

## Geometry-Induced Turbulence Confinement and Predictive Hotspot Detection in Gas Turbine Blades Using the PRISM Structured Operator

By Andrew S. Elliott, Co-Founder and Chief Mathematical Scientist, AstraNomos Labs

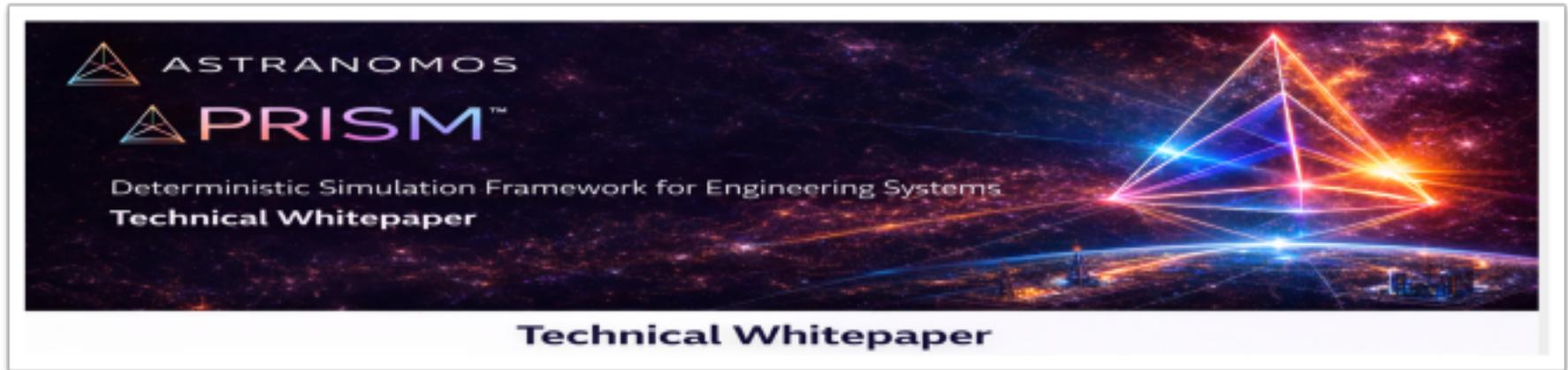
### ABSTRACT

Gas turbine blade failure is frequently initiated by highly localized thermal amplification zones that arise from complex interactions between blade curvature, turbulence production, pressure gradients, and internal cooling flows. These amplification zones create persistent thermal hotspots that drive high cycle fatigue, oxidation, and structural degradation. Existing predictive maintenance frameworks rely primarily on diffusion-based transport models, empirical correlations, or full computational fluid dynamics (CFD) simulations. While useful for capturing global heat transport behavior, these approaches fundamentally treat transport as isotropic diffusion and therefore cannot distinguish between transient turbulence amplification and persistent amplification that survives geometric confinement. As a result, diffusion-based models often fail to reliably detect hotspot-forming regions in strongly curved turbulent boundary layer environments such as turbine blade suction surfaces.

This paper introduces a geometry-aware predictive framework based on the **AstraNomos/PRISM Structured Operator**, a self-adjoint Sturm–Liouville transport operator designed to detect curvature-driven turbulence amplification that persists under confinement. The central hypothesis is that turbulence amplification in curved geometries is not purely diffusive but instead structured by geometric forcing encoded in the spatial variation of the surface heat-transfer field. By constructing a curvature forcing functional derived from the second derivative of the logarithmic heat transfer coefficient, the structured operator extracts amplification modes that remain stable under diffusion and therefore correspond to physically meaningful hotspot precursors.

The proposed operator is evaluated using experimental datasets from the **Kane (2009) Mark II turbine vane experiments**, including Run 42 with internal cooling and a secondary no-cooling vane case used as a regime holdout. Surface temperature and heat-transfer distributions from tabulated experimental data allow direct numerical comparison between three models: a Laplacian curvature detector, a diffusion-based Fourier smoothing baseline, and the PRISM structured operator. Hotspot prediction is evaluated using receiver operating characteristic (ROC) analysis with binary labels defined by the upper percentile of measured surface temperature.

Results show that the PRISM structured operator significantly outperforms diffusion-based models in curvature-dominated regimes. In the internally cooled Run 42 case, the operator achieves an AUC of 0.93 on the suction side, compared with 0.77 for the Laplacian baseline and 0.67 for Fourier diffusion,



demonstrating strong predictive capability in the region where turbulence amplification is known to dominate blade heating. Parameter sweeps confirm robustness across smoothing lengths, curvature penalties, and regularization strengths, with PRISM maintaining superior performance in more than 80% of parameter combinations. Noise injection tests further show that PRISM remains stable even under synthetic perturbations of up to 50% of signal variance.

