

x=a−tx=a-tx=a−t

Limits reverse → same integral.

✓ proved.

Q22. Evaluate

∫0πsinxdx\int_0^\pi \sin x dx∫0πsinxdx [−cosx]0π=2[-\cos x]_0^\pi=2[−cosx]0π=2

Q23. Evaluate

∫01(x+1)2dx\int_0^1 (x+1)^2 dx∫01(x+1)2dx

Expand:

x2+2x+1x^2+2x+1x2+2x+1

Integrate termwise:

=13+1+1=73=\frac13+1+1=\frac73=31+1+1=37

****Q24. Explain transformation in integration.**

Change variable to simplify integrand or exploit symmetry.

Q25. Evaluate

∫dx1+x\int \frac{dx}{1+\sqrt{x}}∫1+xdx

Let u=xu=\sqrt{x}u=x

x=u2, dx=2u dux=u^2,\ dx=2u\,dux=u2, dx=2udu

Solve → logarithmic form.

SECTION F — Mixed Concept Problems

Q26. Evaluate

∫xlnx dx\int x\ln x\,dx∫xlnxdx

Parts:

u=lnx, dv=xdxu=\ln x,\ dv=x dxu=lnx, dv=xdx

Result:

x22lnx−x24+C\frac{x^2}{2}\ln x-\frac{x^2}{4}+C2x2lnx−4x2+C

Q27. Evaluate

∫exsinx dx\int e^x\sin x\,dx∫exsinxdx

Apply parts twice:

=ex2(sinx−cosx)+C=\frac{e^x}{2}(\sin x-\cos x)+C=2ex(sinx−cosx)+C

Q28. Evaluate

∫x2x+1dx\int \frac{x^2}{x+1}dx∫x+1x2dx

Divide first:

x−1+1x+1x-1+\frac1{x+1}x−1+x+11

Integrate:

x22−x+ln|x+1|+C\frac{x^2}{2}-x+\ln|x+1|+C2x2−x+ln|x+1|+C

Q29. Explain difference between substitution and parts.

Method	Used When
Substitution	composite function
Parts	product of functions

****Q30. State applications of integration.**

Used for:

- area
- volume
- displacement
- probability
- physics modelling

LONG QUESTIONS WITH SOLUTIONS (Q31–Q60)

SECTION A — Evaluation of Definite Integrals

Q31. Evaluate

∫01x2 dx\int_0^1 x^2\,dx∫01x2dx

Solution
∫x2dx=x33\int x^2dx=\frac{x^3}{3}∫x2dx=3x3

Apply limits:

[x33]01=13\left[\frac{x^3}{3}\right]_0^1=\frac13[3x3]01=31

✅ Answer:

13\boxed{\frac13}31

Q32. Evaluate

∫0πsinx dx\int_0^\pi \sin x\,dx∫0πsinxdx [−cosx]0π=(−(−1))−(−1)=2[-\cos x]_0^\pi=(-(-1))-(-1)=2[−cosx]0π=(−(−1))−(−1)=2

✅ Answer:

222

Q33. Evaluate

∫0π/2cos2x dx\int_0^{\pi/2}\cos^2x\,dx∫0π/2cos2xdx

Use identity:

cos2x=1+cos2x2\cos^2x=\frac{1+\cos2x}{2}cos2x=21+cos2x
=12∫0π/2(1+cos2x)dx=\frac12\int_0^{\pi/2}(1+\cos2x)dx=21∫0π/2(1+cos2x)dx
=12[x+sin2x2]0π/2=\frac12\left[x+\frac{\sin2x}{2}\right]_0^{\pi/2}=21[x+2sin2x]0π/2
=π4=\frac{\pi}{4}=4π

Q34. Prove

$$\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

Proof

Let $x=a-t$.

Then:

$$dx=-dt$$

Limits interchange → integral unchanged.

✓ proved.

Q35. Evaluate

$$\int_0^1 (3x+2)\,dx =\left[\frac{3x^2}{2}+2x\right]_0^1 =\frac32+2=\frac72$$

SECTION B — Reduction Formulae

Q36. Derive reduction formula for

$$I_n=\int \sin^n x\,dx$$

Solution (Outline)

Write:

$$I_n=\int \sin^{n-1}x\sin x\,dx$$

Apply integration by parts.

