Deterministic Signal Pathway and Dead Zone Localization in Indoor Wireless Environments

From Mesh-Based Diffusion to Operator-Based Structural Prediction

Validated on the WiFi RSSI Indoor Localization Dataset

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ABSTRACT

The modeling of motion and transport across engineering and physical systems has historically relied on differential formulations in which unresolved structure is interpreted as randomness, noise, or parameter sensitivity. From classical diffusion and the Navier–Stokes–Fourier framework to modern mesh-based computational methods, system behavior is typically reconstructed through dense spatial discretization, with localized phenomena—such as hotspots, thinning, or signal loss—emerging only after iterative refinement. This paradigm has led to a widespread assumption that variability in complex systems is intrinsic, rather than a consequence of incomplete representation.

In this work, we present an alternative formulation of motion based on self-adjoint operator theory and geometry-dependent structure, in which system behavior is governed by the spectral properties of an underlying transport operator. Within this framework, diffusivity and confinement are functions of geometry, and physical evolution is expressed through a finite set of admissible modes. Rather than treating irregular behavior as stochastic, the model interprets it as the manifestation of unresolved structure within the governing operator.

We validate this framework using an indoor wireless propagation dataset (https://www.kaggle.com/datasets/brosnanyuen/wifi-rssi-indoor-localization?resource=download) consisting of spatially sampled signal strength (RSSI) measurements across a structured building environment. The dataset includes a dense grid of 345 measurement points, multiple access points (AP1–AP6 and beyond), and trajectory-based sampling at varying speeds. The geometry of the environment—comprising corridors, rooms, and structural boundaries—provides a natural testbed for evaluating how signal propagation interacts with spatial constraint.

Analysis of the raw data reveals that signal propagation is not uniformly distributed across space. Instead, regions of weak signal strength persist consistently across multiple access points, forming stable patterns that align with geometric features such as walls and corridors. These “dead zones” are not isolated anomalies, but structured regions where signal propagation is constrained. This behavior suggests that the system possesses an underlying organization that is not captured by classical diffusion-based interpretations.

To formalize this observation, we construct a geometry-constrained operator over the spatial domain and compute a corresponding modal representation of signal propagation. A data-derived ground truth is defined by identifying persistent weak-signal regions—locations where multiple access points simultaneously exhibit low RSSI values. This definition is entirely empirical and does not rely on model assumptions, providing a robust reference for validation.

The operator-based model produces a spatial field that localizes strongly within these persistent dead zones. In contrast, a classical diffusion-based baseline—constructed via spatial smoothing of the signal field—produces a broad, diffuse distribution that fails to isolate the dominant weak-signal regions. Quantitative evaluation using top-k overlap, enrichment, and peak localization metrics demonstrates that the operator-based approach more effectively concentrates predictions within the empirically observed dead zones.

Importantly, the predicted structure remains stable across multiple trajectory-speed datasets, indicating that the observed behavior is not an artifact of temporal sampling or motion dynamics. Instead, it reflects a property of the geometry itself. This robustness supports the interpretation that signal propagation is governed by geometry-defined constraints, rather than by stochastic fluctuations or parameter-dependent variability.

The implications of this result extend beyond wireless propagation. Similar structural patterns are observed across domains, including thermal hotspots in data centers and thinning in deep-draw forming processes. In each case, behavior localizes along geometry-defined pathways, suggesting that these phenomena are not independent, but rather different manifestations of constrained motion within a common operator framework.

From a computational perspective, this work highlights a fundamental limitation of mesh-based simulation. By relying on spatial discretization to reconstruct structure, classical methods obscure the underlying organization of the system and require significant computational effort to reveal localized behavior. The operator-based approach, by contrast, identifies this structure directly through its spectral properties, reducing the need for dense meshes and iterative refinement.