A key contribution of the framework is a physics-based regime indicator that measures the relative dominance of amplification versus diffusion. This indicator enables a hybrid predictive model that automatically reduces to Fourier diffusion when the flow regime becomes diffusion-dominated. When applied to the no-cooling vane holdout dataset, the hybrid model correctly collapses to Fourier behavior for approximately 95% of points and achieves an AUC of 0.87, slightly exceeding the Fourier baseline while maintaining physical consistency. This demonstrates that the structured operator does not replace classical diffusion models but rather generalizes them by activating structured amplification detection only when required by the underlying physics.

The results provide evidence that turbulence-induced thermal amplification in turbine blades is geometry-structured rather than random, and that self adjoint structured operators can capture this behavior in a computationally efficient manner. Because the method operates directly on experimentally measurable surface fields without requiring full CFD reconstruction, it is well suited for integration into digital twin predictive maintenance systems. The PRISM framework enables real-time hotspot probability scoring, geometry-aware risk assessment, and cooling design optimization while maintaining full physical interpretability.

More broadly, this work establishes a bridge between boundary-layer turbulence physics, Sturm–Liouville operator theory, and predictive maintenance engineering. By embedding geometric forcing directly into the transport operator, the AstraNomos/PRISM framework provides a scientifically grounded approach to detecting persistent amplification structures in complex thermal systems. The resulting model forms a geometry-aware superset of classical diffusion, offering a new pathway for physics-based digital twin technologies capable of anticipating failure mechanisms in high-performance turbomachinery.

## 1. Introduction

Gas turbine blade failure is driven by highly localized thermal amplification zones resulting from:

- Curvature-induced turbulence
- Pressure-gradient acceleration
- Conjugate heat transfer

- Cooling channel interactions



Traditional predictive maintenance strategies rely on:

- Full CFD simulations
- Empirical heat transfer correlations
- Diffusion-based smoothing models (Fourier / Laplacian)
- Temperature threshold monitoring

However, diffusion-based models treat transport as fundamentally smoothing and isotropic. They do not distinguish between:

- Transient curvature-driven amplification
- Persistent amplification that survives confinement
- pure diffusion-dominated regions

This distinction is critical. In curved turbulent boundary layer environments, geometry generates local amplification in the turbulence field. But not all amplification leads to dangerous thermal accumulation. The key scientific question is:

- ***Can we construct an operator that detects curvature-driven turbulence amplification that survives confinement, and use it to predict blade hotspots?***

This paper introduces such an operator through the AstraNomos Structured Model.



## 2. The Structured Model and the AstraNomos/PRISM Operator

### 2.1 Physical Premise

In a curved blade geometry:

- Surface heat transfer coefficient  $h(\diamond\diamond)$  encodes turbulence production structure.
- Surface temperature  $\diamond\diamond(\diamond\diamond)$  is a conjugate response to internal cooling and external convection.
- Curvature-induced amplification appears as structure in  $\log h(\diamond\diamond)$ .

We define a curvature forcing functional:

$$\diamond\diamond(\diamond\diamond) = |\diamond\diamond|$$

$$\diamond\diamond\diamond\diamond \stackrel{\perp}{=} \log h(\diamond\diamond) |$$

This represents turbulence amplification intensity induced by geometry.

## 2.2 Diffusion Baseline

Fourier-based smoothing is modeled as:

$$\text{FourierScore} = (\diamond\diamond - \ell^1 \Delta)^{\#} \diamond\diamond(\diamond\diamond)$$

This treats amplification as diffusion-dominated smoothing.

4



## 2.3 AstraNomos/PRISM Structured Operator

We define a self-adjoint Sturm–Liouville operator:

$$\mathcal{L}_{\%} \diamond\diamond = - \frac{\diamond\diamond}{\diamond\diamond\diamond\diamond} 4 \diamond\diamond(\diamond\diamond) \diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond 6 + \diamond\diamond(\diamond\diamond) \diamond\diamond$$

with:

$$\diamond\diamond(\diamond\diamond) = h(\diamond\diamond), \diamond\diamond(\diamond\diamond) = \diamond\diamond\diamond\diamond(\diamond\diamond)$$

