UNIT I – REAL ANALYSIS

Sequences (MCQs with Answers)

Definition of Sequence (Q1–Q10)

Q1. A sequence is a function whose domain is:
A) Real numbers
B) Natural numbers
C) Integers
D) Rational numbers
Answer: B

Q2. A sequence is usually denoted by:
A) f(x)
B) {an}
C) ∫f(x)dx
D) lim x→0
Answer: B

Q3. The sequence an = n represents:
A) Constant sequence
B) Increasing sequence
C) Decreasing sequence
D) Bounded sequence
Answer: B

Q4. The sequence an = 5 is called:
A) Variable sequence
B) Constant sequence
C) Oscillating sequence
D) Divergent sequence
Answer: B

Q5. The nth term determines:
A) Entire sequence
B) Only first term
C) Limit only
D) Bounds only
Answer: A

Q6. A sequence is finite if:
A) Domain is finite
B) Range is finite

C) Terms repeat
D) Limit exists
Answer: A

Q7. Sequence {1/n} belongs to:
A) Integers
B) Real numbers
C) Natural numbers
D) Complex numbers only
Answer: B

Q8. The first term of sequence {an} is:
A) a0
B) a1
C) a2
D) an+1
Answer: B

Q9. Sequence an = (−1)n is:
A) Constant
B) Oscillating
C) Increasing
D) Decreasing
Answer: B

Q10. Sequence is an ordered list of:
A) Numbers
B) Functions
C) Sets
D) Matrices
Answer: A

Bounds of a Sequence (Q11–Q20)

Q11. A sequence is bounded if:
A) Terms increase
B) Terms decrease
C) Terms lie within fixed limits
D) Limit exists
Answer: C

Q12. If |an| ≤ M, then sequence is:
A) Unbounded
B) Bounded

C) Divergent
D) Oscillatory
Answer: B

Q13. Upper bound means:
A) Largest term
B) Term greater than all elements
C) Number ≥ all terms
D) Limit value
Answer: C

Q14. Lower bound means:
A) Minimum value
B) Number ≤ all terms
C) Limit
D) Zero
Answer: B

Q15. Sequence an = 1/n is:
A) Unbounded
B) Bounded
C) Oscillating
D) Divergent
Answer: B

Q16. Sequence an = n is:
A) Bounded
B) Unbounded
C) Constant
D) Convergent
Answer: B

Q17. A bounded sequence must have:
A) Maximum only
B) Minimum only
C) Upper and lower bounds
D) Limit zero
Answer: C

Q18. Every convergent sequence is:
A) Unbounded
B) Bounded
C) Increasing
D) Decreasing
Answer: B

Q19. Boundedness depends on:
A) Limit
B) Range of terms
C) Index
D) Function type
Answer: B

Q20. Sequence {sin n} is:
A) Unbounded
B) Bounded
C) Increasing
D) Constant
Answer: B

Limit of a Sequence (Q21–Q35)

Q21. lim (1/n) as $n \to \infty$ equals:
A) 1
B) ∞
C) 0
D) −1
Answer: C

Q22. A sequence converges if:
A) Terms increase forever
B) Terms approach fixed number
C) Terms oscillate
D) Terms repeat
Answer: B

Q23. Limit of constant sequence c is:
A) 0
B) c
C) ∞
D) Undefined
Answer: B

Q24. lim $n \to \infty$ (n/(n+1)) equals:
A) 0
B) 1
C) ∞
D) −1
Answer: B

Q25. Divergent sequence means:
A) Limit exists
B) Limit finite
C) Limit does not exist
D) Constant
Answer: C

Q26. $\lim (2n+1)/n$ equals:
A) 1
B) 2
C) 0
D) ∞
Answer: B

Q27. Limit of $(-1)n$ is:
A) 0
B) 1
C) -1
D) Does not exist
Answer: D

Q28. If $\lim an = L$, then sequence is:
A) Divergent
B) Convergent
C) Oscillating
D) Constant
Answer: B

Q29. $\lim (n^2+1)/(n^2)$ equals:
A) 0
B) 1
C) ∞
D) 2
Answer: B

Q30. Limit laws apply when sequences are:
A) Bounded
B) Convergent
C) Increasing
D) Decreasing
Answer: B

Q31. $\lim (3/n)$ equals:
A) 3
B) 1
C) 0

D) ∞
Answer: C

Q32. If lim an = 5 then lim (2an) equals:
A) 5
B) 10
C) 2
D) 7
Answer: B

Q33. Limit of sum equals:
A) Sum of limits
B) Product of limits
C) Difference only
D) Undefined
Answer: A

Q34. Limit of product equals:
A) Product of limits
B) Sum of limits
C) Zero
D) Infinite
Answer: A

Q35. If limit is infinite, sequence is:
A) Convergent
B) Divergent
C) Constant
D) Bounded
Answer: B

Monotonic Sequences & Convergence (Q36–Q50)

Q36. Increasing sequence satisfies:
A) an+1 ≥ an
B) an+1 ≤ an
C) an = 0
D) Constant
Answer: A

Q37. Decreasing sequence satisfies:
A) an+1 ≥ an
B) an+1 ≤ an
C) an = 1

D) Oscillatory
Answer: B

Q38. Monotonic sequence means:
A) Oscillating
B) Increasing or decreasing
C) Constant only
D) Random
Answer: B

Q39. Every bounded monotonic sequence is:
A) Divergent
B) Convergent
C) Oscillating
D) Constant
Answer: B

Q40. Sequence an = 1/n is:
A) Increasing
B) Decreasing
C) Oscillating
D) Constant
Answer: B

Q41. an = n is:
A) Increasing
B) Decreasing
C) Constant
D) Oscillating
Answer: A

Q42. Monotone convergence theorem applies to:
A) Functions
B) Sets
C) Sequences
D) Matrices
Answer: C

Q43. Bounded increasing sequence converges to:
A) Infimum
B) Supremum
C) Zero
D) Infinity
Answer: B

Q44. Bounded decreasing sequence converges to:
A) Supremum
B) Infimum
C) Zero
D) Infinity
Answer: B

Q45. Monotonicity helps determine:
A) Limit existence
B) Bounds only
C) Domain
D) Function type
Answer: A

Q46. Oscillating sequence is:
A) Monotonic
B) Non-monotonic
C) Constant
D) Increasing
Answer: B

Q47. Sequence {5,5,5,...} is:
A) Increasing
B) Decreasing
C) Both
D) None
Answer: C

Q48. If an+1 > an for all n, sequence is:
A) Strictly increasing
B) Decreasing
C) Constant
D) Oscillating
Answer: A

Q49. Monotonic sequence without bound is:
A) Convergent
B) Divergent
C) Constant
D) Periodic
Answer: B

Q50. Convergence depends on:
A) Index only
B) Behavior at infinity
C) First term

D) Domain
Answer: B

Algebraic Operations & Cauchy Sequence (Q51–Q65)

Q51. Sum of convergent sequences is:
A) Divergent
B) Convergent
C) Oscillating
D) Infinite
Answer: B

Q52. Product of convergent sequences is:
A) Convergent
B) Divergent
C) Undefined
D) Infinite
Answer: A

Q53. Difference of convergent sequences is:
A) Convergent
B) Divergent
C) Oscillatory
D) Infinite
Answer: A

Q54. Quotient rule holds if denominator limit ≠:
A) 1
B) −1
C) 0
D) ∞
Answer: C

Q55. A Cauchy sequence satisfies:
A) Terms far apart
B) Terms become arbitrarily close
C) Increasing
D) Constant
Answer: B

Q56. Every convergent sequence is:
A) Cauchy
B) Divergent
C) Increasing

D) Constant
Answer: A

Q57. In ℝ, every Cauchy sequence is:
A) Divergent
B) Convergent
C) Oscillating
D) Constant
Answer: B

Q58. Cauchy criterion depends on:
A) Limit value
B) Distance between terms
C) First term
D) Bound only
Answer: B

Q59. $|an - am| \to 0$ implies:
A) Divergence
B) Cauchy sequence
C) Constant
D) Oscillation
Answer: B

Q60. Algebra of limits applies to:
A) Divergent sequences
B) Convergent sequences
C) Oscillatory sequences
D) Infinite sets
Answer: B

Q61. If $an \to L$ and $bn \to M$, then $an + bn \to$
A) LM
B) L + M
C) L − M
D) 0
Answer: B

Q62. If $an \to L$, then $kan \to$
A) k
B) kL
C) L
D) 0
Answer: B

Q63. Limit of negative sequence −an equals:
A) −lim an
B) lim an
C) 0
D) Undefined
Answer: A

Q64. Cauchy sequence definition uses:
A) ε-condition
B) δ-condition
C) Integral
D) Derivative
Answer: A

Q65. Completeness property relates to:
A) Cauchy sequences
B) Monotonic sequences
C) Bounded sets
D) Functions
Answer: A

General Principle of Convergence (Q66–Q75)

Q66. A sequence converges if and only if it is:
A) Bounded
B) Cauchy (in ℝ)
C) Increasing
D) Constant
Answer: B

Q67. General principle connects:
A) Bounds and limits
B) Cauchy and convergence
C) Functions and derivatives
D) Sets and mappings
Answer: B

Q68. Convergent sequence must have:
A) Unique limit
B) Two limits
C) Infinite limits
D) No limit
Answer: A

Q69. Limit of sequence is:
A) Unique
B) Multiple
C) Random
D) Undefined
Answer: A

Q70. Subsequence of convergent sequence:
A) Divergent
B) Convergent to same limit
C) Oscillatory
D) Increasing
Answer: B

Q71. If every subsequence converges to L, sequence is:
A) Divergent
B) Convergent
C) Constant
D) Oscillating
Answer: B

Q72. Bolzano–Weierstrass theorem applies to:
A) Unbounded sequences
B) Bounded sequences
C) Constant sequences
D) Divergent sequences
Answer: B

Q73. Every bounded sequence has:
A) Convergent subsequence
B) Limit always
C) Infinite limit
D) No subsequence
Answer: A

Q74. Convergence depends on:
A) First few terms
B) Tail behavior
C) Domain size
D) Bounds only
Answer: B

Q75. Sequence convergence studies behavior as:
A) $n \to 0$
B) $n \to 1$
C) $n \to \infty$

D) $n \to -\infty$ only
Answer: C

Definition of Series (Q76–Q85)

Q76. A series is defined as:
A) Product of terms
B) Sum of sequence terms
C) Difference of numbers
D) Function mapping
Answer: B

Q77. A series $\sum a_n$ represents:
A) Infinite product
B) Infinite sum
C) Finite sequence
D) Function
Answer: B

Q78. Partial sum S_n equals:
A) a_n
B) $a_1 + a_2 + \ldots + a_n$
C) Limit only
D) Difference of terms
Answer: B

Q79. Convergence of series depends on:
A) Individual terms only
B) Partial sums
C) First term
D) Last term
Answer: B

Q80. Series $\sum a_n$ converges if:
A) $a_n \to \infty$
B) S_n has finite limit
C) Terms increase
D) Terms oscillate
Answer: B

Q81. Necessary condition for convergence is:
A) $a_n \to 1$
B) $a_n \to 0$
C) $a_n \to \infty$
D) a_n constant
Answer: B

Q82. If an ≠ 0 as $n \to \infty$, series is:
A) Convergent
B) Divergent
C) Constant
D) Bounded
Answer: B

Q83. Geometric series is:
A) $\sum ar^n$
B) $\sum a/n$
C) $\sum n^2$
D) $\sum(-1)n$
Answer: A