Result:

$$\boxed{I_n=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}I_{n-2}}$$

Q37. Evaluate

$$\int_0^{\pi/2}\sin^3x\,dx$$

Using reduction:

$$I_3=\frac23 I_1 \quad I_1=1$$

Hence:

$$\frac23$$

Q38. Derive reduction formula for

$$\int \cos^n x\,dx$$

Result:

$$I_n=\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}I_{n-2}$$

Q39. Evaluate

$$\int_0^{\pi/2}\cos^4x\,dx$$

Using reduction repeatedly:

$$=\frac{3\pi}{16}$$

Q40. Explain importance of reduction formulae.

They convert higher power integrals into simpler lower power integrals.

SECTION C — Curve Tracing

Q41. Trace curve

y=x2y=x^2y=x2

Steps

- Symmetric about y-axis
- Vertex (0,0)
- Opens upward

Parabola obtained.

Q42. Trace

y=x3y=x^3y=x3

Odd function → symmetric about origin.

Inflection at origin.

Q43. Find symmetry of

x2+y2=a2x^2+y^2=a^2x2+y2=a2

Symmetric about both axes and origin → circle.

Q44. Find stationary points of

y=x3−3xy=x^3-3xy=x3−3x y′=3x2−3=0⇒x=±1y'=3x^2-3=0 \Rightarrow x=\pm1y′=3x2−3=0⇒x=±1

Stationary points at (1,−2), (−1,2).

Q45. Explain steps of curve tracing.

1. Symmetry
2. Intercepts
3. Derivatives
4. Asymptotes
5. Sketch

SECTION D — Length of Curve

Q46. Find length of curve

y=x3/2, 0≤x≤1y=x^{3/2},\ 0\le x\le1y=x3/2, 0≤x≤1

Formula:

L=∫1+(y′)2dxL=\int\sqrt{1+(y')^2}dxL=∫1+(y′)2dx y′=32x1/2y'=\frac32x^{1/2}y′=23x1/2
L=∫011+94x dxL=\int_0^1\sqrt{1+\frac94x}\,dxL=∫011+49xdx

Integrate → final numerical value.

Q47. Derive arc length formula.

Using small element:

ds=dx2+dy2ds=\sqrt{dx^2+dy^2}ds=dx2+dy2

Divide by dx:

ds=1+(dy/dx)2dxds=\sqrt{1+(dy/dx)^2}dxds=1+(dy/dx)2dx

✓ derived.

Q48. Find arc length of

y=x, 0≤x≤1y=x,\ 0\le x\le1y=x, 0≤x≤1 y′=1y'=1y′=1 L=∫012 dx=2L=\int_0^1\sqrt2\,dx=\sqrt2L=∫01 2dx=2

SECTION E — Area Under Curve

Q49. Find area under

y=x2, 0≤x≤2y=x^2,\ 0\le x\le2y=x2, 0≤x≤2 A=∫02x2dx=83A=\int_0^2 x^2dx=\frac{8}{3}A=∫02 x2dx=38

Q50. Area between

y=x, y=x2y=x,\ y=x^2y=x, y=x2

Intersection: x=0,1

A=∫01(x−x2)dxA=\int_0^1(x-x^2)dxA=∫01(x−x2)dx =12−13=16=\frac12-\frac13=\frac16=21−31 =61

Q51. Explain geometric meaning of definite integral.

Represents signed area between curve and axis.

SECTION F — Volume of Solids of Revolution

Q52. Volume formed by rotating

y=x, 0≤x≤1y=x,\ 0\le x\le1y=x, 0≤x≤1

about x-axis.

V=π∫01x2dx=π3V=\pi\int_0^1x^2dx=\frac{\pi}{3}V=π∫01x2dx=3π

Q53. Derive disk method formula.

Volume of thin disk:

dV=πy2dxdV=\pi y^2dxdV=πy2dx

Integrate:

V=π∫y2dxV=\pi\int y^2dxV=π∫y2dx

Q54. Volume rotating

y=xy=\sqrt{x}y=x

about x-axis (0→4).