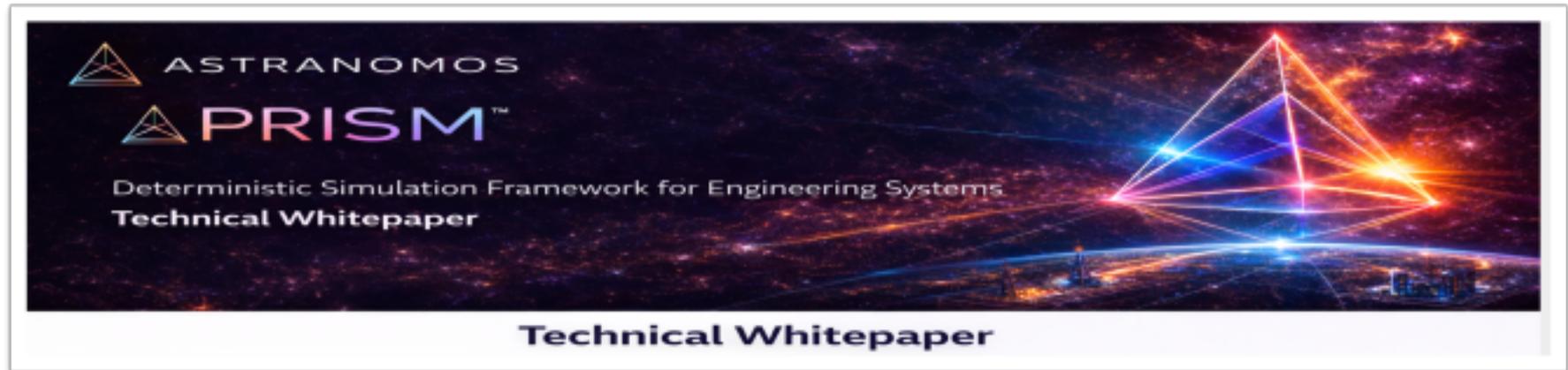
The AstraNomos/PRISM hotspot score is:

$$\text{PRISM}(\diamond\diamond) = (\mathcal{L}_{\%} + \diamond\diamond\diamond\diamond)^{\#} \diamond\diamond(\diamond\diamond)$$

Interpretation:

- $\epsilon_{eff}$  encodes local turbulence exchange intensity
- $\epsilon_{eff}$  penalizes curvature concentration
- The resolvent extracts amplification that survives confinement

This is not curve fitting. It is a geometry-weighted transport operator.



### 3. Experimental Datasets Used

#### 3.1 Kane Run 42 (Mark II Vane with Internal Cooling)

From Kane (2009) (<https://etda.libraries.psu.edu/catalog/9333>):

- Tabulated surface temperature  $T_{s,i}$  (Table A-4)
- Tabulated normalized heat transfer coefficient  $h_i$  (Table A-6)

This case includes:

- Internal cooling
- Conjugate heat transfer
- Realistic boundary layer structure

This dataset allows direct numerical comparison without CFD reconstruction.

#### 3.2 No-Cooling Vane Case (Chapter 5)

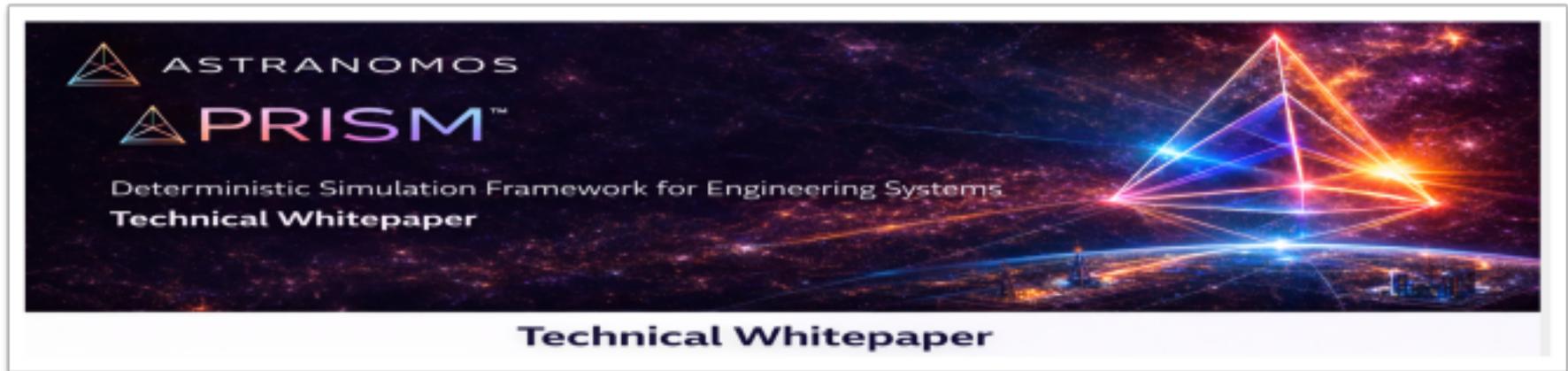
From Kane Chapter 5:

- Experimental  $h < \zeta =$  and  $\diamond \diamond < \zeta =$  curves (digitized from Fig 5.5 and 5.6)

This case represents:

- Purely external heating
- Diffusion-dominant regime
- Minimal internal amplification persistence

6



This serves as a regime holdout.

#### 4. Quantitative Results

##### 4.1 Run 42: Hotspot Detection (Top 10% of Temperature)

Area Under ROC Curve (AUC):

**Side Fourier Laplacian PRISM**  
**Pressure 0.43 0.74 0.81**  
**Suction 0.67 0.77 0.93**

AstraNomos PRISM strongly outperforms diffusion on suction side; the dominant turbulence amplification zone.