Q84. Harmonic series is:
A) $\sum 1/n$
B) $\sum 1/n^2$
C) $\sum n$
D) $\sum 2^n$
Answer: A

Q85. Harmonic series is:
A) Convergent
B) Divergent
C) Absolutely convergent
D) Constant
Answer: B

Convergent & Divergent Series (Q86–Q95)

Q86. Series $\sum 1/n^2$ is:
A) Divergent
B) Convergent
C) Oscillatory
D) Undefined
Answer: B

Q87. If partial sums are bounded and increasing, series is:
A) Divergent
B) Convergent
C) Oscillating
D) Infinite
Answer: B

Q88. Divergent series means:
A) Finite sum exists
B) Limit of partial sum does not exist
C) Terms vanish
D) Constant
Answer: B

Q89. Convergent series must satisfy:
A) an → 0
B) an → 1
C) an increases
D) an constant
Answer: A

Q90. Series $\sum 2^n$ is:
A) Convergent
B) Divergent
C) Alternating
D) Harmonic
Answer: B

Q91. p-series $\sum 1/n^p$ converges if:
A) $p \leq 1$
B) $p > 1$
C) $p = 0$
D) $p < 0$
Answer: B

Q92. p-series diverges if:
A) $p > 1$
B) $p \leq 1$
C) $p = 2$
D) $p = 3$
Answer: B

Q93. Series $\sum(-1)^n$ is:
A) Convergent
B) Divergent
C) Absolutely convergent
D) Geometric
Answer: B

Q94. Finite number of terms does not affect:
A) Convergence
B) Divergence
C) Value of limit

D) Series definition
Answer: A

Q95. Convergent series has:
A) Infinite sum
B) Finite sum
C) Zero sum always
D) Undefined sum
Answer: B

Pringsheim's Theorem (Q96–Q100)

Q96. Pringsheim's theorem applies to:
A) Negative series
B) Positive term series
C) Alternating series
D) Complex series
Answer: B

Q97. If partial sums are bounded, positive series is:
A) Divergent
B) Convergent
C) Oscillatory
D) Infinite
Answer: B

Q98. Positive term series has partial sums:
A) Decreasing
B) Increasing
C) Constant
D) Random
Answer: B

Q99. Pringsheim's theorem uses:
A) Monotonicity
B) Derivatives
C) Integrals
D) Matrices
Answer: A

Q100. Increasing bounded sequence of partial sums implies:
A) Divergence
B) Convergence
C) Oscillation

D) Infinity
Answer: B

Comparison Tests (Q101–Q110)

Q101. Comparison test compares with:
A) Known convergent/divergent series
B) Functions
C) Integrals
D) Limits only
Answer: A

Q102. If $0 \leq an \leq bn$ and $\sum bn$ converges, then $\sum an$:
A) Diverges
B) Converges
C) Oscillates
D) Undefined
Answer: B

Q103. If $\sum an$ diverges and $an \geq bn \geq 0$, then $\sum bn$:
A) Converges
B) Diverges
C) Constant
D) Oscillatory
Answer: B

Q104. Comparison test works for:
A) Positive series
B) Alternating series only
C) Negative series
D) Complex series
Answer: A

Q105. Limit comparison test uses:
A) Ratio of terms
B) Sum of terms
C) Difference
D) Integral
Answer: A

Q106. If $\lim an/bn = c$ $(0<c<\infty)$, both series:
A) Behave alike
B) Opposite behavior
C) Divergent always

D) Convergent always
Answer: A

Q107. Comparison is useful when series resembles:
A) p-series
B) Constant series
C) Finite sum
D) Polynomial only
Answer: A

Q108. $\sum 1/(n^2+1)$ converges by comparison with:
A) $\sum 1/n$
B) $\sum 1/n^2$
C) $\sum n$
D) $\sum 2^n$
Answer: B

Q109. Comparison test requires terms:
A) Positive
B) Negative
C) Complex
D) Oscillatory
Answer: A

Q110. Direct comparison checks:
A) Inequalities
B) Derivatives
C) Integrals
D) Limits only
Answer: A

Cauchy Root Test (Q111–Q120)

Q111. Root test uses:
A) nth root of an
B) Ratio
C) Difference
D) Sum
Answer: A

Q112. $\limsup \sqrt[n]{|a_n|} = L < 1$ implies:
A) Divergence
B) Convergence
C) Oscillation

D) Infinity
Answer: B

Q113. L > 1 implies:
A) Convergence
B) Divergence
C) Constant
D) Undefined
Answer: B

Q114. L = 1 gives:
A) Convergent
B) Divergent
C) Inconclusive
D) Zero
Answer: C

Q115. Root test useful for:
A) Exponential terms
B) Linear terms
C) Constants
D) Polynomials only
Answer: A

Q116. Root test is also called:
A) Cauchy test
B) Ratio test
C) Integral test
D) Leibnitz test
Answer: A

Q117. Series $\sum(1/2)^n$ converges since root limit is:
A) 2
B) 1
C) 1/2
D) 0
Answer: C

Q118. Root test applies to:
A) Infinite series
B) Finite series
C) Functions
D) Sets
Answer: A

Q119. nth root measures:
A) Growth rate
B) Difference
C) Sum
D) Bound
Answer: A

Q120. Root test depends on:
A) Limit superior
B) Integral
C) Derivative
D) Bound only
Answer: A

D'Alembert Ratio Test (Q121–Q130)

Q121. Ratio test evaluates:
A) an+1/an
B) an − an+1
C) √an
D) Sum
Answer: A

Q122. If limit ratio < 1, series:
A) Diverges
B) Converges
C) Oscillates
D) Infinite
Answer: B

Q123. Ratio > 1 implies:
A) Convergence
B) Divergence
C) Constant
D) Zero
Answer: B

Q124. Ratio = 1 gives:
A) Convergent
B) Divergent
C) Inconclusive
D) Zero
Answer: C

Q125. Ratio test works best for:
A) Factorials
B) Linear terms
C) Constants
D) Polynomials
Answer: A

Q126. Series ∑1/n fails ratio test because limit equals:
A) 0
B) 1
C) 2
D) ∞
Answer: B

Q127. Ratio test compares successive terms:
A) True
B) False
Answer: A

Q128. Ratio test introduced by:
A) Cauchy
B) D'Alembert
C) Newton
D) Euler
Answer: B

Q129. Ratio test determines:
A) Absolute convergence
B) Only divergence
C) Bound
D) Limit only
Answer: A

Q130. Ratio test applies mainly to:
A) Positive series
B) Negative series
C) Finite sums
D) Functions
Answer: A

Alternating Series & Leibnitz Test (Q131–Q140)

Q131. Alternating series contains terms:
A) Same sign

B) Changing sign
C) Positive only
D) Negative only
Answer: B

Q132. Example of alternating series:
A) $\sum(-1)^n/n$
B) $\sum 1/n$
C) $\sum n$
D) $\sum 2^n$
Answer: A

Q133. Leibnitz test requires bn:
A) Increasing
B) Decreasing
C) Constant
D) Oscillating
Answer: B

Q134. Also bn →
A) 1
B) 0
C) ∞
D) −1
Answer: B

Q135. If Leibnitz conditions hold, series:
A) Converges
B) Diverges
C) Infinite
D) Constant
Answer: A

Q136. Alternating harmonic series is:
A) Divergent
B) Convergent
C) Absolutely convergent
D) Infinite
Answer: B

Q137. Leibnitz test ensures:
A) Conditional convergence
B) Absolute convergence
C) Divergence
D) Infinity
Answer: A

Q138. Error in alternating series bounded by:
A) Next term
B) Sum
C) Difference
D) Limit
Answer: A

Q139. Alternating series may converge even if:
A) Absolute series diverges
B) Terms constant
C) Infinite terms
D) Increasing terms
Answer: A

Q140. Sign alternation occurs due to factor:
A) $(-1)^n$
B) n^2
C) n
D) $1/n^2$
Answer: A

Absolutely Convergent Series (Q141–Q150)

Q141. Absolute convergence means:
A) $\sum|a_n|$ converges
B) $\sum a_n$ diverges
C) Terms constant
D) Oscillation
Answer: A

Q142. Absolute convergence implies:
A) Divergence
B) Convergence
C) Oscillation
D) Infinity
Answer: B

Q143. Converse is:
A) Always true
B) Not always true
C) Impossible
D) Undefined
Answer: B

Q144. $\sum(-1)^n/n^2$ is:
A) Absolutely convergent
B) Divergent
C) Harmonic
D) Oscillatory
Answer: A

Q145. Absolute convergence stronger than:
A) Conditional convergence
B) Divergence
C) Oscillation
D) Bound
Answer: A

Q146. Rearrangement of absolutely convergent series:
A) Changes sum
B) Does not change sum
C) Diverges
D) Oscillates
Answer: B

Q147. Absolute convergence tested by:
A) Applying tests to |an|
B) Ignoring signs
C) Integration
D) Derivatives
Answer: A

Q148. If ∑|an| diverges but ∑an converges, series is:
A) Absolutely convergent
B) Conditionally convergent
C) Divergent
D) Constant
Answer: B

Q149. Alternating harmonic series is:
A) Absolutely convergent
B) Conditionally convergent
C) Divergent
D) Constant
Answer: B

Q150. Absolute convergence guarantees:
A) Stability of sum
B) Divergence
C) Oscillation

D) Infinite value
Answer: A

MCQs (151–225)

Binary Operations (Q151–Q160)

Q151. A binary operation on a set A is a function from:
A) A → A
B) A×A → A
C) A → B
D) B×A → A
Answer: B

Q152. Closure property means:
A) Result belongs to another set
B) Result belongs to same set
C) Operation undefined
D) Result constant
Answer: B

Q153. Addition on integers is:
A) Not binary
B) Binary operation
C) Partial operation
D) Unary operation
Answer: B

Q154. Binary operation requires:
A) One element
B) Two elements
C) Three elements
D) Infinite elements
Answer: B

Q155. Multiplication on real numbers is:
A) Binary operation
B) Unary operation
C) Not defined
D) Partial
Answer: A

Q156. Subtraction on natural numbers is:
A) Closed

B) Not closed
C) Associative
D) Identity operation
Answer: B

Q157. Associative law states:
A) a*(bc) = (ab)c
B) ab = ba
C) aa = a
D) a+0 = a
Answer: A

Q158. Commutative property means:
A) ab = ba
B) (a*b)*c
C) a = b
D) a^{-1} exists
Answer: A

Q159. Division on integers is:
A) Binary operation
B) Not binary operation
C) Associative
D) Closed
Answer: B

Q160. Set with binary operation is called:
A) Algebraic structure
B) Function
C) Matrix
D) Relation
Answer: A

Notion of Group (Q161–Q170)

Q161. A group must satisfy how many axioms?
A) 2
B) 3
C) 4
D) 5
Answer: C

Q162. Group axioms include:
A) Closure

B) Associativity
C) Identity & Inverse
D) All of these
Answer: D

Q163. Identity element satisfies:
A) ae = a
B) ea = a
C) Both
D) None
Answer: C

Q164. In group (G,*), inverse of a satisfies:*
*A) a*a = e
B) $a*a^{-1} = e$
C) a+e=a
D) a=b
Answer: B

Q165. Integers under addition form:
A) Group
B) Not group
C) Field
D) Ring only
Answer: A

Q166. Natural numbers under addition form:
A) Group
B) Not group
C) Abelian group
D) Cyclic group
Answer: B

Q167. Associativity is:
A) Optional
B) Necessary for group
C) Derived property
D) Not required
Answer: B

