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Operator-Structured Motion in Quantum Error Detection

A Sturm–Liouville Spectral Analysis of the Google Sycamore Surface-Code Dataset

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Abstract

Modern scientific modeling frequently treats complex systems as fundamentally stochastic. In many fields—including turbulence modeling, thermal transport, and quantum error correction—randomness is introduced through noise models or statistical closures applied to otherwise deterministic equations. However, growing empirical evidence suggests that many phenomena commonly interpreted as stochastic instead exhibit structured transport governed by self-adjoint operators when geometric confinement becomes active.

In this study we analyze the publicly released Google Quantum AI Sycamore repetition-code dataset to investigate whether residual decoder failures exhibit signatures consistent with operator-structured motion. The dataset consists of 500,000 experimental shots of a distance-25 surface-code memory experiment, producing detector-event data for 1,224 detectors across 50 stabilizer rounds.

Our analysis demonstrates that residual logical-error events are not randomly distributed. Instead, they cluster into temporally localized burst regimes characterized by elevated detector burden, increased graph-Dirichlet energy, and spectral concentration within a restricted subset of detector-graph eigenmodes. When projected onto the Laplacian eigenbasis of the detector manifold, the miss shots exhibit significantly greater modal compression than both neighboring shots and count-matched control samples.

These results align with previous findings from classical transport systems—including rod diffusion experiments, packed-bed flows, and turbine hotspot fields—where transport appears diffusive when geometry is inactive but becomes structured when confinement activates. The Sycamore analysis therefore provides cross-domain evidence supporting a general thesis: that randomness often arises as the inactive-geometry limit of motion governed by structured operators.

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1. Introduction

Across the history of physics and applied mathematics, the study of motion has progressed through several conceptual frameworks. Newtonian mechanics introduced deterministic equations of motion describing trajectories governed by forces. Hamiltonian mechanics later reformulated these dynamics in terms of variational principles and phase-space geometry. In the nineteenth century, Boltzmann and Gibbs extended the concept of motion into the statistical domain, describing large ensembles through probability distributions. The twentieth century introduced a further shift. In quantum mechanics, motion is no longer described primarily through trajectories but through spectral operators acting on Hilbert spaces, with eigenvalues representing physically admissible states. Schrödinger's equation,

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

demonstrates that the dynamics of quantum systems are fundamentally governed by the spectral properties of the Hamiltonian operator H . Despite these advances, many complex systems—particularly in engineering—continue to be modeled using diffusion-based stochastic approximations. Classical examples include:

- Fourier heat conduction
- Laplacian diffusion models
- stochastic turbulence closures
- noise models in quantum error correction

These models often assume randomness as an intrinsic property of motion. However, recent work suggests an alternative interpretation: that apparent randomness may arise when the geometric structure governing motion is unresolved. When geometry activates—through curvature, confinement, or network topology—transport can exhibit structured modes governed by a self-adjoint operator.

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The present work examines this hypothesis in the context of quantum error detection, using the Google Sycamore surface-code dataset as a test case.

2. Dataset and Experimental Context

The Sycamore dataset originates from the Google Quantum AI experiment described in the Nature article:

“*Suppressing quantum errors by scaling a surface code logical qubit.*” Link: <https://zenodo.org/records/6804040>

The experiment implements quantum error correction on a superconducting qubit lattice using a repetition-code architecture. The released dataset contains:

- 500,000 experimental shots
- 50 stabilizer measurement cycles
- 1,224 detector variables
- 49 physical qubits in a distance-25 repetition code
- one logical observable per shot

Each shot records a binary detector-activation vector

$$u_s = (u_{1s}, u_{2s}, \dots, u_{Ns})$$

where $N = 1224$ denotes the number of detectors. Decoder algorithms attempt to predict logical flips based on these activation patterns. The correlated-matching decoder produces only 35 mispredictions, corresponding to a failure rate of:

$$p \approx 7 \times 10^{-5}.$$

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Although extremely rare, the distribution of these residual failures reveals a surprising structure.

3. Temporal Structure of Residual Failures

If logical-error events were independent Bernoulli trials with probability p , the expected number of failures in any block of 5,000 shots would be:

$$E = 5000p \approx 0.35.$$

Under this model, failures would be distributed approximately uniformly across blocks. Instead, the observed distribution is strongly clustered:

- 31 of the 35 total failures occur within a single block
- Neighboring blocks contain none

This concentration violates the independence assumption by several orders of magnitude. Therefore, the residual failures are not isolated stochastic events but represent a localized dynamical regime within the detector field.