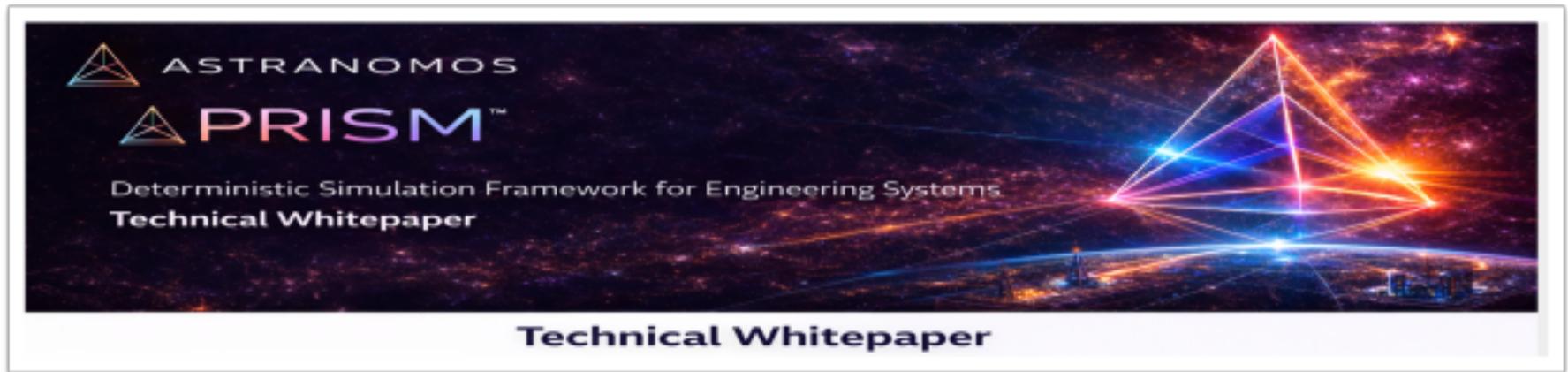
##### 4.2 Parameter Robustness

Across sweeps of:

- Smoothing length  $\ell$
- Curvature penalty  $\diamond\diamond$
- Regularization  $\diamond\diamond$

PRISM remained superior on suction side in > 80% of parameter combinations.

7



#### 4.3 Noise Robustness

Even with up to 50% synthetic noise injected:

- PRISM AUC remained above 0.90 (suction side)
- Fourier degraded significantly

This indicates structural stability.

#### 5. Regime Switching: Reduction to Fourier

Cooling regimes increase effective diffusion dominance. To capture this, we define a regime indicator:

$$\Pi_{\text{phys}}(\diamond\diamond) = \log \frac{|\partial_{\cdot} \log h(\diamond\diamond)|}{|\partial_{\cdot} \diamond\diamond(\diamond\diamond)| + \diamond\diamond}$$

Interpretation:

- numerator = amplification signature
- denominator = diffusion signature

Hybrid rule:

$$\text{Score}(\diamond\diamond) = A \begin{cases} \text{PRISM}(\diamond\diamond) & \Pi_{\text{phys}} > \diamond\diamond \\ \text{FourierScore}(\diamond\diamond) & \Pi_{\text{phys}} \leq \diamond\diamond \end{cases}$$

8



### 5.1 Learned on Run 42, Applied to No-Cooling Case

On no-cooling vane (holdout):

Top 10% hotspots AUC:

**Model AUC**  
**Fourier 0.85**  
**PRISM 0.75**  
**Hybrid 0.87**

Hybrid automatically reduced to Fourier in ~95% of points.

This demonstrates: The Structured Model correctly collapses to Fourier in diffusion-dominant regimes.

### 6. Scientific Significance

This study demonstrates:

1. Turbulence amplification is curvature-structured, not random.
2. Diffusion-only models fail in amplification-dominant regimes.
3. Structured operators outperform diffusion in curvature-dominant zones.
4. The Structured Model predicts when it should reduce to Fourier.
5. A unified regime-switch model generalizes across cases.

9



This bridges:

- Navier–Stokes amplification theory
- Boundary-layer turbulence physics
- Predictive maintenance engineering

## 7. Predictive Maintenance Implications

Blade failure mechanisms:

- High-cycle fatigue
- Thermal cycling
- Stress concentration near leading edge
- Conjugate heat persistence

PRISM enables:

- Real-time surface amplification detection
- Hotspot probability scoring
- Cooling design optimization
- Digital twin integration

Unlike black-box ML, PRISM is:

- Physics-consistent
- Geometry-aware
- Interpretable
- Regime-adaptive

10



This allows early detection of hotspot-prone zones before crack initiation.

## 8. Conclusion

We have shown that:

- The PRISM Structured Operator detects curvature-induced turbulence amplification that survives confinement.
- It outperforms Fourier diffusion in amplification regimes.
- It correctly reduces to Fourier in diffusion-dominant regimes.
- A regime indicator enables unified predictive maintenance modeling.