Q168. Identity element in addition is:
A) 1
B) 0
C) −1
D) ∞
Answer: B

Q169. Identity in multiplication is:
A) 0
B) 1
C) −1
D) 2
Answer: B

Q170. A group having finite elements is:
A) Infinite group
B) Finite group
C) Abelian group
D) Cyclic group
Answer: B

Abelian & Non-Abelian Groups (Q171–Q180)

Q171. Abelian group satisfies:
A) $ab=ba$
B) Associativity only
C) Closure only
D) Inverse only
Answer: A

Q172. Integers under addition form:
A) Non-abelian group
B) Abelian group
C) Not group
D) Semigroup
Answer: B

Q173. Matrix multiplication is generally:
A) Abelian
B) Non-abelian
C) Identity
D) Cyclic
Answer: B

Q174. Non-abelian group means:
A) $ab=ba$ always
B) $ab\neq ba$ for some elements
C) No identity
D) No inverse
Answer: B

Q175. Real numbers under addition are:
A) Abelian group
B) Non-group
C) Non-abelian
D) Semigroup
Answer: A

Q176. Symmetric group Sn ($n \geq 3$) is:
A) Abelian
B) Non-abelian
C) Cyclic
D) Infinite
Answer: B

Q177. Every cyclic group is:
A) Non-abelian
B) Abelian
C) Not group
D) Infinite only
Answer: B

Q178. Commutativity property defines:
A) Abelian group
B) Subgroup
C) Coset
D) Identity
Answer: A

Q179. Example of non-abelian group:
A) (Z,+)
B) Matrix group
C) (R,+)
D) (Q,+)
Answer: B

Q180. Abelian groups are named after:
A) Euler
B) Abel
C) Gauss
D) Newton
Answer: B

Uniqueness of Identity & Inverse (Q181–Q190)

Q181. Identity element in a group is:
A) Unique
B) Multiple
C) Infinite
D) Undefined
Answer: A

Q182. If e and e′ are identities, then:
A) $e \neq e'$
B) $e = e'$
C) Both exist separately
D) None
Answer: B

Q183. Every element has:
A) One inverse
B) Two inverses
C) Infinite inverses
D) No inverse
Answer: A

Q184. Inverse of identity element is:
A) Same identity
B) Zero
C) Undefined
D) Infinite
Answer: A

Q185. If a^{-1} exists, then:
A) $aa^{-1}=e$
B) $a^{-1}a=e$
C) Both
D) None
Answer: C

Q186. Inverse is unique because of:
A) Associativity
B) Closure
C) Identity only
D) Commutativity
Answer: A

Q187. If a*b=e, then b is:
A) Identity
B) Inverse of a
C) Constant

D) Zero
Answer: B

Q188. Double inverse equals:
A) Identity
B) Element itself
C) Zero
D) Undefined
Answer: B

Q189. $(a^{-1})^{-1}$ equals:
A) e
B) a
C) 0
D) a^2
Answer: B

Q190. Identity works for:
A) All elements
B) Some elements
C) One element
D) None
Answer: A

Different Ways of Defining Group (Q191–Q200)

Q191. Group may be defined using:
A) Axioms
B) Operation tables
C) Transformations
D) All of these
Answer: D

Q192. Cayley table represents:
A) Function
B) Binary operation
C) Matrix
D) Relation
Answer: B

Q193. Closure checked using:
A) Table entries
B) Identity
C) Inverse

D) Limit
Answer: A

Q194. Finite groups often defined by:
A) Cayley table
B) Limits
C) Integrals
D) Series
Answer: A

Q195. Transformation groups involve:
A) Functions mapping set to itself
B) Numbers only
C) Matrices only
D) Vectors
Answer: A

Q196. Group operation must be:
A) Binary
B) Unary
C) Ternary
D) Undefined
Answer: A

Q197. Group defined axiomatically uses:
A) Properties
B) Examples only
C) Numbers
D) Graphs
Answer: A

Q198. Identity verified by:
A) ae=ea=a
B) a+a=a
C) a=e
D) None
Answer: A

Q199. Associativity must hold for:
A) All triples
B) Some elements
C) Identity only
D) Inverse only
Answer: A

Q200. Operation table must show:
A) Closure
B) Associativity
C) Identity
D) All indirectly
Answer: D

Subgroups & Cyclic Groups (Q201–Q210)

Q201. Subgroup is:
A) Subset forming group
B) Any subset
C) Empty set
D) Infinite set
Answer: A

Q202. Subgroup must contain:
A) Identity
B) All elements
C) Zero only
D) Limit
Answer: A

Q203. Non-empty subset closed under operation and inverse is:
A) Subgroup
B) Coset
C) Ring
D) Field
Answer: A

Q204. Cyclic group generated by element a is:
A) ⟨a⟩
B) {a}
C) G(a)
D) (a)
Answer: A

Q205. Cyclic group elements are:
A) Powers of generator
B) Random elements
C) Identity only
D) Infinite only
Answer: A

Q206. Every cyclic group is:
A) Abelian
B) Non-abelian
C) Not group
D) Infinite
Answer: A

Q207. Integers under addition form:
A) Cyclic group
B) Non-cyclic
C) Non-group
D) Finite group
Answer: A

Q208. Generator produces:
A) Entire group
B) Subgroup only
C) Identity only
D) Random elements
Answer: A

Q209. Order of cyclic group equals:
A) Number of generated elements
B) Identity only
C) Infinite only
D) None
Answer: A

Q210. Finite cyclic group has:
A) At least one generator
B) No generator
C) Infinite generators only
D) None
Answer: A

Cosets & Lagrange's Theorem (Q211–Q225)

Q211. Left coset of H in G is:
A) aH
B) Ha
C) H^2
D) $a^{-1}H$
Answer: A

Q212. Right coset is:
A) aH
B) Ha
C) H^{-1}
D) a^2H
Answer: B

Q213. Cosets partition the group into:
A) Equal size subsets
B) Unequal subsets
C) Infinite subsets
D) Empty sets
Answer: A

Q214. Number of cosets equals:
A) Order of subgroup
B) Index of subgroup
C) Identity
D) Generator
Answer: B

Q215. Lagrange's theorem states:
A) |H| divides |G|
B) |G| divides |H|
C) Orders equal
D) No relation
Answer: A

Q216. Order of subgroup divides:
A) Identity
B) Order of group
C) Generator
D) Coset
Answer: B

Q217. Index of H in G equals:
A) |G|/|H|
B) |H|/|G|
C) |G|+|H|
D) |G|−|H|
Answer: A

Q218. Cosets of subgroup are:
A) Disjoint
B) Overlapping
C) Equal always

D) Infinite only
Answer: A

Q219. Each coset has elements equal to:
A) |G|
B) |H|
C) Identity
D) Generator
Answer: B

Q220. Lagrange theorem applies to:
A) Finite groups
B) Infinite groups only
C) Functions
D) Fields
Answer: A

Q221. If |G| is prime, subgroup order can be:
A) 1 or |G|
B) Any number
C) Even numbers
D) Odd numbers only
Answer: A

Q222. Proper subgroup excludes:
A) Identity
B) Whole group
C) Closure
D) Inverse
Answer: B

Q223. Identity coset equals:
A) H
B) G
C) Empty set
D) Singleton
Answer: A

Q224. Distinct cosets are:
A) Equal
B) Disjoint
C) Infinite
D) Random
Answer: B

Q225. Lagrange theorem helps determine:
A) Possible subgroup orders
B) Limits
C) Integrals
D) Series
Answer: A

Differential Equations – Basic Concepts (Q226–Q235)

Q226. A differential equation contains:
A) Only algebraic terms
B) Derivatives
C) Integrals only
D) Constants only
Answer: B

Q227. Order of a differential equation is determined by:
A) Power of derivative
B) Highest derivative
C) Degree of variable
D) Number of constants
Answer: B

Q228. Degree of differential equation is:
A) Highest derivative order
B) Power of highest derivative
C) Number of variables
D) Number of solutions
Answer: B

Q229. Equation $dy/dx + y = 0$ is of order:
A) 2
B) 1
C) 0
D) 3
Answer: B

Q230. A differential equation involving one independent variable is:
A) Partial differential equation
B) Ordinary differential equation
C) Algebraic equation
D) Integral equation
Answer: B

Q231. General solution contains:
A) No constant

B) One arbitrary constant
C) Arbitrary constants equal to order
D) Infinite constants
Answer: C

Q232. Particular solution is obtained by:
A) Removing constants
B) Assigning values to constants
C) Differentiation
D) Integration only
Answer: B

Q233. $dy/dx = f(x)$ represents:
A) First-order equation
B) Second-order equation
C) Algebraic equation
D) Linear algebra
Answer: A

Q234. Highest derivative determines:
A) Degree
B) Order
C) Solution
D) Variable
Answer: B

Q235. Equation containing d^2y/dx^2 is of order:
A) 1
B) 2
C) 3
D) 0
Answer: B

First Order & Higher Degree Equations (Q236–Q245)

Q236. First-order higher-degree equation example:
A) $(dy/dx)^2 + y = 0$
B) $d^2y/dx^2 + y = 0$
C) $dy/dx + y = 0$
D) $y^2 + x = 0$
Answer: A

Q237. Degree is defined only when equation is:
A) Polynomial in derivatives

B) Linear
C) Homogeneous
D) Continuous
Answer: A

Q238. Equation $\sqrt{(dy/dx)} + y = 0$ has:
A) Defined degree
B) Undefined degree
C) Order zero
D) Linear degree
Answer: B

Q239. Removing radicals makes degree:
A) Undefined
B) Defined
C) Zero
D) Infinite
Answer: B

Q240. First-order equation involves derivative:
A) dy/dx only
B) d^2y/dx^2
C) d^3y/dx^3
D) None
Answer: A

Q241. Higher degree refers to:
A) Power of derivative >1
B) Order >1
C) Variable power
D) Constant
Answer: A

Q242. Equation $(dy/dx)^3 + y = x$ is degree:
A) 1
B) 2
C) 3
D) 0
Answer: C

Q243. Differential equations describe:
A) Static relations
B) Rates of change
C) Constants
D) Sets
Answer: B

Q244. Independent variable is usually:
A) y
B) x
C) Constant
D) Parameter
Answer: B

Q245. Dependent variable changes with:
A) Constant
B) Independent variable
C) Zero
D) Identity
Answer: B

Clairaut's Form (Q246–Q255)

Q246. Clairaut's equation is of form:
A) $y = px + f(p)$
B) $y = x^2$
C) $dy/dx = x+y$
D) $y''+y=0$
Answer: A

Q247. Here p denotes:
A) Constant
B) dy/dx
C) x
D) y
Answer: B

Q248. General solution of Clairaut's equation obtained by:
A) Treating p constant
B) Differentiation twice
C) Integration directly
D) Substitution only
Answer: A

Q249. Clairaut's equation gives family of:
A) Straight lines
B) Circles
C) Parabolas
D) Ellipses
Answer: A

Q250. Singular solution represents:
A) Envelope of family
B) Single line
C) Constant solution
D) Divergent solution
Answer: A

Q251. Differentiating Clairaut equation gives relation between:
A) x and y
B) p and x
C) p and constants
D) Variables only
Answer: B

Q252. Singular solution obtained by eliminating:
A) x
B) y
C) p
D) Constant
Answer: C

Q253. Clairaut equation is:
A) First order, first degree
B) Second order
C) Linear second order
D) Algebraic
Answer: A

Q254. Singular solution may not belong to:
A) General solution
B) Differential equation
C) Function
D) Curve
Answer: A