V=π∫04x dx=8πV=\pi\int_0^4x\,dx=8\piV=π∫04xdx=8π

Q55. Explain washer method.

Used when region has inner and outer radii:

V=π∫(R2−r2)dxV=\pi\int(R^2-r^2)dxV=π∫(R2−r2)dx

SECTION G — Surface Area of Revolution

Q56. Derive surface area formula.

Surface strip:

dS=2πy dsdS=2\pi y\,dsdS=2πyds S=2π∫y1+(y′)2dxS=2\pi\int y\sqrt{1+(y')^2}dxS=2π∫y1+(y′)2dx

---

Q57. Find surface area rotating

y=x, 0≤x≤1y=x,\ 0\le x\le1y=x, 0≤x≤1 y′=1y'=1y′=1 S=2π∫01x2 dx=π2S=2\pi\int_0^1x\sqrt2\,dx =\pi\sqrt2S=2π∫01x2dx=π2

Q58. Explain applications of solids of revolution.

Used in:

- engineering design
- tanks & vessels
- manufacturing shapes

Q59. Compare area and volume integrals.

Quantity	Formula
Area	$\int y\,dx$
Volume	$\pi\int y^2 dx$

Q60. State real-life applications of definite integrals.

- area calculation
- volume measurement
- physics motion problems
- economics accumulation models

SECTION A — Scalars and Vectors (Theory + Numerical)

Q61. Define scalar and vector quantities with examples.

Solution

Scalar: Quantity having magnitude only.
Examples: mass, temperature, time.

Vector: Quantity having magnitude and direction.
Examples: velocity, force, displacement.

Vector representation:

A⃗=ai+bj+ck\vec{A}=ai+bj+ckA=ai+bj+ck

Magnitude:

|A⃗|=a2+b2+c2|\vec A|=\sqrt{a^2+b^2+c^2}|A|=a2+b2+c2

Q62. Find magnitude and unit vector of

A⃗=3i−4j+12k\vec A=3i-4j+12kA=3i−4j+12k

Solution

Magnitude:

|A⃗|=9+16+144=13|\vec A|=\sqrt{9+16+144}=13|A|=9+16+144=13

Unit vector:

A^=A⃗|A⃗|=3i−4j+12k13\hat A=\frac{\vec A}{|\vec A|} =\frac{3i-4j+12k}{13}A^=|A|A =133i−4j+12k

---

Q63. Show that two vectors are perpendicular if their dot product is zero.

Solution
A⃗·B⃗=|A⃗||B⃗|cosθ\vec A\cdot\vec B=|\vec A||\vec B|\cos\thetaA·B=|A||B|cosθ

If dot product = 0:

cosθ=0⇒θ=90∘\cos\theta=0\Rightarrow \theta=90^\circcosθ=0⇒θ=90∘

Hence perpendicular.

Q64. Find angle between

A⃗=i+2j+2k,B⃗=2i−j+2k\vec A=i+2j+2k,\quad \vec B=2i-j+2kA=i+2j+2k,B=2i−j+2k

Dot product:

A·B=2−2+4=4A\cdot B=2-2+4=4A·B=2−2+4=4

Magnitudes:

|A|=3,|B|=3|A|=3,\quad |B|=3|A|=3,|B|=3 cosθ=49\cos\theta=\frac{4}{9}cosθ=94

Q65. Explain vector point function geometrically.

A vector function assigns a vector to each value of parameter ttt.
It represents motion of a particle or curve in space.

SECTION B — Vector Function of Scalar Variable

Q66. Define vector function of scalar variable.

A function:

r⃗(t)=x(t)i+y(t)j+z(t)k\vec r(t)=x(t)i+y(t)j+z(t)kr(t)=x(t)i+y(t)j+z(t)k

where t is scalar parameter.

Q67. Given

r⃗(t)=t2i+t3j+tk\vec r(t)=t^2i+t^3j+tkr(t)=t2i+t3j+tk

find position at t=2t=2t=2.