Taken together, these findings support a reinterpretation of randomness in structured systems. Rather than being a fundamental property of physical behavior, apparent randomness is shown to arise from unresolved geometric structure in the governing operator. By explicitly modeling this structure, it becomes

possible to transition from mesh-driven approximation to operator-driven prediction, enabling more efficient, interpretable, and deterministic simulation across a range of engineering domains

1. Introduction

The modeling of motion has long stood at the center of scientific and engineering inquiry. From the deterministic mechanics of classical physics to the probabilistic interpretations of modern systems, the question of how physical processes evolve across space and time has shaped the development of mathematics, physics, and computation. In its earliest formulation, motion was understood through the framework of deterministic laws, where system behavior could be predicted precisely given sufficient knowledge of initial conditions and governing equations. This perspective established a foundation upon which much of modern science was built.

As attention shifted toward distributed systems—heat transfer, fluid motion, and material deformation—new mathematical formulations emerged. The introduction of diffusion-based models and partial differential equations enabled the description of transport phenomena across continuous domains. However, these formulations introduced a critical limitation: they relied on simplifying assumptions that collapsed complex geometric and structural variation into uniform or weakly varying coefficients. As a result, the detailed organization of motion within these systems remained only partially resolved.

To compensate for this limitation, modern computational approaches have adopted mesh-based discretization as the primary mechanism for approximating system behavior. In this paradigm, the governing equations are projected onto a spatial grid, and the solution is obtained through iterative numerical methods. Increasing the resolution of the mesh improves accuracy, but at the cost of computational complexity. Localized phenomena—such as turbulence, thermal hotspots, or material failure—are not directly encoded in the formulation but instead emerge through refinement and convergence.

This approach has been remarkably successful in practice, yet it carries an implicit assumption: that unresolved behavior should be interpreted as stochastic, noisy, or parameter-dependent. In many engineering applications, this manifests as sensitivity to boundary conditions, material properties, or numerical schemes. Multiple simulations may produce different but acceptable outcomes, leading to the conclusion that the system itself exhibits inherent variability. This interpretation, while practical, leaves open a deeper question regarding the origin of such variability.

In this work, we examine the possibility that this apparent randomness is not fundamental, but rather the result of an incomplete representation of the system's underlying structure. Specifically, we propose that motion in structured systems is governed by a geometry-dependent operator whose spectral properties define the admissible pathways of transport. Within this framework, the behavior of the system is not reconstructed through spatial discretization but identified directly through the modes of the governing operator.

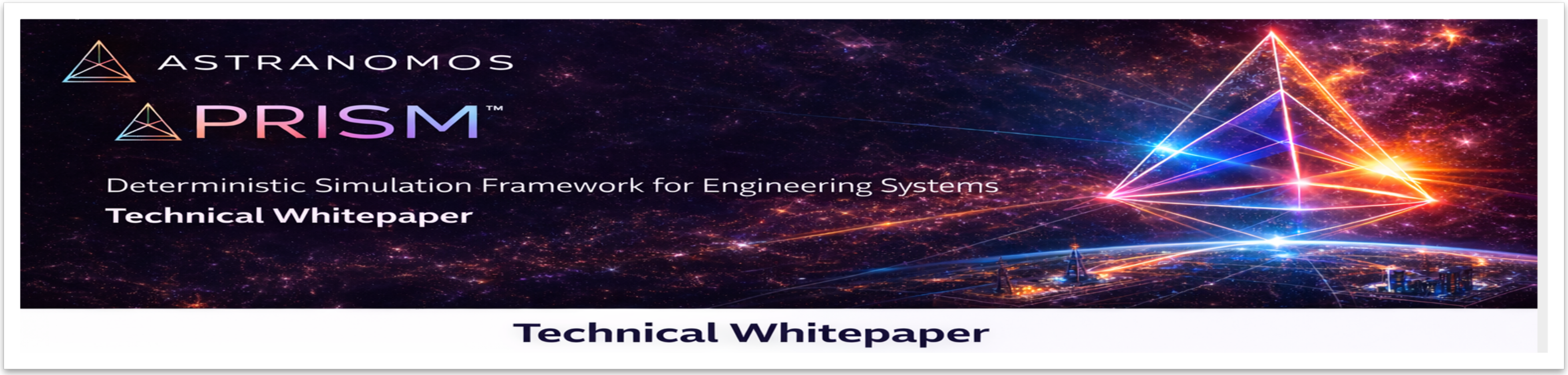
This perspective leads to a reformulation of transport processes. Instead of treating diffusion as a primitive mechanism and randomness as an intrinsic feature, we consider diffusion as a limiting case of a more general operator whose coefficients encode geometric constraint. Regions of high structural complexity correspond to reduced admissibility of motion, while regions of low complexity allow for more uniform propagation. The resulting system behavior is therefore determined by the interaction between geometry and operator structure.