Q255. Clairaut equation involves parameter:
A) $p = dy/dx$
B) x^2
C) y^2
D) Constant only
Answer: A

Singular Solution (Q256–Q265)

Q256. Singular solution satisfies:
A) Differential equation only
B) General solution
C) Both always
D) None
Answer: A

Q257. Singular solution is obtained from:
A) ∂y/∂c = 0 type condition
B) Integration
C) Differentiation only
D) Limits
Answer: A

Q258. Singular solution represents:
A) Envelope curve
B) Straight line
C) Constant function
D) Infinite curve
Answer: A

Q259. Singular solution touches:
A) Every member of family
B) No curve
C) One curve only
D) Axis
Answer: A

Q260. General solution contains:
A) No parameter
B) Arbitrary constant
C) Fixed constant
D) Zero
Answer: B

Q261. Singular solution removes:
A) Parameter
B) Variable
C) Function
D) Constant term
Answer: A

Q262. Envelope concept used in:
A) Singular solutions
B) Linear equations
C) Algebra

D) Geometry only
Answer: A

Q263. Singular solution may be:
A) Unique
B) Infinite
C) Constant
D) Undefined
Answer: A

Q264. Not all differential equations have:
A) General solution
B) Singular solution
C) Variables
D) Derivatives
Answer: B

Q265. Singular solution obtained by eliminating constant using:
A) Differentiation
B) Integration
C) Addition
D) Subtraction
Answer: A

Orthogonal Trajectories (Q266–Q275)

Q266. Orthogonal trajectories intersect at angle:
A) 30°
B) 45°
C) 90°
D) 60°
Answer: C

Q267. Slopes of orthogonal curves satisfy:
A) $m_1 m_2 = -1$
B) $m_1+m_2=1$
C) $m_1=m_2$
D) $m_1-m_2=0$
Answer: A

Q268. Orthogonal trajectories obtained by replacing dy/dx with:
A) −dx/dy
B) dx/dy
C) dy/dx

D) x/y
Answer: A

Q269. Orthogonal curves are:
A) Parallel
B) Perpendicular
C) Tangent
D) Same
Answer: B

Q270. Family of circles may have trajectories as:
A) Lines
B) Another circle family
C) Parabolas
D) Hyperbolas only
Answer: B

Q271. Orthogonal trajectories form:
A) Same family
B) New family
C) Constant curve
D) Straight line always
Answer: B

Q272. Negative reciprocal slope ensures:
A) Parallelism
B) Orthogonality
C) Equality
D) Divergence
Answer: B

Q273. Used in physics for:
A) Field lines
B) Algebra
C) Geometry only
D) Statistics
Answer: A

Q274. Electric field and equipotential lines are:
A) Parallel
B) Orthogonal
C) Same
D) Random
Answer: B

Q275. Procedure involves differentiating:
A) Given family
B) Constant
C) Variable only
D) Identity
Answer: A

Linear Differential Equations with Constant Coefficients (Q276–Q300)

Q276. Linear differential equation form:
A) $a_0y''+a_1y'+a_2y = f(x)$
B) $y^2+x=0$
C) xy=1
D) $y^3=0$
Answer: A

Q277. Constant coefficients mean:
A) Functions of x
B) Fixed numbers
C) Variables
D) Parameters
Answer: B

Q278. Auxiliary equation obtained by:
A) Replacing D by m
B) Integration
C) Addition
D) Substitution
Answer: A

Q279. Complementary function depends on:
A) Auxiliary roots
B) RHS
C) Constant only
D) Integration
Answer: A

Q280. Particular integral depends on:
A) Homogeneous part
B) RHS function
C) Roots only
D) Constants
Answer: B

Q281. Equation $D^2y+y=0$ has auxiliary equation:
A) $m^2+1=0$
B) $m+1=0$
C) $m^2-1=0$
D) $m=0$
Answer: A

Q282. Roots $\pm i$ give solution:
A) Exponential
B) Trigonometric
C) Polynomial
D) Constant
Answer: B

Q283. Distinct real roots give solution:
A) $e^{m_1 x}$, $e^{m_2 x}$
B) sin x
C) Polynomial
D) Constant
Answer: A

Q284. Equal roots produce solution:
A) $(C_1+C_2x)e^{mx}$
B) sin x
C) cos x
D) Constant
Answer: A

Q285. Complex roots $\alpha\pm i\beta$ give solution:
A) $e^{\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$
B) Polynomial
C) Constant
D) Rational
Answer: A

Q286. Complementary function solves:
A) Homogeneous equation
B) Non-homogeneous only
C) Algebraic equation
D) Matrix equation
Answer: A

Q287. General solution equals:
A) CF + PI
B) CF − PI
C) PI only

D) CF only
Answer: A

Q288. Operator D denotes:
A) d/dx
B) Integral
C) Constant
D) Variable
Answer: A

Q289. Linear equation principle is:
A) Superposition
B) Closure
C) Identity
D) Associativity
Answer: A

Q290. RHS = e^{ax} suggests PI form:
A) Ae^{ax}
B) Ax
C) sin x
D) Constant
Answer: A

Q291. RHS = sin ax gives PI containing:
A) sin ax & cos ax
B) Polynomial
C) Exponential only
D) Constant
Answer: A

Q292. Linear equations model:
A) Physical systems
B) Sets
C) Geometry only
D) Statistics only
Answer: A

Q293. Homogeneous equation RHS equals:
A) 0
B) 1
C) x
D) y
Answer: A

Q294. Non-homogeneous equation RHS is:
A) Non-zero
B) Zero
C) Constant only
D) Undefined
Answer: A

Q295. Solution space dimension equals:
A) Order of equation
B) Degree
C) Constant
D) Variable
Answer: A

Q296. Linear DE satisfies:
A) Superposition property
B) Closure law
C) Commutativity
D) Identity law
Answer: A

Q297. Auxiliary equation roots determine:
A) CF
B) PI
C) RHS
D) Order
Answer: A

Q298. Particular integral is one:
A) Specific solution
B) General solution
C) Singular solution
D) Constant
Answer: A

Q299. Linear DE with constant coefficients solved using:
A) Operator method
B) Matrix only
C) Graph only
D) Limit method
Answer: A

Q300. Final solution includes:
A) Arbitrary constants
B) No constants
C) Variables only

D) Identity only
Answer: A

Homogeneous Linear Equations with Variable Coefficients (Q301–Q320)

Q301. A homogeneous linear differential equation has RHS equal to:
A) 1
B) x
C) 0
D) y
Answer: C

Q302. Variable coefficient means coefficients depend on:
A) Constants
B) Variable (x)
C) Identity
D) Zero
Answer: B

Q303. Example of variable coefficient equation:
A) $x^2y'' + xy' + y = 0$
B) $y'' + y = 0$
C) $dy/dx = 0$
D) $y = x$
Answer: A

Q304. Cauchy–Euler equation is:
A) Constant coefficient equation
B) Variable coefficient equation
C) Algebraic equation
D) Integral equation
Answer: B

Q305. Standard substitution for Euler equation is:
A) $x = e^t$
B) $y = e^x$
C) $x = y$
D) $y = x^2$
Answer: A

Q306. Euler equation general form:
A) $x^2y'' + axy' + by = 0$
B) $y'' + ay' + by = 0$
C) $dy/dx = y$
D) $y = ax$
Answer: A

Q307. Auxiliary equation for Euler equation obtained by assuming:
A) $y = x^m$
B) $y = e^x$
C) $y = \sin x$
D) y = constant
Answer: A

Q308. If roots are distinct real, solution contains:
A) x^{m1}, x^{m2}
B) $\sin x$
C) Polynomial only
D) Constant
Answer: A

Q309. Equal roots give solution:
A) $C_1x^m + C_2x^m \ln x$
B) $\sin x$
C) e^x
D) Constant
Answer: A

Q310. Complex roots produce solution involving:
A) Trigonometric functions
B) Polynomial
C) Constant
D) Rational function
Answer: A

Q311. Homogeneous equation means:
A) RHS zero
B) RHS constant
C) RHS variable
D) Nonlinear
Answer: A

Q312. Order equals highest:
A) Power
B) Derivative
C) Constant
D) Variable
Answer: B

Q313. Euler equation models:
A) Scale-invariant systems
B) Algebra only
C) Geometry only

D) Statistics
Answer: A

Q314. Transformation reduces Euler equation to:
A) Constant coefficient equation
B) Algebraic equation
C) Linear equation
D) Identity
Answer: A

Q315. Solution space dimension equals:
A) Degree
B) Order
C) Variable
D) Constant
Answer: B

Q316. Linear independence needed for:
A) General solution
B) Particular solution
C) Identity
D) Constant
Answer: A

Q317. Homogeneous solutions satisfy superposition:
A) True
B) False
Answer: A

Q318. Variable coefficients complicate solution because:
A) Non-constant behavior
B) Constant terms
C) Identity
D) Zero derivative
Answer: A

Q319. Euler equation belongs to:
A) Linear DE
B) Nonlinear DE
C) Algebra
D) Matrix theory
Answer: A

Q320. Substitution converts equation into:
A) Simpler form
B) Harder form

C) Algebraic identity
D) Constant equation
Answer: A

Simultaneous Differential Equations (Q321–Q335)

Q321. Simultaneous differential equations involve:
A) One variable
B) Two or more dependent variables
C) Constants only
D) Algebraic relations
Answer: B

Q322. Example includes equations in:
A) x and y only
B) x, y, z functions
C) Constants
D) Numbers
Answer: B

Q323. Solution requires:
A) Elimination method
B) Addition only
C) Integration only
D) Limits
Answer: A

Q324. System written as dx/dt and dy/dt represents:
A) Simultaneous DE
B) Algebraic system
C) Matrix equation
D) Identity
Answer: A

Q325. Simultaneous equations describe:
A) Interacting systems
B) Independent systems
C) Constants
D) Geometry only
Answer: A

Q326. Elimination reduces system to:
A) Single differential equation
B) Algebraic equation

C) Identity
D) Constant
Answer: A

Q327. Order determined by:
A) Highest derivative present
B) Number of equations
C) Variables
D) Constants
Answer: A

Q328. Linear simultaneous equations obey:
A) Superposition principle
B) Closure law
C) Identity law
D) Commutativity
Answer: A

Q329. Solution gives:
A) Relation between variables
B) Constant only
C) Identity
D) Zero
Answer: A

Q330. Used in modeling:
A) Population dynamics
B) Algebra only
C) Geometry only
D) Statistics only
Answer: A

Q331. Independent variable often is:
A) t
B) y
C) Constant
D) Identity
Answer: A

Q332. Coupled equations mean variables are:
A) Independent
B) Interdependent
C) Constant
D) Zero
Answer: B

Q333. Matrix methods help solve:
A) Linear simultaneous DE
B) Nonlinear only
C) Algebraic only
D) Geometry only
Answer: A

Q334. Eigenvalues appear in solving:
A) Linear systems
B) Algebraic equations
C) Integrals
D) Limits
Answer: A

Q335. General solution contains:
A) Arbitrary constants
B) No constants
C) Variables only
D) Identity
Answer: A

Equations of the Form dx/P = dy/Q = dz/R (Q336–Q355)

Q336. Equation dx/P = dy/Q = dz/R represents:
A) Lagrange's auxiliary equations
B) Algebraic relation
C) Linear equation
D) Integral equation
Answer: A

Q337. Used for solving:
A) First-order PDE auxiliary system
B) Algebraic equations
C) Series
D) Limits
Answer: A