=4i+8j+2k=4i+8j+2k=4i+8j+2k

Q68. Find magnitude of

r⃗(t)=ti+2tj\vec r(t)=ti+2tjr(t)=ti+2tj |r⃗|=t2+4t2=5 t|\vec r|=\sqrt{t^2+4t^2}=\sqrt5\,t|r|=t2+4t2=5t

---

Q69. Show vector function represents a curve.

Since coordinates depend on parameter:

x=f(t),y=g(t),z=h(t)x=f(t),y=g(t),z=h(t)x=f(t),y=g(t),z=h(t)

Eliminating t gives equation of curve.

Q70. Find velocity vector if

r(t)=t2i+3tjr(t)=t^2i+3tjr(t)=t2i+3tj

Velocity:

v=drdt=2ti+3jv=\frac{dr}{dt}=2ti+3jv=dtdr=2ti+3j

SECTION C — Continuity of Vector Function

Q71. Define continuity of vector function.

Vector function continuous at t=at=at=a if:

lim⁡t→ar(t)=r(a)\lim_{t\to a} r(t)=r(a)t→alimr(t)=r(a)

All components must be continuous.

Q72. Test continuity of

r(t)=(t2)i+(3t)jr(t)=(t^2)i+(3t)jr(t)=(t2)i+(3t)j

Since polynomial functions continuous → vector function continuous.

Q73. Prove continuity depends on component functions.

Limit:

lim⁡r(t)=lim⁡xi+lim⁡yj+lim⁡zk\lim r(t)=\lim xi+\lim yj+\lim zklimr(t)=limxi+limyj+limzk

Exists only if each component limit exists.

✓ proved.

Q74. Explain geometrical meaning of continuity.

Particle moves smoothly without jumps.

Q75. Check continuity at t=0t=0t=0:

r(t)=(sin⁡t)i+t2jr(t)=(\sin t)i+t^2jr(t)=(sint)i+t2j

Both components continuous ⇒ function continuous.

SECTION D — Differentiation of Vector Function

Q76. Define derivative of vector function.

drdt=lim⁡Δt→0r(t+Δt)−r(t)Δt\frac{dr}{dt}=\lim_{\Delta t\to0} \frac{r(t+\Delta t)-r(t)}{\Delta t}dtdr=Δt→0limΔtr(t+Δt)−r(t)

Q77. Differentiate

r(t)=t3i+t2j+tkr(t)=t^3i+t^2j+tkr(t)=t3i+t2j+tk r′(t)=3t2i+2tj+kr'(t)=3t^2i+2tj+kr′(t)=3t2i+2tj+k

---

Q78. Find velocity and acceleration.

Given above:

Velocity:

v=r′(t)v=r'(t)v=r′(t)

Acceleration:

a=r′′(t)=6ti+2ja=r''(t)=6ti+2ja=r′′(t)=6ti+2j

Q79. Differentiate

r(t)=eti+sintjr(t)=e^ti+\sin tjr(t)=eti+sintj r′(t)=eti+costjr'(t)=e^ti+\cos tjr′(t)=eti+costj

---

Q80. Show derivative of constant vector is zero.

Let AAA constant:

dAdt=0\frac{dA}{dt}=0dtdA=0

since no change with t.

SECTION E — Rules of Differentiation

Q81. Prove derivative of sum rule.

ddt(A+B)=dAdt+dBdt\frac{d}{dt}(A+B) =\frac{dA}{dt}+\frac{dB}{dt}dtd(A+B)=dtdA+dtdB

Follows from limit definition.

Q82. Differentiate

(ti+t2j)+(etk)(ti+t^2j)+(e^tk)(ti+t2j)+(etk) =i+2tj+etk=i+2tj+e^tk=i+2tj+etk

---

Q83. Differentiate scalar multiple

ddt[t2(i+j)]\frac{d}{dt}[t^2(i+j)]dtd[t2(i+j)] =2t(i+j)=2t(i+j)=2t(i+j)

Q84. If

r(t)=ai+btjr(t)=ai+btjr(t)=ai+btj

find derivative.

r′(t)=bjr'(t)=bjr′(t)=bj

****Q85. Explain physical meaning of derivative of vector.**

Represents velocity — instantaneous rate of change of position.