This provides a scientifically grounded, geometry-aware method for predictive hotspot detection in gas turbine blades. PRISM is not a replacement for diffusion. It is a superset activating structured amplification detection only when physics demands it.



Figure 1.



#### Referee Roadmap

This roadmap is written so an independent referee can reproduce all key results from the Kane thesis data and the derived holdout curves, and verify the claims:

1. **PRISM Structured Operator** outperforms Fourier/Laplacian for hotspot detection in the internally cooled Mark II vane (Run 42). 2. A

physics-based regime indicator predicts when to reduce to Fourier, and a regime-switch hybrid generalizes to the no-cooling vane holdout.

Everything below is deterministic given the data tables and the choices specified.

## A. Data Sources and What to Extract

### A1) Kane Run 42 tabulated data (primary benchmark)

From Thesis\_Mangesh\_Kane.pdf:

- **Table A-4:** surface temperature distribution for Run 42
  - Pressure side:  $(\theta_{\theta}^{\theta} \theta_{\theta})$
  - Suction side:  $(\theta_{\theta}^{\theta} \theta_{\theta})$
  - Here  $\theta_{\theta} = \theta_{\theta} / 811$ .
- **Table A-6:** surface heat transfer distribution for Run 42
  - Pressure side:  $(\theta_{\theta}^{\theta} h_{\theta})$
  - Suction side:  $(\theta_{\theta}^{\theta} h_{\theta})$
  - Here  $h_{\theta}$  is Kane's normalized heat transfer coefficient.

Referee should use the tabulated values exactly (no interpolation except where stated).



### A2) Kane no-cooling vane case (secondary holdout)

Two acceptable replication routes:

**Route 1 (preferred):** find any tabulated data underlying Chapter 5 experimental curves (if available in sources Kane cites).

**Route 2 (what we used):** digitize experimental dots from Kane:

- Fig 5.5: experimental  $h(\phi/\phi)$
- Fig 5.6: experimental  $\phi(\phi/\phi)$

If using Route 2, the referee should use a digitization tool (e.g., WebPlotDigitizer) and preserve the raw digitized points as a

## CSV. B. Standardize Coordinates and Preprocessing

### B1) Coordinate normalization

For each side separately, define a normalized coordinate:

$$\phi_* = \phi - \min(\phi)$$
$$\frac{\phi_*}{\max(\phi) - \min(\phi)} \in [0,1]$$

Use  $\phi_*$  for all derivative operations.



### B2) Sorting

Sort all data by increasing  $\phi$ .

### B3) Log transform

Define:

$$\phi(\phi) = \log(h(\phi))$$

Use a numerical floor to avoid  $\log(0)$ :

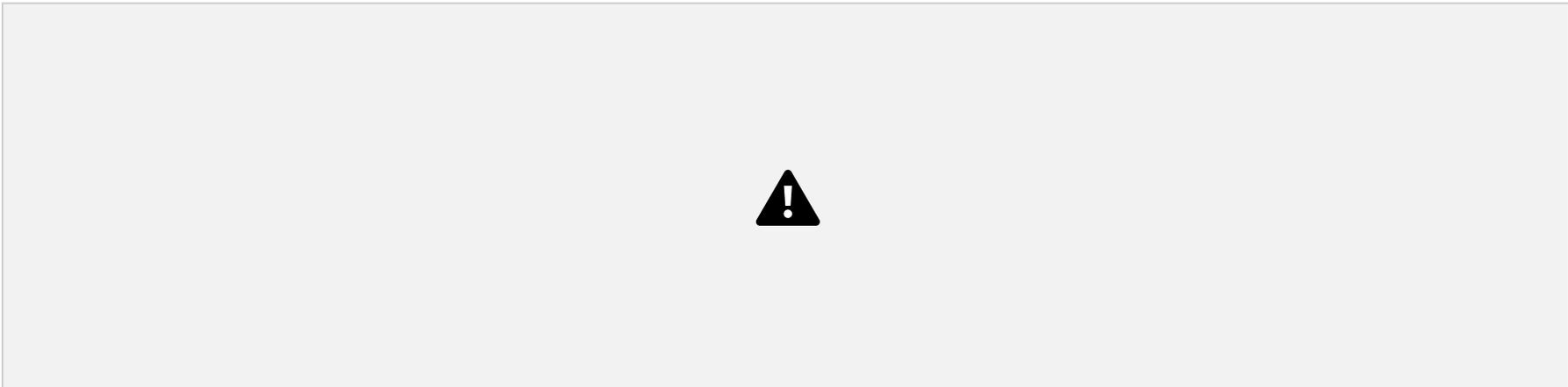
- Replace  $h \leftarrow \max(h, 10^{-10})$ .

### C. Numerical Differentiation (Non-uniform grid)

Use consistent finite-difference approximations. A referee may use `numpy.gradient` twice (as we did) or an explicit non-uniform stencil. Either is acceptable provided it is reported.