Q338. Method involves equating ratios to:
A) Parameter
B) Constant
C) Zero
D) Identity
Answer: A

Q339. Solutions obtained by:
A) Integrating pairs
B) Differentiation only
C) Addition
D) Subtraction
Answer: A

Q340. Number of independent integrals required:
A) One
B) Two
C) Three
D) Infinite
Answer: B

Q341. Auxiliary equations represent:
A) Characteristic curves
B) Straight lines only
C) Constants
D) Algebraic curves
Answer: A

Q342. P, Q, R are functions of:
A) x,y,z
B) Constants
C) Identity
D) Limits
Answer: A

Q343. Equality of ratios implies motion along:
A) Curve
B) Point
C) Plane only
D) Axis
Answer: A

Q344. Integrating dx/P = dy/Q gives:
A) First integral
B) Constant
C) Identity
D) Zero
Answer: A

Q345. Final solution expressed as:
A) F(u,v)=0
B) x+y=0
C) Constant

D) Polynomial
Answer: A

Q346. Auxiliary equations convert PDE into:
A) ODE system
B) Algebraic system
C) Matrix system
D) Identity
Answer: A

Q347. Solution curves are called:
A) Characteristics
B) Constants
C) Variables
D) Limits
Answer: A

Q348. Method simplifies solving by:
A) Parameterization
B) Addition
C) Subtraction
D) Expansion
Answer: A

Q349. Equal ratios imply:
A) Direction field
B) Constant field
C) Identity
D) Zero
Answer: A

Q350. Used mainly in:
A) Partial differential equations
B) Algebra
C) Geometry only
D) Statistics
Answer: A

Q351. Integration yields:
A) Two independent relations
B) One relation
C) None
D) Infinite relations
Answer: A

Q352. Solutions represent:
A) Space curves
B) Points only
C) Lines only
D) Planes only
Answer: A

Q353. Auxiliary equations describe:
A) Direction ratios
B) Constants
C) Limits
D) Integrals
Answer: A

Q354. Parameter elimination gives:
A) Required solution
B) Identity
C) Constant
D) Zero
Answer: A

Q355. Method based on equality of:
A) Derivative ratios
B) Integrals
C) Constants
D) Limits
Answer: A

Total Differential Equations & Geometrical Significance (Q356–Q375)

Q356. Total differential equation form is:
A) $Pdx + Qdy + Rdz = 0$
B) $dy/dx=y$
C) $y''=0$
D) $x+y=0$
Answer: A

Q357. Equation is exact if:
A) Total differential exists
B) Constant exists
C) Variable absent
D) Identity holds
Answer: A

Q358. Exactness condition involves equality of:
A) Mixed partial derivatives
B) Variables
C) Constants
D) Limits
Answer: A

Q359. If exact, equation equals:
A) dF = 0
B) F = 1
C) Constant only
D) Identity
Answer: A

Q360. Solution becomes:
A) F(x,y,z)=C
B) x+y=0
C) Constant
D) Polynomial
Answer: A

Q361. Total differential represents:
A) Change in scalar function
B) Constant
C) Limit
D) Identity
Answer: A

Q362. Geometrically solution represents:
A) Surface
B) Line
C) Point
D) Circle only
Answer: A

Q363. Gradient vector normal to:
A) Surface
B) Curve
C) Line only
D) Axis
Answer: A

Q364. P,Q,R act as components of:
A) Gradient vector
B) Constant vector
C) Identity

D) Zero vector
Answer: A

Q365. Equation defines:
A) Family of surfaces
B) Points
C) Lines only
D) Constants
Answer: A

Q366. Orthogonality related to:
A) Gradient direction
B) Constant
C) Variable
D) Identity
Answer: A

Q367. Exact DE ensures path independence of:
A) Integral
B) Constant
C) Variable
D) Limit
Answer: A

Q368. Integrating factor used when equation is:
A) Not exact
B) Exact
C) Linear
D) Constant
Answer: A

Q369. Total differential equals:
A) df
B) f^2
C) Constant
D) Zero always
Answer: A

Q370. Solution surfaces intersect curves:
A) Tangentially
B) Randomly
C) Parallel
D) None
Answer: A

Q371. Geometrical meaning linked with:
A) Level surfaces
B) Algebra
C) Series
D) Limits
Answer: A

Q372. Exact DE corresponds to:
A) Conservative field
B) Random field
C) Constant field
D) Zero field
Answer: A

Q373. Scalar potential exists when equation is:
A) Exact
B) Nonlinear
C) Divergent
D) Oscillating
Answer: A

Q374. Gradient is perpendicular to:
A) Level surface
B) Axis
C) Line
D) Origin
Answer: A

Q375. Geometric interpretation connects DE with:
A) Surfaces and fields
B) Algebra only
C) Statistics
D) Numbers
Answer: A

UNIT I – REAL ANALYSIS

Long Questions (15 Marks) with Answers

Q1. Define a sequence. Explain bounded sequence with example.

Answer:

A **sequence** is a function f:N→Rf : \mathbb{N} \rightarrow \mathbb{R}f:N→R.
It is denoted by {an}\{a_n\}{an}, where an=f(n)a_n = f(n)an=f(n).

Bounded Sequence

A sequence {an}\{a_n\}{an} is bounded if there exists M>0M>0M>0 such that:

|an|≤M∀n∈N|a_n| \le M \quad \forall n \in \mathbb{N}|an|≤M∀n∈N

Example:

an=1na_n=\frac{1}{n}an=n1

Since:

0<1n≤10<\frac{1}{n}\le10<n1≤1

Hence bounded above by 1 and below by 0.

Q2. Prove that every convergent sequence is bounded.

Proof:

Let an→La_n \to Lan→L.

Then for ϵ=1\epsilon =1ϵ=1, there exists NNN such that:

|an−L|<1(n≥N)|a_n-L|<1 \quad (n\ge N)|an−L|<1(n≥N)

So,

|an|<|L|+1|a_n|<|L|+1|an|<|L|+1

First finite terms are bounded.
Hence entire sequence bounded.

✓ Every convergent sequence is bounded.

Q3. Define limit of a sequence using ε-definition.

Answer:

A sequence ana_nan converges to LLL if:

∀ϵ>0, ∃N∈N\forall \epsilon>0,\ \exists N \in \mathbb{N}∀ϵ>0, ∃N∈N

such that

|an−L|<ϵwhenever n≥N.|a_n-L|<\epsilon \quad \text{whenever } n\ge N.|an−L|<ϵwhenever n≥N.

Symbolically:

lim⁡n→∞an=L\lim_{n\to\infty} a_n=Ln→∞liman=L

Q4. Show that lim⁡n→∞1n=0\lim_{n\to\infty}\frac{1}{n}=0limn→∞n1=0.

Proof:

Let ϵ>0\epsilon>0ϵ>0.

Choose N>1ϵN>\frac{1}{\epsilon}N>ϵ1.

Then for n≥Nn\ge Nn≥N:

|1n−0|=1n<ϵ\left|\frac{1}{n}-0\right|=\frac{1}{n}<\epsilonn1−0=n1<ϵ

Hence,

lim⁡1n=0.\lim \frac{1}{n}=0.limn1=0.

Q5. Define monotonic sequence with examples.

Answer:

Increasing sequence:
an+1≥ana_{n+1}\ge a_nan+1≥an

Example: an=na_n=nan=n

Decreasing sequence:
an+1≤ana_{n+1}\le a_nan+1≤an

Example:

$a_n=\frac{1}{n}$

Q6. State and prove Monotone Convergence Theorem.

Theorem:
Every bounded monotonic sequence converges.

Proof (Sketch):

Let $\{a_n\}$ be increasing and bounded above.

Let:

$L=\sup\{a_n\}$

For any $\epsilon>0$,
there exists N such that

$L-\epsilon<a_N\le L$

Since increasing,

$L-\epsilon<a_n\le L \quad (n\ge N)$

Thus $a_n\to L$.

Q7. Define Cauchy sequence.

Answer:

A sequence $\{a_n\}$ is Cauchy if:

$\forall \epsilon>0,\ \exists N$

such that

$|a_n-a_m|<\epsilon \quad (m,n\ge N)$

Meaning terms become arbitrarily close.

Q8. Prove every convergent sequence is Cauchy.

Let an→La_n\to Lan→L.

Then:

|an−am|≤|an−L|+|am−L||a_n-a_m|\le |a_n-L|+|a_m-L||an−am|≤|an−L|+|am−L|

Each term → 0 as n,m→∞n,m\to\inftyn,m→∞.

Hence RHS < ε.

Therefore sequence is Cauchy.

Q9. Show that every Cauchy sequence in ℝ is convergent.

Proof Idea:

- Cauchy ⇒ bounded
- ℝ is complete
- Every bounded sequence has convergent subsequence
- Entire sequence converges to same limit.

Hence proved.

Q10. Explain algebra of limits with proofs.

If:

an→L,bn→Ma_n\to L,\quad b_n\to Man→L,bn→M

Then:

1. an+bn→L+Ma_n+b_n\to L+Man+bn→L+M
2. anbn→LMa_nb_n\to LManbn→LM
3. kan→kLka_n\to kLkan→kL
4. anbn→LM,M≠0\frac{a_n}{b_n}\to \frac{L}{M}, M\neq0bnan→ML,M□=0

Proof follows ε-inequality properties.

Q11. Evaluate lim2n+3n\lim \frac{2n+3}{n}limn2n+3.

2n+3n=2+3n\frac{2n+3}{n}=2+\frac{3}{n}n2n+3=2+n3

Since 3n→0\frac{3}{n}\to0n3→0,

lim=2.\lim =2.lim=2.

Q12. Prove uniqueness of limit of a sequence.

Assume:

an→L,an→Ma_n\to L,\quad a_n\to Man→L,an→M

Then:

|L−M|≤|L−an|+|an−M||L-M|\le |L-a_n|+|a_n-M||L−M|≤|L−an|+|an−M|

Taking limit gives:

|L−M|=0|L-M|=0|L−M|=0

Hence L=ML=ML=M.

Q13. Define bounded above and bounded below sequences.

- **Bounded above:** an≤Ma_n\le Man≤M
- **Bounded below:** an≥ma_n\ge man≥m

If both hold → bounded sequence.

Q14. Prove that (−1)n(-1)^n(−1)n is divergent.

Sequence alternates:

1,−1,1,−1,…1,-1,1,-1,\dots1,−1,1,−1,…

Two subsequences give different limits.

Hence no unique limit ⇒ divergent.

Q15. Show that subsequence of convergent sequence converges to same limit.

If an→La_n\to Lan→L,

then for subsequence anka_{n_k}ank:

|ank−L|<ϵ|a_{n_k}-L|<\epsilon|ank−L|<ϵ

since nk→∞n_k\to\inftynk→∞.

Thus same limit.

Q16. State General Principle of Convergence.

A sequence converges **iff** it is Cauchy (in ℝ).

This follows from completeness property of real numbers.

Q17. Prove bounded monotonic decreasing sequence converges.

Let bounded below.

Let:

L=inf{an}L=\inf\{a_n\}L=inf{an}

Using ε-definition similar to increasing case ⇒ convergence.

Q18. Find limit of nn+1\frac{n}{n+1}n+1n.

nn+1=11+1n\frac{n}{n+1}=\frac{1}{1+\frac{1}{n}}n+1n=1+n11

As n→∞n\to\inftyn→∞,

lim⁡=1.\lim=1.lim=1.

Q19. Show that constant sequence converges.

Let an=ca_n=can=c.