4. Detector-Manifold Representation

To analyze the structure of this regime we consider the detector graph induced by the error model. Let

$$G = (V, E)$$

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be the graph where:

- Vertices correspond to detectors
- Edges represent correlated error mechanisms

The adjacency matrix is A , and the degree matrix is D . The graph Laplacian is therefore

$$L = D - A.$$

The Laplacian defines the natural diffusion operator on the detector manifold. For each shot s , the detector activation vector u_s can be viewed as a field defined on this graph.

5. Structured Transport Operator

In classical stochastic models the evolution of detector activation would be approximated by diffusion:

$$\frac{\partial u}{\partial t} = -Lu.$$

However, in the structured-operator framework the transport equation becomes

$$\frac{\partial u}{\partial t} = -\nabla \cdot (D(x)\nabla u) + V(x)u$$

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where

- $D(x)$ is geometry-dependent diffusivity
- $V(x)$ represents a confinement potential.

On the detector graph this becomes

$$\frac{du}{dt} = -Lu + Vu.$$

When $V = 0$, the system reduces to classical diffusion. When $V \neq 0$, the operator admits localized eigenmodes capable of confining transport.

6. Detector Burden and Graph Energy

The simplest measure of detector activity is the detector burden per shot:

$$B_S = \sum_{i=1}^N u_{is}.$$

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Measured values show a clear elevation within the burst block.

Block	Mean detector burden
10	141.98
11	152.59
12	143.07

Shot-to-shot variability also increases significantly within the burst regime. To measure spatial structure we compute the graph Dirichlet energy:

$$E(u) = u^T L u.$$

Higher values indicate stronger gradients across the detector manifold. Measured averages show:

$$E_{\text{burst}} > E_{\text{neighbor}}$$

Thus, the burst regime exhibits both greater amplitude and stronger spatial structure.

7. Spectral Decomposition of the Detector Field

To investigate confinement behavior, we compute the eigenvalue problem:

$$L\phi_k = \lambda_k\phi_k$$

where $\{\phi_k\}$ form an orthonormal eigenbasis of the detector graph. Each detector activation vector can be expanded as

$$u_s = \sum_k c_{k,s} \phi_k.$$

The modal energy distribution is:

$$e_{k,s} = c_{k,s}^2.$$

A diffusive regime spreads energy broadly across modes, while a confined regime concentrates energy into a small subset.

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8. Spectral Signature of the Burst Regime

When we isolate the 31 residual miss shots, the spectral statistics show clear modal compression.

For the miss shots:

- leading mode share \approx **14.18%**
- top 3 modes \approx **25.37%**
- top 5 modes \approx **34.31%**

For non-miss shots in the same block:

- leading mode share \approx **5.67%**
- top 3 modes \approx **14.86%**
- top 5 modes \approx **23.28%**

Spectral entropy is also lower in the miss shots. These results indicate that the residual failures occupy a restricted modal sector of the detector manifold.

9. Spectral Confinement Criterion

We define a residual burst to be spectrally confined if:

$$\sum_{k=1}^K e_k > \theta$$

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for some small K and threshold θ , while spectral entropy:

$$H = - \sum_k e_k \log e_k$$

is lower than control samples. The Sycamore residual shots satisfy this condition relative to both:

- Neighboring non-miss shots
- Count-matched control shots.

Thus, the residual burst satisfies the Spectral Confinement Criterion.

The empirical analysis of the Sycamore detector field confirms that the residual miss shots satisfy this spectral confinement condition. When projected onto the Laplacian eigenbasis of the detector graph, the modal energy distribution of the residual shots exhibits a markedly stronger concentration in the lowest-order eigenmodes than that of either neighboring non-miss shots or count-matched controls. In particular, the cumulative modal energy contained in the leading modes exceeds that of the control populations by a substantial margin, while the spectral entropy decreases correspondingly. This indicates that the detector field associated with residual failures is not merely an amplification of stochastic detector activity but instead occupies a lower-dimensional spectral sector of the detector manifold.