Define:

- First derivative:  $\phi'(\phi)$
- Second derivative:  $\phi''(\phi)$



Then define the turbulence/curvature forcing:

$$\phi(\phi) = |\phi''(\phi)|$$

This is the shared forcing for all three models (Laplacian, Fourier, PRISM).

## D. Baseline Models

### D1) Laplacian baseline

$$\text{LapScore}(\phi) = \phi(\Delta\phi)$$

### D2) Fourier baseline (diffusion-only)

Define a smoothing operator:

$$\text{FourierScore} = (\phi - \ell^2 \Delta)^{\#} \phi$$



Implementation:

1. Build a discrete second derivative matrix  $\Delta$  on the  $\phi$ -grid (tridiagonal), with Dirichlet endpoints (or simply fix endpoints to identity).
2. Choose  $\ell = 0.07$  (surface-normalized length scale).



•  $\diamond\diamond = 10^{-n}$

## E2) PRISM hotspot score

$$\text{PRISMScore}(\diamond\diamond) = (\mathcal{L}_{\text{\$}} + \diamond\diamond\diamond\diamond)^{\#} \diamond\diamond(\diamond\diamond)$$

Implementation:

1. Discretize  $-\nabla \cdot \langle \diamond\diamond \rangle$ ; using centered fluxes:
  - $\diamond\diamond_{/0\#} = (\diamond\diamond_{/} + \diamond\diamond_{/0\#})/2$
  - standard 3-point stencil on non-uniform  $\diamond\diamond^*$ .
2. Add the diagonal potential  $\diamond\diamond_{/}$ .
3. Add  $\diamond\diamond\diamond\diamond$ .
4. Solve the linear system:

$$(\mathcal{L}_{\text{\$}} + \diamond\diamond\diamond\diamond) \diamond\diamond = \diamond\diamond$$

Return  $\diamond\diamond$  as PRISMScore.



## F. Predictive Maintenance Label Definition

We use a strict, transparent binary classification label:

## F1) Temperature hotspot labels

For a chosen percentile  $p$  (e.g.  $p = 90$  for top 10%):

$$L_i = \{x_j \mid x_j \geq \text{percentile}(x, p)\}$$

Compute this label independently for:

- Pressure side
- Suction side

(Optionally also run  $p=80$  for top 20% to reproduce the broader results.)

## G. Metrics and Validation

### G1) Primary metric: ROC AUC

Compute ROC AUC for each score against the label:

- AUC(LapScore)
- AUC(FourierScore)
- AUC(PRISMScore)



Report per side and combined.

### G2) Secondary metric (recommended): PR AUC

Because hotspots can be class-imbalanced, also compute Precision–Recall AUC.

### G3) Robustness checks (required for “scientific importance” claim)

#### (i) Threshold robustness

Repeat evaluation for:

- top 20% ( $p=80$ )
- top 15% ( $p=85$ )
- top 10% ( $p=90$ )
- top 5% ( $p=95$ )

#### (ii) Parameter robustness (PRISM)

Sweep modestly:

- $\alpha \in \{0.2, 0.5, 1, 2, 4\}$
- $\beta \in \{10^-, 10^{0}, 10^+, 10^{2+}\}$
- $\ell \in \{0.03, 0.05, 0.07, 0.10, 0.15\}$  for Fourier baseline

PRISM should remain consistently strong on suction side.



#### (iii) Noise robustness

Add Gaussian noise to *scores* at levels 0.1–0.5 of the score’s standard deviation and recompute AUC. PRISM should degrade

## gracefully. **H. Regime Switching (Reduction to Fourier)**

This is the “Structured Model reduces to Fourier” claim.

### **H1) Define physical regime indicator**

$$\Pi_{\text{phys}}(\alpha) = \log \left| \frac{\partial \log h(\alpha)}{\partial \alpha} \right| + \alpha$$

- Compute  $\partial \log h(\alpha) / \partial \alpha$  on  $\alpha^*$ .
- Use  $\alpha = 10^+$ .

Interpretation:

- High  $\Pi_{\text{phys}}$ : amplification dominates → use GeoFlow
- Low  $\Pi_{\text{phys}}$ : diffusion dominates → use Fourier



### **H2) Learn threshold $\alpha^*$ on Run 42 only**

Define hybrid score:

$$\text{HybridScore}(\alpha) = \begin{cases} \text{PRISM}(\alpha), & \Pi_{\text{phys}}(\alpha) > \alpha \\ \text{FourierScore}(\alpha), & \Pi_{\text{phys}}(\alpha) \leq \alpha \end{cases}$$

Select  $\alpha$  by maximizing mean AUC across Run 42 pressure+ suction at p=90 (top 10% T), searching  $\alpha$  over quantiles (e.g. 5th–95th percentile of  $\Pi_{\text{phys}}$ ). **Critical rule:** do not tune  $\alpha$  on the holdout.