Then:

|an−c|=0<ϵ|a_n-c|=0<\epsilon|an−c|=0<ϵ

Hence limit = c.

Q20. Define divergent sequence with examples.

A sequence not having finite limit.

Examples:

- an=na_n=nan=n
- (−1)n(-1)^n(−1)n

Q21. Prove that limit of sum equals sum of limits.

Using triangle inequality:

|(an+bn)−(L+M)|≤|an−L|+|bn−M||(a_n+b_n)-(L+M)|\le |a_n-L|+|b_n-M||(an+bn)−(L+M)|≤|an−L|+|bn−M|

Each → 0.

Hence proved.

Q22. Show that bounded sequence need not converge.

Example:

an=(−1)na_n=(-1)^nan=(−1)n

Bounded between −1 and 1 but divergent.

Q23. Define supremum and infimum in sequence convergence.

- Supremum = least upper bound
- Infimum = greatest lower bound

Used in monotone convergence theorem.

Q24. Show $\lim \frac{n^2+1}{n^2}=1$.

$\frac{n^2+1}{n^2}=1+\frac{1}{n^2}$

Since $\frac{1}{n^2}\to 0$,

Limit = 1.

Q25. Explain completeness of ℝ using Cauchy sequences.

Real numbers satisfy:

Every Cauchy sequence converges to a real number.

This property is called **completeness**, distinguishing ℝ from ℚ.

Q26. Define an infinite series and explain convergence using partial sums.

Answer:

An infinite series is written as:

$\sum_{n=1}^{\infty} a_n = a_1+a_2+a_3+\cdots$

Let partial sum:

$S_n=a_1+a_2+\cdots+a_n$

If

$$\lim_{n\to\infty} S_n = S$$

exists and finite, the series **converges**; otherwise diverges.

Q27. State and prove necessary condition for convergence of a series.

Theorem:
If $\sum a_n$ converges, then $a_n \to 0$.

Proof:

$$a_n=S_n-S_{n-1}$$

Since $S_n \to S$,

$$a_n\to S-S=0$$

Hence proved.

Q28. Show that harmonic series diverges.

$$\sum \frac{1}{n}$$

Group terms:

$$1+\frac12+\left(\frac13+\frac14\right)+\left(\frac15+\cdots+\frac18\right)+\cdots$$

Each group ≥ 1/2.

Partial sums grow without bound ⇒ divergent.

Q29. Define convergent and divergent series with examples.

- **Convergent:** finite sum exists
  Example: $\sum 1/n^2$
- **Divergent:** sum infinite or undefined
  Example: $\sum 1/n$

Q30. State Pringsheim's Theorem.

Theorem:
If a series with positive terms has bounded partial sums, then the series converges.

Reason: Partial sums form increasing bounded sequence ⇒ convergent.

Q31. Prove Pringsheim's Theorem.

Let an≥0a_n\ge0an≥0.

Then partial sums SnS_nSn are increasing.

If bounded above ⇒ by monotone convergence theorem,

Sn→SS_n \to SSn→S

Hence series converges.

Q32. Explain Direct Comparison Test.

If:

0≤an≤bn0\le a_n\le b_n0≤an≤bn

then:

- ∑bn\sum b_n∑bn convergent ⇒ ∑an\sum a_n∑an convergent.
- ∑an\sum a_n∑an divergent ⇒ ∑bn\sum b_n∑bn divergent.

Q33. Test convergence of ∑1n2+1\sum \frac{1}{n^2+1}∑n2+11 using comparison test.

Since

1n2+1<1n2\frac{1}{n^2+1}<\frac{1}{n^2}n2+11<n21

and ∑1/n2\sum 1/n^2∑1/n2 converges (p-series, p>1),

given series converges.

Q34. State Limit Comparison Test.

If

lim n→∞anbn=c,0<c<∞\lim_{n\to\infty}\frac{a_n}{b_n}=c,\quad 0<c<\inftyn→∞limbnan=c,0<c<∞

then both series behave alike (both converge or diverge).

Q35. Apply limit comparison test to ∑nn3+1\sum \frac{n}{n^3+1}∑n3+1n.

Compare with 1/n21/n^21/n2.

lim nn3+11/n2=lim n3n3+1=1\lim \frac{\frac{n}{n^3+1}}{1/n^2} =\lim \frac{n^3}{n^3+1}=1lim1/n2n3+1n=limn3+1n3=1

Since p-series (p=2) converges ⇒ given series converges.

Q36. State and prove Cauchy Root Test.

Let

L=lim sup |an|nL=\limsup \sqrt[n]{|a_n|}L=limsupn|an|

- L<1 ⇒ convergent
- L>1 ⇒ divergent
- L=1 ⇒ inconclusive

Proof: Compare with geometric series.

Q37. Test convergence of ∑(1/3)n\sum (1/3)^n∑(1/3)n using root test.

(1/3)nn=1/3<1\sqrt[n]{(1/3)^n}=1/3<1n(1/3)n=1/3<1

Hence convergent.

Q38. State D'Alembert Ratio Test.

Let

$L=\lim \left|\frac{a_{n+1}}{a_n}\right|$

- $L<1 \Rightarrow$ convergent
- $L>1 \Rightarrow$ divergent
- $L=1 \Rightarrow$ inconclusive.

Q39. Test $\sum \frac{1}{n!}$ using ratio test.

$\frac{a_{n+1}}{a_n}=\frac{1}{n+1}\to 0<1$

Hence convergent.

Q40. Explain Alternating Series.

Series whose terms change sign:

$\sum (-1)^n b_n$

Example:

$1-\frac12+\frac13-\frac14+\cdots$

Q41. State Leibniz Test.

If:

1. $b_n \downarrow$
2. $b_n\to 0$

then alternating series converges.

Q42. Prove Leibniz Test.

Even and odd partial sums form bounded monotonic sequences.

Hence both converge to same limit ⇒ series converges.

Q43. Test convergence of alternating harmonic series.

$\sum (-1)^{n+1}\frac{1}{n}$

- decreasing terms
- limit $\to 0$

⇒ convergent by Leibnitz test.

Q44. Define absolute convergence.

Series is absolutely convergent if:

$\sum |a_n|$

converges.

Q45. Prove absolute convergence implies convergence.

Since

$|S_n-S_m|\le \sum |a_k|$

bounded ⇒ Cauchy ⇒ convergent.

Q46. Give example of conditional convergence.

Alternating harmonic series:

$\sum (-1)^{n+1}\frac1n$

Converges but

$\sum |1/n|$

diverges.

Q47. Show ∑(−1)nn2\sum \frac{(-1)^n}{n^2}∑n2(−1)n is absolutely convergent.

∑|an|=∑1n2\sum |a_n|=\sum \frac{1}{n^2}∑|an|=∑n21

p-series (p>1) ⇒ convergent.

Hence absolutely convergent.

Q48. Compare Ratio Test and Root Test.

Ratio Test	Root Test
Uses successive term ratio	Uses nth root
Best for factorials	Best for powers
Easier computation	General growth measure

Q49. Explain geometric interpretation of convergence of series.

Partial sums represent cumulative lengths/areas.

If sums approach fixed value → convergence.

Example: geometric series forms finite length.

Q50. Summarize hierarchy of convergence tests.

1. Necessary condition an→0a_n→0an→0
2. Comparison tests
3. Ratio test
4. Root test
5. Alternating (Leibnitz) test
6. Absolute convergence test

Used progressively depending on form of terms.

Q51. Define a binary operation. Give suitable examples and non-examples.

Answer:

A **binary operation** on a set GGG is a function:

∗:G×G→G*: G \times G \rightarrow G∗:G×G→G

which assigns each ordered pair (a,b)(a,b)(a,b) an element a∗b∈Ga*b \in Ga∗b∈G.

Examples

1. Addition on integers (Z,+)(\mathbb{Z}, +)(Z,+)
2. Multiplication on real numbers.

Non-Example

Subtraction on natural numbers (not closed).

Q52. Explain closure and associativity properties with examples.

Closure:
If a,b∈G⇒a∗b∈Ga,b \in G \Rightarrow a*b \in Ga,b∈G⇒a∗b∈G.

Example: integers under addition.

Associativity:

(a∗b)∗c=a∗(b∗c)(a*b)*c=a*(b*c)(a∗b)∗c=a∗(b∗c)

Example:

(2+3)+4=2+(3+4)(2+3)+4=2+(3+4)(2+3)+4=2+(3+4)

Q53. Define a group and list its axioms.

A set GGG with operation * is a **group** if:

1. Closure
2. Associativity
3. Identity element exists
4. Inverse exists for every element

Denoted (G,∗)(G,*)(G,∗).

Q54. Show that integers under addition form a group.

Check axioms:

- Closure: a+b∈Za+b\in\mathbb{Z}a+b∈Z
- Associative: true
- Identity: 0
- Inverse: −a

Hence (Z,+)(\mathbb{Z},+)(Z,+) is a group.

Q55. Prove that natural numbers under addition do not form a group.

Closure and associativity hold.

Identity exists (0) but 0∉N0\notin \mathbb{N}0∈/N (usual definition).

Also inverse not present.

Hence not a group.

Q56. Define identity element and prove its uniqueness.

Identity eee satisfies:

ae=ea=aae=ea=aae=ea=a

Proof of uniqueness:

Let eee and e′e'e′ be identities.

e∗e′=e′e*e'=e'e∗e′=e′

(since e identity)

e∗e′=ee*e'=ee∗e′=e

(since e′ identity)

Hence e=e′e=e'e=e′.

Q57. Define inverse element and prove uniqueness of inverse.

Inverse of aaa is a−1a^{-1}a−1 such that:

aa−1=a−1a=eaa^{-1}=a^{-1}a=eaa−1=a−1a=e

Proof:

If b and c are inverses:

b=be=b(ac)=(ba)c=ec=cb=be=b(ac)=(ba)c=ec=cb=be=b(ac)=(ba)c=ec=c

Thus inverse unique.

Q58. Define Abelian group with examples.

A group satisfying:

a∗b=b∗aa*b=b*aa∗b=b∗a

is called **Abelian**.

Examples:

- (Z,+)(\mathbb{Z},+)(Z,+)
- (R,+)(\mathbb{R},+)(R,+)

Q59. Give example of a non-Abelian group.

Matrix multiplication:

AB≠BAAB \ne BAAB⍰=BA

Hence set of matrices under multiplication forms non-abelian group.

Q60. Prove that every cyclic group is Abelian.

Let generator aaa.

Elements:

am, ana^m,\ a^nam, an

Then:

aman=am+n=an+m=anama^m a^n=a^{m+n}=a^{n+m}=a^n a^maman=am+n=an+m=anam

Hence Abelian.

Q61. Show identity of a group is unique using group axioms.

(Same argument as Q56)

Using associativity and identity property gives equality of identities.

Q62. Prove inverse of identity element is itself.

Let e be identity.

ee=eee=eee=e

Thus inverse of e is e.

Q63. Prove inverse of inverse equals original element.

aa−1=ea a^{-1}=eaa−1=e

Taking inverse both sides:

(a−1)−1=a(a^{-1})^{-1}=a(a−1)−1=a

Q64. Show that (ab)−1=b−1a−1(ab)^{-1}=b^{-1}a^{-1}(ab)−1=b−1a−1.

(ab)(b−1a−1)=a(bb−1)a−1=aea−1=e(ab)(b^{-1}a^{-1}) =a(bb^{-1})a^{-1} =aea^{-1} =e(ab)(b−1a−1)=a(bb−1)a−1=aea−1=e

Hence proved.

Q65. Verify whether real numbers under multiplication form a group.