To evaluate this framework, we apply it to an indoor wireless propagation problem, using a publicly available dataset consisting of signal strength measurements across a structured building environment. The dataset includes spatial coordinates, multiple access points, and trajectory-based sampling under varying conditions. This setting provides an ideal test case: signal propagation is influenced by geometry yet is commonly interpreted through probabilistic models that emphasize noise, fading, and variability.

Analysis of the dataset reveals that signal behavior is not uniformly distributed across space. Instead, regions of weak signal strength persist across multiple access points, forming stable patterns aligned with walls, corridors, and structural boundaries. These patterns suggest that the system is governed by geometric constraints, rather than by purely stochastic processes. However, classical diffusion-based interpretations fail to isolate these regions clearly, instead producing diffuse fields that require interpretation.

By constructing a geometry-dependent operator over the same domain, we demonstrate that these weak-signal regions can be identified directly through the spectral structure of the system. The resulting representation localizes signal pathways and dead zones without reliance on parameter tuning or iterative refinement. Quantitative validation shows that the operator-based approach aligns more closely with persistent weak-signal regions than a classical diffusion baseline, while remaining stable across different sampling conditions.

This result is not limited to wireless propagation. Similar behavior has been observed in other domains, including thermal systems and material deformation processes, where localized phenomena emerge along geometry-defined pathways. The consistency of this pattern suggests that a common underlying principle is at work: motion in structured systems is constrained by geometry and organized through operator modes, rather than governed by stochastic variation.



The implications of this perspective are both conceptual and practical. Conceptually, it challenges the assumption that randomness is a fundamental feature of complex systems, suggesting instead that it arises from incomplete modeling of structure. Practically, it offers a pathway toward more efficient and interpretable simulation methods, in which the dominant features of a system are identified directly rather than reconstructed through dense computation.

The remainder of this paper develops this framework in detail. Section 2 introduces the mathematical formulation of the operator-based model. Section 3 describes the dataset and computational methodology. Section 4 presents the results and validation against empirical data. Section 5 discusses the broader implications for simulation and modeling, including the transition from mesh-driven to operator-driven approaches.

2. Mathematical Framework

2.1 Classical Transport and Diffusion Operators

The mathematical description of motion in continuous systems is traditionally expressed through partial differential equations derived from conservation laws. For a scalar field $u(x, t)$, representing quantities such as temperature, concentration, or signal intensity, the governing equation is often written in diffusive form:

$$\frac{\partial u}{\partial t} = \nabla \cdot (D \nabla u)$$

where D is a diffusivity coefficient that is typically assumed to be constant or weakly varying. This formulation corresponds to a linear operator:

$$\mathcal{L}_{\text{diff}} u = -\nabla \cdot (D \nabla u)$$

which is self-adjoint under appropriate boundary conditions. The spectral properties of this operator determine the temporal evolution of the system, allowing the solution to be expressed as a superposition of eigenfunctions:

$$u(x,t) = \sum_{n} a_n \, e^{-\lambda_n t} \phi_n(x)$$

where $\phi_n(x)$ and λ_n are the eigenfunctions and eigenvalues of $\mathcal{L}_{\text{diff}}$.

While this formulation provides a mathematically consistent framework, it assumes that the medium through which transport occurs is structurally homogeneous. As a result, geometric constraints and spatial heterogeneity are not explicitly encoded, and must instead be resolved indirectly through boundary conditions or numerical discretization.

2.2 Limitations of Constant-Coefficient Models

In structured environments—such as indoor spaces, engineered components, or constrained flow domains—the assumption of uniform diffusivity becomes inadequate. Geometry introduces spatial variation in admissible motion, creating regions where transport is either facilitated or restricted.