SECTION F — Higher Order Derivatives

Q86. Find second derivative

r(t)=t4i+t3jr(t)=t^4i+t^3jr(t)=t4i+t3j r′′(t)=12t2i+6tjr''(t)=12t^2i+6tjr′′(t)=12t2i+6tj

---

Q87. Show acceleration is derivative of velocity.

a=dvdt=d2rdt2a=\frac{dv}{dt} =\frac{d^2r}{dt^2}a=dtdv=dt2d2r

✓ proved.

Q88. Find unit tangent vector for

r(t)=ti+t2jr(t)=ti+t^2jr(t)=ti+t2j

Velocity:

v=i+2tjv=i+2tjv=i+2tj

Unit tangent:

T=v∣v∣T=\frac{v}{|v|}T=∣v∣v

Q89. Show velocity tangent to path.

Direction of motion given by derivative ⇒ tangent direction.

Q90. Explain applications of vector differentiation.

Used in:

- particle motion
- mechanics
- robotics
- space trajectories
- fluid dynamics

LONG QUESTIONS WITH SOLUTIONS (Q91–Q120)

SECTION A — Vector Function of Scalar Variable

Q91. Define a vector function of a scalar variable and give an example.

Solution

A vector function depending on scalar ttt is:

r⃗(t)=x(t)i+y(t)j+z(t)k\vec r(t)=x(t)i+y(t)j+z(t)kr(t)=x(t)i+y(t)j+z(t)k

Example:

r(t)=t2i+sin⁡t j+tkr(t)=t^2 i+\sin t\,j+t kr(t)=t2i+sintj+tk

It represents motion of a particle in space.

Q92. Find position vector at t=1t=1t=1 for

r(t)=t3i+2tj+cos⁡t kr(t)=t^3 i+2t j+\cos t\,kr(t)=t3i+2tj+costk

Substitute t=1t=1t=1:

r(1)=i+2j+cos⁡1 kr(1)=i+2j+\cos1\,kr(1)=i+2j+cos1k

Q93. Show that vector function represents a space curve.

Since:

x=f(t), y=g(t), z=h(t)x=f(t),\; y=g(t),\; z=h(t)x=f(t),y=g(t),z=h(t)

Eliminating parameter ttt gives relation among x,y,zx,y,zx,y,z → curve in space.

Q94. Find magnitude of

r(t)=2ti+3tj+tkr(t)=2ti+3tj+tkr(t)=2ti+3tj+tk |r|=4t2+9t2+t2=14 t|r|=\sqrt{4t^2+9t^2+t^2} =\sqrt{14}\,t|r|=4t2+9t2+t2=14t

---

Q95. Explain geometrical meaning of parameter ttt.

Parameter represents time or position index describing motion along a path.

SECTION B — Continuity of Vector Function

Q96. Define continuity of a vector function.

A vector function is continuous at t=at=at=a if:

lim t→ar(t)=r(a)\lim_{t\to a} r(t)=r(a)t→alimr(t)=r(a)

i.e., all components are continuous.

Q97. Test continuity of

r(t)=t2i+sin t j+etkr(t)=t^2i+\sin t\,j+e^t kr(t)=t2i+sintj+etk

All component functions continuous ⇒ vector function continuous.

Q98. Prove continuity depends on component continuity.

$$\lim r(t)= (\lim x)i+(\lim y)j+(\lim z)k$$

Exists only if each limit exists.

✓ proved.

Q99. Explain physical meaning of continuity.

Particle moves smoothly without sudden jumps.

Q100. Check continuity at $t=0$:

$$r(t)=\frac{\sin t}{t}i+t^2j$$ $$\lim_{t\to0}\frac{\sin t}{t}=1$$

Hence continuous.

SECTION C — Differentiation w.r.t Scalar Variable

Q101. Define derivative of vector function.

$$\frac{dr}{dt}= \lim_{\Delta t\to0} \frac{r(t+\Delta t)-r(t)}{\Delta t}$$

Q102. Differentiate

$$r(t)=t^2i+t^3j+t^4k$$ $$r'(t)=2ti+3t^2j+4t^3k$$

Q103. Find velocity and acceleration.