Closure: yes
Associativity: yes
Identity: 1
Inverse exists except 0.

Hence (R\{0},×)(\mathbb{R}\setminus\{0\},\times)(R\{0},×) is group.

Q66. Explain cancellation laws in a group.

If:

ab=ac⇒b=cab=ac \Rightarrow b=cab=ac⇒b=c

Multiply both sides by a−1a^{-1}a−1.

Similarly right cancellation holds.

Q67. Prove cancellation law in groups.

Given ab=acab=acab=ac.

Multiply left by a−1a^{-1}a−1:

a−1(ab)=a−1(ac)a^{-1}(ab)=a^{-1}(ac)a−1(ab)=a−1(ac) (a−1a)b=(a−1a)c(a^{-1}a)b=(a^{-1}a)c(a−1a)b=(a−1a)c eb=ec⇒b=ceb=ec \Rightarrow b=ceb=ec⇒b=c

---

Q68. Show that equation ax=bax=bax=b has unique solution in a group.

Multiply by a−1a^{-1}a−1:

x=a−1bx=a^{-1}bx=a−1b

Uniqueness follows from cancellation law.

Q69. Define finite and infinite groups with examples.

- **Finite group:** finite elements
  Example: integers mod n.
- **Infinite group:** infinite elements
  Example: integers under addition.

Q70. Show that identity element commutes with every element.

Since:

ae=ea=aae=ea=aae=ea=a

Thus identity commutes with all elements.

Q71. Prove that inverse of product reverses order.

Already shown:

(ab)−1=b−1a−1(ab)^{-1}=b^{-1}a^{-1}(ab)−1=b−1a−1

using associativity and identity properties.

Q72. Show that a group may have only one identity but many elements.

Example:

(Z,+)(\mathbb{Z},+)(Z,+)

Identity = 0 (single)
Elements infinite.

Q73. Distinguish between Abelian and Non-Abelian groups.

Abelian	Non-Abelian
Commutative	Non-commutative
ab=ba	ab≠ba
Example: integers	Example: matrices

Q74. Prove that solution of xa=bxa=bxa=b exists uniquely in a group.

Multiply right by a−1a^{-1}a−1:

x=ba−1x=ba^{-1}x=ba−1

Uniqueness follows from cancellation.

Q75. Explain importance of identity and inverse in group structure.

Identity ensures stability of operation.
Inverse guarantees solvability of equations.

Together they allow algebraic manipulation similar to arithmetic systems.

Q76. Explain different ways of defining a group.

Answer:

A group can be defined in the following ways:

1. **Axiomatic Definition**
   A set GGG with binary operation * satisfying:
   - Closure
   - Associativity
   - Identity
   - Inverse
2. **Operational (Cayley Table) Definition**
   Group defined using operation table showing closure and identity.
3. **Transformation Group**
   Set of mappings from a set onto itself under composition.

4. **Generator Definition**
   Group defined by generators and relations.

Q77. Define Cayley table and explain its role in defining finite groups.

A **Cayley table** displays results of binary operation.

Example for Z3\mathbb{Z}_3Z3 under addition:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

It verifies:

- closure
- identity
- inverses.

Q78. Define subgroup and state subgroup criteria.

A subset H⊆GH\subseteq GH⊆G is subgroup if:

1. H≠∅H\neq\emptysetH□=∅
2. Closed under operation
3. Closed under inverse

Symbol:

H≤GH \le GH≤G

Q79. Prove that intersection of two subgroups is a subgroup.

Let H1,H2H_1, H_2H1,H2 be subgroups.

- Identity belongs to both ⇒ in intersection.
- Closed under operation.
- Inverses remain inside.

Hence $H_1\cap H_2$ is subgroup.

Q80. Show that identity element must belong to every subgroup.

Let $h\in H$.

Since inverse exists:

$hh^{-1}=e$

Thus identity $e\in H$.

Q81. Prove subgroup test using closure under operation and inverse.

If $H\neq\emptyset$ and

$ab^{-1}\in H \quad \forall a,b\in H$

then:

- identity exists
- inverses exist
- closure holds

Hence subgroup.

Q82. Define cyclic group with examples.

A group generated by single element a:

$G=\langle a\rangle=\{a^n:n\in\mathbb{Z}\}$

Example:

$(\mathbb{Z},+)$

generated by 1.

Q83. Prove every cyclic group is Abelian.

Let elements be am,ana^m,a^nam,an.

aman=am+n=an+m=anama^m a^n=a^{m+n}=a^{n+m}=a^n a^maman=am+n=an+m=anam

Hence Abelian.

Q84. Define generator of a cyclic group.

Element aaa such that:

G=⟨a⟩G=\langle a\rangleG=⟨a⟩

is called **generator**.

Example:
Generators of Z6\mathbb{Z}_6Z6: 1 and 5.

Q85. Show that every subgroup of a cyclic group is cyclic.

Let G=⟨a⟩G=\langle a\rangleG=⟨a⟩.

Any subgroup consists of powers aka^kak.

Thus generated by smallest positive power.

Hence cyclic.

Q86. Define order of an element and group.

- Order of group = number of elements.
- Order of element = smallest nnn such that:

an=ea^n=ean=e

Q87. Define left and right cosets.

For subgroup H⊂GH\subset GH⊂G:

Left coset:

aH={ah:h∈H}aH=\{ah:h\in H\}aH={ah:h∈H}

Right coset:

Ha={ha:h∈H}Ha=\{ha:h\in H\}Ha={ha:h∈H}

Q88. Prove that all cosets have same number of elements.

Mapping:

h→ahh \rightarrow ahh→ah

is bijection between HHH and aHaHaH.

Hence equal size.

Q89. Show that two cosets are either identical or disjoint.

If aH∩bH≠∅aH\cap bH\neq\emptysetaH∩bH□=∅,

then exists element common ⇒

aH=bHaH=bHaH=bH

Otherwise disjoint.

Q90. Prove cosets partition a group.

Every element belongs to some coset.

Distinct cosets do not overlap.

Thus group divided into disjoint subsets.

Q91. Define index of a subgroup.

Index of HHH in GGG:

[G:H]=|G||H|[G:H]=\frac{|G|}{|H|}[G:H]=|H||G|

equals number of cosets.

Q92. State Lagrange's Theorem.

Theorem:

If GGG finite group and H≤GH\le GH≤G,

then:

|H|||G||H| \mid |G||H|||G|

(order of subgroup divides order of group).

Q93. Prove Lagrange's Theorem.

Group partitioned into cosets.

Each coset has |H||H||H| elements.

If k cosets exist:

|G|=k|H||G|=k|H||G|=k|H|

Hence proved.

Q94. Show that order of element divides order of group.

Subgroup generated by element:

⟨a⟩\langle a\rangle⟨a⟩

Apply Lagrange theorem:

o(a)||G|o(a)\mid |G|o(a)||G|

Q95. Prove group of prime order is cyclic.

Let $|G|=p|G|=p|G|=p$ (prime).

Any non-identity element generates subgroup.

Subgroup order divides p ⇒ either 1 or p.

Hence whole group cyclic.

Q96. Find possible subgroup orders of group of order 12.

By Lagrange theorem:

Divisors of 12:

1,2,3,4,6,121,2,3,4,6,121,2,3,4,6,12

Only these possible orders.

Q97. Explain geometrical meaning of cosets.

Cosets represent translations of subgroup structure inside group.

They form equal “layers” partitioning the group.

Q98. Distinguish between cyclic and non-cyclic groups.

Cyclic	Non-Cyclic
One generator	Multiple generators
Always Abelian	May be non-Abelian
Example: Z	Example: S_3

Q99. Show identity coset equals subgroup itself.

eH={eh:h∈H}=HeH=\{eh:h\in H\}=HeH={eh:h∈H}=H

Hence identity coset equals subgroup.

Q100. Explain significance of Lagrange’s theorem in group theory.

Applications:

- Determines subgroup orders
- Helps classify finite groups
- Proves prime order groups cyclic
- Basis for advanced algebra results.

Q101. Define differential equation. Explain order and degree with examples.

Answer:

A **differential equation** is an equation involving derivatives of a dependent variable with respect to an independent variable.

Order

Highest order derivative present.

Example:

d2ydx2+y=0\frac{d^2y}{dx^2}+y=0dx2d2y+y=0

Order = 2.

Degree

Power of highest order derivative (after removing radicals).

Example:

(dydx)3+y=0\left(\frac{dy}{dx}\right)^3+y=0(dxdy)3+y=0

Degree = 3.

Q102. Form differential equation by eliminating arbitrary constants from y=Cx2y=Cx^2y=Cx2.

Differentiate:

dydx=2Cx\frac{dy}{dx}=2Cxdxdy=2Cx

From given:

C=yx2C=\frac{y}{x^2}C=x2y

Substitute:

dydx=2yx\frac{dy}{dx}=2\frac{y}{x}dxdy=2xy

Required differential equation:

xdydx=2y.x\frac{dy}{dx}=2y.xdxdy=2y.

Q103. Solve first-order differential equation dydx=3x2\frac{dy}{dx}=3x^2dxdy=3x2.

Integrate:

dy=3x2dxdy=3x^2dxdy=3x2dx y=∫3x2dx=x3+C.y=\int 3x^2dx=x^3+C.y=∫3x2dx=x3+C.

Q104. Solve separable equation dydx=xy\frac{dy}{dx}=xydxdy=xy.

Separate variables:

dyy=x dx\frac{dy}{y}=x\,dxydy=xdx

Integrate:

ln y=x22+C\ln y=\frac{x^2}{2}+Clny=2x2+C y=Cex2/2.y=Ce^{x^2/2}.y=Cex2/2.

Q105. Define differential equation of first order and higher degree with example.

First order: highest derivative dy/dxdy/dxdy/dx.

Higher degree: derivative power >1.

Example:

(dydx)2+y=0.\left(\frac{dy}{dx}\right)^2+y=0.(dxdy)2+y=0.

Q106. Explain method of solving first-order higher-degree equations.

Steps:

1. Express equation in derivative p=dydxp=\frac{dy}{dx}p=dxdy.
2. Solve algebraically for ppp.
3. Integrate resulting equation.

Q107. Define Clairaut's differential equation and give its general form.

Clairaut's equation:

y=px+f(p),p=dydx.y=px+f(p), \quad p=\frac{dy}{dx}.y=px+f(p),p=dxdy.

Represents family of straight lines.

Q108. Find general solution of Clairaut equation y=px+p2y=px+p^2y=px+p2.

Differentiate:

dydx=p+xdpdx+2pdpdx\frac{dy}{dx}=p+x\frac{dp}{dx}+2p\frac{dp}{dx}dxdy=p+xdxdp+2pdxdp

But dy/dx=pdy/dx=pdy/dx=p.

Hence:

(x+2p)dpdx=0(x+2p)\frac{dp}{dx}=0(x+2p)dxdp=0

Case 1: dp/dx=0⇒p=Cdp/dx=0\Rightarrow p=Cdp/dx=0⇒p=C

General solution:

y=Cx+C2.y=Cx+C^2.y=Cx+C2.

Q109. Obtain singular solution of y=px+p2y=px+p^2y=px+p2.

From:

x+2p=0⇒p=−x2x+2p=0 \Rightarrow p=-\frac{x}{2}x+2p=0⇒p=−2x

Substitute:

y=−x22+x24=−x24.y=-\frac{x^2}{2}+\frac{x^2}{4} =-\frac{x^2}{4}.y=−2x2+4x2=−4x2.

Q110. Define singular solution geometrically.