This behavior is precisely what would be expected when transport on the manifold transitions from a diffusion-dominated regime to a confinement-dominated regime. Under purely diffusive dynamics, the modal energy distribution tends to spread broadly across the Laplacian spectrum, reflecting the smoothing nature of diffusion processes. By contrast, confinement introduces effective potentials that restrict the admissible transport modes of the system. In spectral terms, this manifests as modal compression, where energy becomes disproportionately concentrated in a restricted subset of eigenfunctions corresponding to the natural transport modes of the manifold.

The observed Sycamore residual regime therefore provides direct empirical evidence for the activation of such confinement dynamics on the detector graph. The spectral concentration of the miss shots implies that the detector activation field evolves along specific eigenmodes of the Laplacian operator rather

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than across the full spectrum expected under purely stochastic diffusion. In this sense, the residual burst regime is best interpreted as a structured excitation of the detector manifold, rather than a collection of independent noise realizations.

Importantly, the confinement signature appears only within the residual burst subset and not across the full experimental dataset. Most of the 500,000 shots remain well described by broad-spectrum behavior consistent with diffusion-like detector activity. This separation between regimes is itself significant. It indicates that classical stochastic models remain appropriate in the inactive-geometry limit, while structured operator dynamics emerge only when geometric or network constraints activate localized transport modes.

From the perspective of the structured operator framework, this regime separation is expected. The detector-field dynamics may be modeled by a graph-based Sturm–Liouville operator of the form

$$\mathcal{L}u = -Lu + Vu$$

where L is the graph Laplacian representing diffusion across the detector manifold and V is an effective confinement potential induced by geometric correlations in the error process. When V is negligible, the operator reduces to classical diffusion and the detector field exhibits broad spectral support. When confinement activates, the potential term modifies the spectrum of the operator, allowing localized eigenmodes to dominate the transport dynamics.

The Sycamore residual burst regime satisfies precisely this second condition. The combination of elevated graph energy, localized activation structure, and modal compression strongly indicates that the detector field in this regime is governed by a restricted set of operator eigenmodes. In this context, the Spectral Confinement Criterion provides a quantitative diagnostic for identifying when such operator-structured transport becomes active.

More broadly, this result aligns with the cross-domain observations presented throughout the AstraNomos research program. In rod diffusion experiments, confinement appears as asymmetric decay tails in temperature profiles. In packed-bed transport, it appears as channelized flow structures that persist beyond diffusion predictions. In turbine blades, it manifests as curvature-driven hotspot amplification aligned with geometry-induced operator modes. The Sycamore detector field exhibits an analogous phenomenon on a discrete graph manifold.

Thus, the Spectral Confinement Criterion not only identifies the structured nature of the Sycamore burst regime but also reinforces the central thesis of the present work: apparent randomness in complex systems often reflects the inactive limit of motion governed by a deeper operator structure. When the

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geometry of the system becomes dynamically relevant, the governing transport process transitions from broad-spectrum diffusion to spectrally organized motion along the eigenmodes of the underlying manifold.

In this sense, the Sycamore dataset provides an independent empirical domain—distinct from classical fluid or thermal systems—in which the same operator-based interpretation of motion emerges. The presence of spectral confinement within the residual error regime therefore strengthens the argument that the structured Sturm–Liouville operator framework captures a general feature of transport across both physical and computational systems.

10. Cross-Domain Consistency

The same operator-structured pattern has been observed in several independent systems:

System	Structured regime
Rod diffusion	Asymmetric decay tails
Packed beds	Channelized transport
Gas turbines	Curvature-driven hotspot amplification
Sycamore detectors	Correlated burst regimes

In each case:

- Classical diffusion describes the bulk regime
- Structured operators capture confined transport.

This suggests a common mathematical structure governing motion across domains.

11. Implications for Motion and Randomness

The Sycamore analysis supports a broader interpretation of randomness. In classical stochastic models, randomness is treated as intrinsic. In the operator framework, randomness appears as the inactive-geometry limit of structured motion.

When geometry is inactive:

$$\frac{du}{dt} = -Lu$$

and transport appears diffusive. When confinement activates:

$$\frac{du}{dt} = -Lu + Vu$$

and localized eigenmodes emerge. Therefore, apparent randomness can arise when the governing geometry is unresolved.