Velocity:

v=r′(t)v=r'(t)v=r′(t)

Acceleration:

a=r′′(t)a=r''(t)a=r′′(t)

Q104. Differentiate

r(t)=eti+cos⁡t j+sin⁡t kr(t)=e^ti+\cos t\,j+\sin t\,kr(t)=eti+costj+sintk
r′(t)=eti−sin⁡t j+cos⁡t kr'(t)=e^ti-\sin t\,j+\cos t\,kr′(t)=eti−sintj+costk

---

Q105. Show derivative of constant vector is zero.

If AAA constant:

dAdt=0\frac{dA}{dt}=0dtdA=0

since no change occurs.

SECTION D — Derivative of Sum of Vectors

Q106. Prove

ddt(A+B)=A′+B′\frac{d}{dt}(A+B)=A'+B'dtd(A+B)=A′+B′

Using limit definition:

ddt(A+B)=lim⁡A(t+h)+B(t+h)−A(t)−B(t)h\frac{d}{dt}(A+B) =\lim\frac{A(t+h)+B(t+h)-A(t)-B(t)}{h}dtd(A+B)=limhA(t+h)+B(t+h)−A(t)−B(t)

Separate terms ⇒ result proved.

Q107. Differentiate

(ti+t2j)+(etk)(ti+t^2j)+(e^tk)(ti+t2j)+(etk) =i+2tj+etk=i+2tj+e^tk=i+2tj+etk

---

Q108. Differentiate

(3t2i+4tj)+(5k)(3t^2i+4tj)+(5k)(3t2i+4tj)+(5k) =6ti+4j=6ti+4j=6ti+4j

---

Q109. Explain linearity of vector differentiation.

Derivative distributes over addition and scalar multiplication.

Q110. If

A(t)=ti, B(t)=t2jA(t)=ti,\ B(t)=t^2jA(t)=ti, B(t)=t2j

find derivative of A+BA+BA+B.

=A′+B′=i+2tj=A'+B'=i+2tj=A′+B′=i+2tj

SECTION E — Derivative of Scalar Product

Q111. Prove

ddt(A·B)=A′·B+A·B′\frac{d}{dt}(A\cdot B)=A'\cdot B+A\cdot B'dtd(A·B)=A′·B+A·B′

Using product rule and limit definition.

✓ proved.

Q112. If

A=ti, B=tjA=ti,\ B=tjA=ti, B=tj

find derivative of A·BA\cdot BA·B.

A·B=0⇒derivative=0A\cdot B=0 \Rightarrow \text{derivative}=0A·B=0⇒derivative=0

Q113. Differentiate

(ti+t2j)·(i+j)(ti+t^2j)\cdot(i+j)(ti+t2j)·(i+j)

Dot product:

=t+t2=t+t^2=t+t2

Derivative:

1+2t1+2t1+2t

Q114. Show derivative of A·AA\cdot AA·A equals 2A·A′2A\cdot A'2A·A′.

ddt(A·A)=A′·A+A·A′=2A·A′\frac{d}{dt}(A\cdot A) =A'\cdot A+A\cdot A' =2A\cdot A'dtd (A·A)=A′·A+A·A′=2A·A′

Q115. Explain geometric meaning of scalar product derivative.

Represents rate of change of projection of one vector on another.

SECTION F — Derivative of Vector Product

Q116. Prove

ddt(A×B)=A′×B+A×B′\frac{d}{dt}(A\times B)=A'\times B+A\times B'dtd(A×B)=A′×B+A×B′

Derived using limit definition similar to product rule.

✓ proved.

Q117. Differentiate

(ti)×(tj)(ti)\times(tj)(ti)×(tj) =t2k=t^2k=t2k

Derivative:

2tk2tk2tk

Q118. If

A=ti, B=t2jA=ti,\ B=t^2jA=ti, B=t2j

find derivative of A×BA\times BA×B.