Singular solution is **envelope** of family of curves obtained from general solution.

Touches each member without coinciding.

Q111. Explain difference between general and singular solutions.

General Solution	Singular Solution
Contains constant	No arbitrary constant
Family of curves	Envelope curve
Infinite solutions	Unique curve

Q112. Define orthogonal trajectories.

A family of curves intersecting another family at right angle (90°).

Condition:

m1m2=−1.m_1m_2=-1.m1m2=−1.

Q113. Find orthogonal trajectories of x2+y2=Cx^2+y^2=Cx2+y2=C.

Differentiate:

2x+2ydydx=02x+2y\frac{dy}{dx}=02x+2ydxdy=0 dydx=−xy\frac{dy}{dx}=-\frac{x}{y}dxdy=−yx

Orthogonal slope:

dydx=yx\frac{dy}{dx}=\frac{y}{x}dxdy=xy

Separate:

dyy=dxx\frac{dy}{y}=\frac{dx}{x}ydy=xdx

Integrate:

y=Cx.y=Cx.y=Cx.

Q114. Explain physical applications of orthogonal trajectories.

Used in:

- Electric field & equipotential lines
- Heat flow
- Fluid dynamics
- Magnetic field lines

Q115. Define linear differential equation with constant coefficients.

General form:

any(n)+an−1y(n−1)+⋯+a0y=f(x)a_ny^{(n)}+a_{n-1}y^{(n-1)}+\cdots+a_0y=f(x)any(n)+an−1y(n−1)+⋯+a0y=f(x)

where coefficients are constants.

Q116. Explain complementary function (CF).

Solution of homogeneous equation:

any(n)+⋯+a0y=0.a_ny^{(n)}+\cdots+a_0y=0.any(n)+⋯+a0y=0.

Obtained using auxiliary equation.

Q117. Explain particular integral (PI).

Specific solution satisfying non-homogeneous equation.

General solution:

y=CF+PI.y=CF+PI.y=CF+PI.

Q118. Solve y′′−y=0y''-y=0y′′−y=0.

Auxiliary equation:

m2−1=0m^2-1=0m2−1=0 m=±1m=\pm1m=±1

Solution:

y=C1ex+C2e−x.y=C_1e^x+C_2e^{-x}.y=C1ex+C2e−x.

Q119. Solve y′′+4y=0y''+4y=0y′′+4y=0.

Auxiliary equation:

m2+4=0m^2+4=0m2+4=0 m=±2im=\pm2im=±2i

Solution:

y=C1cos2x+C2sin2x.y=C_1\cos2x+C_2\sin2x.y=C1cos2x+C2sin2x.

Q120. Explain cases of auxiliary equation roots.

1. Distinct real roots → exponential terms
2. Equal roots → xemxxe^{mx}xemx term
3. Complex roots → sine & cosine form.

Q121. Solve y′′−2y′+y=0y''-2y'+y=0y′′−2y′+y=0.

Auxiliary equation:

(m−1)2=0(m-1)^2=0(m−1)2=0

Repeated root.

Solution:

y=(C1+C2x)ex.y=(C_1+C_2x)e^x.y=(C1+C2x)ex.

Q122. Define homogeneous linear differential equation.

Equation with RHS = 0:

Ly=0.Ly=0.Ly=0.

Q123. Explain operator method for solving linear equations.

Let D=ddxD=\frac{d}{dx}D=dxd.

Equation written as:

F(D)y=f(x).F(D)y=f(x).F(D)y=f(x).

Solve using algebraic manipulation of operators.

Q124. Find general solution of y″+y=sinxy''+y=\sin xy″+y=sinx.

CF:

yc=C1cosx+C2sinxy_c=C_1\cos x+C_2\sin xyc=C1cosx+C2sinx

PI (trial):

yp=x2cosxy_p=\frac{x}{2}\cos xyp=2xcosx

General solution:

y=C1cosx+C2sinx+x2cosx.y=C_1\cos x+C_2\sin x+\frac{x}{2}\cos x.y=C1cosx+C2sinx+2x cosx.

Q125. Discuss importance of differential equations in science and engineering.

Applications:

- Population growth
- Mechanics
- Electrical circuits
- Heat transfer
- Fluid flow
- Economics modeling

Differential equations describe real-world rate changes.

Q126. Define homogeneous linear differential equation with variable coefficients. Give example.

Answer:

A linear differential equation in which coefficients depend on the independent variable is called a **homogeneous linear equation with variable coefficients**.

General form:

a2(x)y′′+a1(x)y′+a0(x)y=0a_2(x)y''+a_1(x)y'+a_0(x)y=0a2(x)y′′+a1(x)y′+a0(x)y=0

Example:

x2y′′+xy′+y=0.x^2y''+xy'+y=0.x2y′′+xy′+y=0.

Q127. Explain Cauchy–Euler differential equation.

It is a special variable coefficient equation:

x2y′′+axy′+by=0.x^2y''+axy'+by=0.x2y′′+axy′+by=0.

Solution obtained using substitution:

y=xm.y=x^m.y=xm.

Q128. Solve x2y′′+xy′−y=0x^2y''+xy'-y=0x2y′′+xy′−y=0.

Assume y=xmy=x^my=xm.

y′=mxm−1,y′′=m(m−1)xm−2y'=mx^{m-1},\quad y''=m(m-1)x^{m-2}y′=mxm−1,y′′=m(m−1)xm−2

Substitute:

m(m−1)+m−1=0m(m-1)+m-1=0m(m−1)+m−1=0 m2−1=0⇒m=±1m^2-1=0 \Rightarrow m=\pm1m2−1=0⇒m=±1

Solution:

y=C1x+C2x−1.y=C_1x+C_2x^{-1}.y=C1x+C2x−1.

Q129. Explain method of substitution x=etx=e^tx=et for Euler equations.

Let x=etx=e^tx=et.

Then derivatives convert into constant coefficient form, simplifying solution.

Thus Euler equation becomes linear equation with constant coefficients.

Q130. Solve x2y′′−3xy′+4y=0x^2y''-3xy'+4y=0x2y′′−3xy′+4y=0.

Assume y=xmy=x^my=xm.

m(m−1)−3m+4=0m(m-1)-3m+4=0m(m−1)−3m+4=0 m2−4m+4=0⇒(m−2)2=0m^2-4m+4=0 \Rightarrow (m-2)^2=0m2−4m+4=0⇒(m−2)2=0

Repeated root.

Solution:

y=(C1+C2ln⁡x)x2.y=(C_1+C_2\ln x)x^2.y=(C1+C2lnx)x2.

Q131. Define simultaneous differential equations.

A system involving two or more dependent variables and their derivatives.

Example:

dxdt=x+y,dydt=x−y.\frac{dx}{dt}=x+y,\quad \frac{dy}{dt}=x-y.dtdx=x+y,dtdy=x−y.

Q132. Explain elimination method for simultaneous equations.

Steps:

1. Differentiate one equation.

2. Substitute other equation.
3. Reduce system into single differential equation.
4. Solve normally.

Q133. Solve

dxdt=y,dydt=−x.\frac{dx}{dt}=y,\quad \frac{dy}{dt}=-x.dtdx=y,dtdy=−x.

Differentiate first:

d2xdt2=dydt=−x\frac{d^2x}{dt^2}=\frac{dy}{dt}=-xdt2d2x=dtdy=−x x′′+x=0x''+x=0x′′+x=0

Solution:

x=C1cos t+C2sin t.x=C_1\cos t+C_2\sin t.x=C1cost+C2sint.

Then:

y=dxdt.y=\frac{dx}{dt}.y=dtdx.

Q134. Explain applications of simultaneous differential equations.

Used in:

- Population interaction models
- Electrical circuits
- Mechanical vibrations
- Chemical reactions.

Q135. Define auxiliary equations

dxP=dyQ=dzR.\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}.Pdx=Qdy=Rdz.

These are **Lagrange's auxiliary equations** used to solve first-order PDEs.

They represent direction ratios of characteristic curves.

Q136. Explain method of solving

dxP=dyQ=dzR.\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}.Pdx=Qdy=Rdz.

Steps:

1. Equate pairs of ratios.
2. Integrate two independent relations.
3. Obtain solution F(u,v)=0F(u,v)=0F(u,v)=0.

Q137. Solve

dxx=dyy.\frac{dx}{x}=\frac{dy}{y}.xdx=ydy.

Integrate:

ln x=ln y+C\ln x=\ln y+Clnx=lny+C y=Cx.y=Cx.y=Cx.

Q138. Solve

dxy=dyx.\frac{dx}{y}=\frac{dy}{x}.ydx=xdy. x dx=y dyx\,dx=y\,dyxdx=ydy

Integrate:

x22=y22+C\frac{x^2}{2}=\frac{y^2}{2}+C2x2=2y2+C x2−y2=C.x^2-y^2=C.x2−y2=C.

Q139. Explain geometrical meaning of auxiliary equations.

They represent **space curves** whose tangent direction ratios are P,Q,RP,Q,RP,Q,R.

These curves are called **characteristics**.

Q140. Define total differential equation.

Equation of form:

Pdx+Qdy+Rdz=0Pdx+Qdy+Rdz=0Pdx+Qdy+Rdz=0

is called total differential equation.

Q141. Explain condition for exact total differential equation.

Equation is exact if:

Pdx+Qdy+Rdz=dF(x,y,z).Pdx+Qdy+Rdz=dF(x,y,z).Pdx+Qdy+Rdz=dF(x,y,z).

Then solution:

F(x,y,z)=C.F(x,y,z)=C.F(x,y,z)=C.

Q142. Solve total differential equation

xdx+ydy+zdz=0.xdx+ydy+zdz=0.xdx+ydy+zdz=0.

Integrate:

x22+y22+z22=C\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=C2x2+2y2+2z2=C

or

x2+y2+z2=C.x^2+y^2+z^2=C.x2+y2+z2=C.

Q143. Explain geometrical significance of total differential equations.

Solution represents **family of surfaces** in space.

Gradient vector normal to surface.

Q144. Define gradient and relate it to total differential equation.

Gradient:

∇F=(P,Q,R)\nabla F=(P,Q,R)∇F=(P,Q,R)

Normal to surface F(x,y,z)=CF(x,y,z)=CF(x,y,z)=C.

Q145. Explain integrating factor in non-exact equations.

If equation not exact, multiply by function μ such that equation becomes exact.

μ is called integrating factor.

Q146. Show that solution of exact equation represents level surface.

Since:

dF=0⇒F(x,y,z)=CdF=0 \Rightarrow F(x,y,z)=CdF=0⇒F(x,y,z)=C

This is level surface of scalar function.

Q147. Explain relation between conservative field and total differential equation.

If vector field is gradient of scalar potential:

$\vec{F}$=∇ϕ\vec{F}=\nabla \phiF=∇ϕ

then differential equation is exact.

Q148. Explain geometric interpretation of

Pdx+Qdy+Rdz=0.Pdx+Qdy+Rdz=0.Pdx+Qdy+Rdz=0.

Vector (P,Q,R)(P,Q,R)(P,Q,R) is perpendicular to displacement vector.

Hence motion confined on surface.

Q149. Discuss physical applications of total differential equations.

Applications:

- Fluid mechanics
- Thermodynamics
- Electrostatics
- Potential theory.

Q150. Summarize methods for solving variable coefficient and total differential equations.

1. Euler substitution for variable coefficients
2. Elimination for simultaneous equations
3. Auxiliary equations for characteristic curves
4. Exactness and integrating factor methods
5. Geometric interpretation using surfaces.