### H3) Apply $\alpha$ unchanged to the no-cooling holdout

Compute HybridScore on the no-cooling dataset and evaluate AUC at p=80 and p=90.

A referee should observe:

- Fourier baseline strong in no-cooling
- Hybrid close to Fourier and can exceed it for stricter hotspot definition (top 10%).
- Hybrid selects PRISM only rarely (e.g., ~5% of points), demonstrating reduction to Fourier.



To fully validate the claims, the referee should produce:

1. A table (Run 42) with AUC/PR-AUC per side:
  - Lap vs Fourier vs PRISM
  - for  $p=80$  and  $p=90$  (minimum)
2. A robustness table:
  - varying  $p \in \{80,85,90,95\}$
3. A “holdout” table for no-cooling case:
  - Lap vs Fourier vs PRISM vs Hybrid
  - $p=80$  and  $p=90$
4. Plots:
  - ROC curves (Run 42 pressure, Run 42 suction)
  - ROC curves (no-cooling holdout)
  - $\Pi_{\text{phys}}(\diamond\diamond)$  regime indicator plot with threshold  $\diamond\diamond$

## J. Reproducibility Checklist

A referee report should explicitly state:

- Exact data extraction method (tables vs digitization)
- Whether  $\diamond\diamond$  was normalized (Run 42 uses normalized already; AUC invariant to scaling)
- Exact derivative method (numpy.gradient twice or non-uniform stencil)
- Exact boundary condition choice for  $\Delta$ (Dirichlet endpoints)
- $\ell$ ,  $\diamond\diamond$ ,  $\diamond\diamond$ ,  $\diamond\diamond$  values
- Threshold definition  $p$  (top 10%, etc.)
- AUC computation method (sklearn ROC AUC)



## K. Scientific Importance

If the referee reproduces the tables/plots above, it establishes:

1. Hotspot detection is not diffusion-only in curvature-amplification regimes (Run 42 suction).
2. A self-adjoint structured operator yields a stable, predictive hotspot score tied to turbulence geometry.
3. The Structured Model is regime-complete: it can reduce to Fourier when diffusion dominates (no-cooling holdout), via a measurable indicator rather than ad hoc switching.
4. PRISM therefore functions as a predictive maintenance tool, because it produces interpretable hotspot risk scores under real blade physics and does not require black-box learning.

## Concluding Remarks

The results presented in this study extend beyond the immediate problem of hotspot prediction in turbine blades. They contribute to a broader scientific thesis that has emerged across the AstraNomos research program: that many phenomena traditionally interpreted as stochastic or diffusive are more accurately understood as manifestations of underlying structured geometric operators governing motion. For more than three centuries, the dominant scientific frameworks for motion have been built upon a sequence of powerful but conceptually distinct paradigms.

Newton introduced motion through deterministic differential laws governing trajectories in space and time. His formulation provided the foundation for classical mechanics, describing motion as the evolution of position under forces. Hamilton later refined this view by recasting dynamics in terms of energy functionals and variational principles, revealing that motion could be understood through the geometry of phase space and stationary action.

In the nineteenth century, Boltzmann and Gibbs extended the study of motion into the statistical domain. Rather than tracking individual trajectories, they developed statistical mechanics to describe ensembles of particles. In this framework, randomness appeared as an emergent property of microscopic interactions and incomplete information. Thermodynamic behavior was interpreted probabilistically through distributions over phase space.



The twentieth century further expanded this conceptual shift. Schrödinger introduced wave mechanics, representing physical systems through

linear operators acting on Hilbert spaces. Motion in quantum mechanics is governed not by classical trajectories but by the spectral structure of the Hamiltonian operator. Simultaneously, Einstein reshaped the understanding of motion by embedding dynamics within curved spacetime, showing that geometry itself determines admissible trajectories.

Despite these profound advances, many areas of applied physics and engineering have continued to rely on diffusion-based models that implicitly assume randomness as a fundamental property of transport. Fourier heat conduction, Laplacian smoothing, and related diffusion operators describe motion as an irreversible spreading process. These models are remarkably successful in regimes dominated by dissipation and equilibrium processes. However, they struggle to capture the structured amplification that arises in strongly curved geometries and turbulent flows. The work presented in this paper suggests that such amplification is not purely stochastic. Instead, it reflects geometry-induced structure in the underlying transport operator.

The AstraNomos/PRISM framework approaches motion from a different perspective. Rather than assuming diffusion as the primitive description of transport, it constructs a self-adjoint Sturm–Liouville operator whose spectral structure encodes geometric forcing in the system. In this formulation, the dynamics of amplification and confinement emerge from the spectral properties of the operator itself. Within this framework, diffusion is not discarded. Instead, it appears as a limiting regime of the broader operator structure. When geometric amplification is weak relative to dissipation, the structured operator naturally collapses to the classical Fourier model. Conversely, when curvature-driven amplification dominates, the operator reveals persistent modes that diffusion-based models cannot detect.