A×B=t3kA\times B=t^3kA×B=t3k

Derivative:

3t2k3t^2k3t2k

Q119. Show cross product derivative gives rotational change.

Since cross product gives perpendicular direction, its derivative measures change in rotational orientation.

Q120. Explain applications of vector product differentiation.

Used in:

- angular velocity
- rotational motion
- electromagnetism
- fluid flow analysis

LONG QUESTIONS WITH SOLUTIONS (Q121–Q150)

SECTION A — Gradient

Q121. Define gradient of a scalar function and find gradient of

φ=x2+y2+z2\phi=x^2+y^2+z^2φ=x2+y2+z2

Solution

Gradient:

∇φ=i∂φ∂x+j∂φ∂y+k∂φ∂z\nabla \phi=i\frac{\partial \phi}{\partial x} +j\frac{\partial \phi}{\partial y} +k\frac{\partial \phi}{\partial z}∇φ=i∂x∂φ+j∂y∂φ+k∂z∂φ

Compute derivatives:

∂φ∂x=2x, ∂φ∂y=2y, ∂φ∂z=2z\frac{\partial \phi}{\partial x}=2x,\; \frac{\partial \phi}{\partial y}=2y,\; \frac{\partial \phi}{\partial z}=2z∂x∂φ=2x,∂y∂φ=2y,∂z∂φ=2z
∇φ=2xi+2yj+2zk\boxed{\nabla\phi=2xi+2yj+2zk}∇φ=2xi+2yj+2zk

Q122. Show gradient points in direction of maximum increase.

Directional derivative:

Duφ=∇φ·u^D_u\phi=\nabla\phi\cdot \hat uDuφ=∇φ·u^

Maximum occurs when u^\hat uu^ parallel to ∇ϕ\nabla\phi∇ϕ.

Hence gradient gives direction of greatest increase.

Q123. Find gradient of

φ=xyz\phi=xyzφ=xyz ∇φ=yz i+xz j+xy k\nabla\phi=yz\,i+xz\,j+xy\,k∇φ=yzi+xzj+xyk

---

Q124. Find directional derivative of

φ=x2+y2+z2\phi=x^2+y^2+z^2φ=x2+y2+z2

at (1,1,1) along vector i+j+ki+j+ki+j+k.

Unit vector:

i+j+k3\frac{i+j+k}{\sqrt3}3i+j+k

Gradient at point:

2i+2j+2k2i+2j+2k2i+2j+2k

Directional derivative:

=63=23=\frac{6}{\sqrt3}=2\sqrt3=36=23

Q125. Explain physical meaning of gradient.

Represents rate and direction of maximum change (temperature, potential fields).

SECTION B — Divergence

Q126. Define divergence and find divergence of

A=xi+yj+zkA=xi+yj+zkA=xi+yj+zk ∇·A=∂x∂x+∂y∂y+∂z∂z=3\nabla\cdot A= \frac{\partial x}{\partial x} +\frac{\partial y}{\partial y} +\frac{\partial z}{\partial z}=3∇·A=∂x∂x+∂y∂y+∂z∂z=3

Q127. Interpret physical meaning of divergence.

Measures source or sink strength of vector field.

Positive → source
Negative → sink.

Q128. Find divergence of

A=x2i+y2j+z2kA=x^2i+y^2j+z^2kA=x2i+y2j+z2k ∇·A=2x+2y+2z\nabla\cdot A=2x+2y+2z∇·A=2x+2y+2z

---

Q129. Show divergence of constant vector is zero.

All partial derivatives vanish ⇒ divergence = 0.

Q130. Determine whether field is solenoidal

A=yi+zj+xkA=yi+zj+xkA=yi+zj+xk ∇·A=0+0+0=0\nabla\cdot A=0+0+0=0∇·A=0+0+0=0

Hence solenoidal.

SECTION C — Curl

Q131. Define curl and compute curl of

A=xi+yj+zkA=xi+yj+zkA=xi+yj+zk ∇×A=0\nabla\times A=0∇×A=0

Hence irrotational field.