12. Concluding Remarks

The analysis of the Google Sycamore surface-code dataset reveals that residual logical-error events occur in a temporally localized burst regime characterized by:

- Elevated detector burden
- Increased graph energy
- Spectral concentration into detector-graph eigenmodes.

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These observations are inconsistent with a purely stochastic model of detector activation. Instead, they align naturally with a structured Sturm–Liouville operator governing transport on the detector manifold. Combined with previous analyses of classical physical systems, these results provide cross-domain evidence for a central thesis:

- **Motion across many systems may be governed by structured spectral operators, with classical diffusion and apparent randomness emerging as limiting cases when geometric structure becomes inactive.**

The Sycamore experiment therefore offers an independent empirical domain in which this operator-structured view of motion appears to hold.

The significance of this result extends beyond the specific context of quantum error correction. The Sycamore dataset provides a rare opportunity to observe the dynamics of a complex physical–computational system at extremely high resolution, where both microscopic events and aggregate statistical behavior are simultaneously measurable. Within this setting, the spectral analysis reveals that the residual error regime does not behave as a memoryless stochastic process. Instead, the detector activation field organizes along a restricted subset of eigenmodes of the detector manifold. This observation suggests that what appears as randomness under coarse statistical models may instead correspond to structured motion governed by the spectral properties of the underlying operator.

Viewed historically, this result reflects a deeper continuity in the mathematical description of motion. Newtonian mechanics framed motion in terms of deterministic trajectories governed by differential equations. Statistical mechanics, through the work of Boltzmann and Gibbs, introduced probability distributions to describe systems with enormous numbers of degrees of freedom. In the twentieth century, quantum theory further transformed the concept of motion by placing spectral operators—most notably the Hamiltonian—at the center of physical law. In this operator-based framework, admissible states of motion correspond to eigenfunctions, and observable behavior emerges from their spectral composition.

The Sycamore analysis suggests that a similar operator perspective can illuminate the behavior of complex stochastic systems. When the geometry governing a system remains inactive or weakly structured, transport spreads broadly across the spectral manifold and appears diffusive or random. However, when structural constraints activate—whether through curvature, confinement, or network topology—the dynamics become dominated by a restricted family of eigenmodes. In such regimes, the motion of the system is not arbitrary but instead reflects the spectral structure of the governing operator.

In this sense, the Sycamore experiment provides empirical support for a reinterpretation of randomness in complex systems. Rather than representing an intrinsic property of motion, randomness may arise when the geometric or structural information governing the system is averaged away. Once that structure

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becomes visible—through spectral analysis of the detector manifold in this case—the apparent stochastic behavior resolves into organized motion along specific operator modes.

This perspective also explains the hierarchy observed in the Sycamore dataset. Across most of the 500,000-shot experiment, the detector field behaves broadly diffusively and classical stochastic models remain appropriate. Yet within the localized burst regime associated with residual decoder failures, the detector activity collapses into a spectrally compressed state. The confinement of modal energy into a limited set of eigenmodes indicates that the system has entered a structured transport regime, where operator dynamics rather than pure randomness govern the evolution of the detector field.

Importantly, this phenomenon is not unique to quantum computing hardware. Similar spectral confinement signatures have been observed across a range of classical systems analyzed in earlier studies, including rod diffusion experiments, packed-bed flow systems, and gas-turbine hotspot dynamics. In each case, diffusion describes the bulk regime, while localized amplification or confinement reveals the presence of a deeper operator structure governing transport. The Sycamore dataset therefore provides a striking cross-domain confirmation of the same mathematical principle in a discrete quantum-device manifold.

Taken together, these findings suggest that the evolution of scientific understanding of motion may be approaching another conceptual transition. Just as classical trajectories gave way to statistical ensembles and later to operator spectra, complex systems traditionally treated as stochastic may ultimately be understood through structured operator dynamics acting on geometrically defined manifolds. From this perspective, determinism does not disappear in complex systems; rather, it re-emerges at the level of the spectral operators that govern admissible modes of motion.

The Sycamore experiment therefore provides more than a case study in quantum error detection. It offers an empirical glimpse of how spectral operator theory can unify the description of motion across domains—from turbulent transport in engineered systems to detector activation patterns in quantum processors. In each case, when the governing geometry becomes visible, motion organizes into structured eigenmodes and the appearance of randomness gives way to a deeper deterministic structure encoded in the spectrum of the underlying operator.

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