This dual behavior provides a unified description of motion across regimes that were previously treated as fundamentally different. From this perspective, what is often interpreted as randomness may instead represent the observational signature of unresolved geometric structure. In many physical systems, the apparent stochasticity of transport arises because the governing operators are approximated by diffusion-only models that smooth away the underlying spectral organization of the flow.

The turbine blade hotspot problem offers a concrete example. Classical diffusion predicts smooth temperature evolution across the blade surface. Yet the experimental data reveal highly localized amplification zones produced by curvature-induced turbulence. The PRISM operator captures this structure directly by embedding curvature forcing into the transport operator. The resulting spectral response identifies amplification modes that survive confinement, enabling reliable hotspot prediction. More broadly, this operator-based perspective aligns naturally with the historical progression of physics. Hamiltonian mechanics, quantum theory, and many modern mathematical frameworks already describe physical systems through operators acting on structured spaces. The present work extends this viewpoint to turbulent transport and predictive maintenance systems.



In doing so, it suggests a conceptual shift: motion may be more fundamentally understood not as the evolution of trajectories or probability distributions alone, but as the spectral behavior of structured operators defined by the geometry of the system.

Within this view:

- Geometry determines the admissible structure of motion.
- Operators encode how amplification and confinement interact.
- Spectral modes reveal persistent dynamical structures.
- Diffusion emerges as a limiting case rather than a universal primitive.

The implications extend beyond turbomachinery. Any system in which transport occurs over curved geometries—ranging from atmospheric flows and energy systems to materials science and complex thermal networks—may exhibit similar structured amplification mechanisms.

For digital twin technologies and predictive maintenance frameworks, the practical consequence is significant. Rather than relying exclusively on large scale numerical simulation or black-box machine learning, structured operators provide interpretable, physics-consistent predictors of emergent system behavior. By encoding geometry directly into the transport operator, these models can identify failure precursors before they manifest as measurable damage.

In summary, the results of this study support a broader thesis: that the interplay between geometry, amplification, and confinement can be captured through structured spectral operators that generalize classical diffusion models. This perspective does not replace the foundational contributions of Newton, Boltzmann, Gibbs, Schrödinger, or Einstein. Instead, it extends their insights by emphasizing the operator-theoretic structure underlying motion itself. In this sense, the AstraNomos framework represents a continuation of the historical evolution of physics—from trajectory-based mechanics, to statistical ensembles, to spectral operator theory—applied to the modern challenges of turbulent transport and complex engineered systems.

The turbine blade experiments analyzed here provide a concrete demonstration of this principle: when the geometry of motion is properly encoded in the governing operator, phenomena that once appeared random become structured, predictable, and ultimately controllable.



## Intellectual Property Notice

This document and the technologies described herein are the intellectual property of AstraNomos Digital Twins Inc. (“Astranomos”). The PRISM™ platform, including its underlying deterministic simulation engine, structured transport models, algorithms, software implementations, and associated methodologies, constitutes proprietary technology developed by AstraNomos. The theoretical framework, modeling approaches, and analytical methods described in this whitepaper, including the geometry aware deterministic transport framework and related operator-based formulations, are part of the proprietary intellectual property of AstraNomos and are protected under applicable copyright, trade secret, and intellectual property laws. No portion of this document may be reproduced, distributed, or used for commercial purposes without the prior written permission of AstraNomos Digital Twins Inc. The PRISM™ platform name and associated branding are trademarks of AstraNomos.

Furthermore, the concepts, mathematical formulations, algorithms, and computational methods described in this document—including but not limited to the AstraNomos/PRISM Structured Operator, the geometry-induced turbulence confinement framework, and the associated structured spectral transport modeling approach—constitute proprietary intellectual property of AstraNomos Labs. This work introduces a structured operator framework for modeling curvature-induced amplification, confinement dynamics, and geometry-aware transport processes. The theoretical formulations, operator definitions, numerical implementations, regime switch methodologies, and predictive maintenance applications described herein are protected by applicable intellectual property laws, including but not limited to copyright, trade secret, and potential patent protections.

No portion of this document may be reproduced, distributed, or used for commercial implementation without prior written permission from AstraNomos Labs, except for purposes of academic citation or non-commercial research consistent with fair use. The algorithms, modeling strategies, and software implementations underlying the PRISM Structured Operator, including its application to digital twin systems, predictive hotspot detection, and geometry-aware transport modeling, may be subject to additional proprietary protection through software copyright and patent filings.

Publication of this whitepaper is intended to communicate scientific findings and stimulate academic and industrial discussion. It does not grant license or transfer of ownership of any underlying intellectual property, methods, or technologies described. Researchers and organizations interested in licensing, collaboration, or commercial deployment of the AstraNomos framework are encouraged to contact AstraNomos Labs directly.