Q132. Find curl of

A=yz i+zx j+xy kA=yz\,i+zx\,j+xy\,kA=yzi+zxj+xyk

Using determinant form:

∇×A=0\nabla\times A=0∇×A=0

Q133. Show curl of gradient is zero.

∇×(∇φ)=0\nabla\times(\nabla\phi)=0∇×(∇φ)=0

Mixed partial derivatives cancel.

✓ proved.

Q134. Explain physical meaning of curl.

Measures rotational tendency of vector field.

Q135. Determine if vector field is irrotational

$$A=2xi+2yj+2zk$$

Curl = 0 ⇒ irrotational.

SECTION D — Second Order Vector Operators

Q136. Define Laplacian operator.

$$\nabla^2=\frac{\partial^2}{\partial x^2} +\frac{\partial^2}{\partial y^2} +\frac{\partial^2}{\partial z^2}$$

Q137. Find Laplacian of

$$\phi=x^2+y^2+z^2 \quad \nabla^2\phi=2+2+2=6$$

Q138. Prove

$$\nabla\cdot(\nabla\times A)=0$$

Divergence of curl always zero due to equality of mixed derivatives.

Q139. Show

$$\nabla\times(\nabla\phi)=0$$

Gradient has no rotation ⇒ curl zero.

Q140. Explain relation

$$\nabla^2\phi=\nabla\cdot(\nabla\phi)$$

Laplacian equals divergence of gradient.

SECTION E — Cartesian Coordinate System

Q141. Write gradient, divergence and curl in Cartesian form.

$$\nabla=i\partial_x+j\partial_y+k\partial_z$$

Divergence:

$$\partial_xA_x+\partial_yA_y+\partial_zA_z$$

Curl:
determinant form.

Q142. Show gradient operator is vector operator.

Contains unit vectors i,j,k with derivatives ⇒ vector quantity.

Q143. Explain advantages of Cartesian coordinates.

- Simple derivatives
- Orthogonal axes
- Easy vector operations

SECTION F — De Moivre’s Theorem

Q144. State and prove De Moivre’s theorem.

Statement

(cosθ+isinθ)n=cosnθ+isinnθ(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta(cosθ+isinθ)n=cosnθ+isinnθ

Proof (by induction)

True for n=1.

Assume true for n=k.

Multiply by (cosθ+isinθ)(\cos\theta+i\sin\theta)(cosθ+isinθ) → obtain result for k+1.

✓ proved.

Q145. Use De Moivre to expand

(cosθ+isinθ)3(\cos\theta+i\sin\theta)^3(cosθ+isinθ)3
=cos3θ+isin3θ=\cos3\theta+i\sin3\theta=cos3θ+isin3θ

Q146. Find cos3θ\cos3\thetacos3θ using De Moivre.

Compare real parts:

cos3θ=4cos3θ−3cosθ\cos3\theta=4\cos^3\theta-3\cos\thetacos3θ=4cos3θ−3cosθ

Q147. Find cube roots of unity.

z3=1z^3=1z3=1 z=cos2kπ3+isin2kπ3z=\cos\frac{2k\pi}{3}+i\sin\frac{2k\pi}{3}z=cos32kπ+isin32kπ

Roots:

1, −12±i321,\; -\frac12\pm i\frac{\sqrt3}{2}1,−21±i23

Q148. Express complex number 1+i1+i1+i in polar form.

Magnitude:

r=2r=\sqrt2r=2

Argument:

θ=π/4\theta=\pi/4θ=π/4 2(cosπ4+isinπ4)\boxed{\sqrt2(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}2(cos4π+isin4π)

Q149. Find (1+i)4(1+i)^4(1+i)4 using De Moivre.

1+i=2(cosπ4+isinπ4)1+i=\sqrt2(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})1+i=2(cos4π+isin4π)

Raise power:

(2)4(cosπ+isinπ)=4(−1)=−4(\sqrt2)^4(\cos\pi+i\sin\pi) =4(-1)= -4(2)4(cosπ+isinπ)=4(−1)=−4

Q150. State applications of De Moivre's theorem.

Used in:

- finding roots of complex numbers
- trigonometric identities
- signal processing
- electrical engineering
- wave analysis