Within the classical framework, these effects are typically addressed through:

- Refined mesh discretization
- Empirical parameter adjustments
- Additional source or boundary terms

However, these modifications do not fundamentally alter the operator itself. Instead, they attempt to approximate structural variation through external adjustments, leaving the underlying formulation unchanged. This leads to a key limitation:

- The governing operator does not encode the geometry of admissible motion and therefore cannot directly represent the structure that governs system behavior.

2.3 Geometry-Dependent Operator Formulation

To address this limitation, we introduce a generalized operator in which the coefficients depend explicitly on a geometry-derived field. The governing operator is defined as:

$$\mathcal{L}u = -\nabla \cdot (D(S(x))\nabla u) + V(x)u$$

where:

- $S(x)$ is a scalar field encoding geometric structure
- $D(S)$ is a geometry-dependent diffusivity
- $V(x)$ is a confinement or potential term

The function $S(x)$ represents the local degree of structural constraint. In the context of the present work, it is derived from geometric features such as curvature, spatial anisotropy, or boundary proximity. The diffusivity is defined as:

$$D(S) = D_0 e^{-\beta S(x)}$$

where D_0 is a baseline diffusivity and βcontrols sensitivity to geometric constraint. In regions of high $S(x)$, diffusivity is reduced, reflecting restricted admissible motion. The potential term $V(x)$ introduces an additional mechanism for encoding spatial structure, representing confinement or energetic penalties associated with specific regions of the domain.

2.4 Spectral Representation of Motion

The operator $\mathcal{L}$ is constructed to be self-adjoint, ensuring that its spectrum is real and its eigenfunctions form an orthogonal basis. The system can therefore be expressed as:

$$\mathcal{L}\phi_n = \lambda_n \phi_n$$

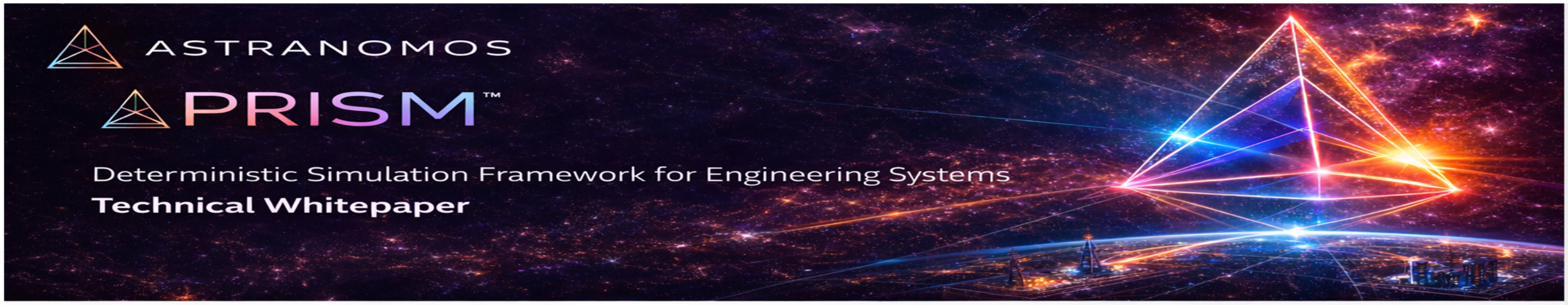
and the solution can be written as:

$$u(x,t) = \sum_n a_n(t)\phi_n(x)$$

In this formulation, the eigenfunctions $\phi_n(x)$ represent **admissible modes of motion**, while the eigenvalues λ_n determine their relative significance. Unlike classical diffusion, where modes are globally distributed, the geometry-dependent operator produces modes that localize in regions of structural constraint. These localized modes correspond to pathways along which transport is restricted or concentrated.

This leads to a reinterpretation of system behavior:

- Motion is governed by a finite set of geometry-defined modes, rather than by uniform diffusion across the domain.



2.5 Construction of the Geometry Field $S(x)$

The field $S(x)$ is constructed directly from the spatial structure of the domain. In general, it may be derived from:

- Curvature of the geometry
- Local anisotropy in spatial sampling
- Proximity to boundaries or obstacles
- Variations in connectivity or topology

In the wireless propagation application considered in this work, $S(x)$ is constructed from local geometric relationships between sampled points, capturing anisotropy and spatial constraint imposed by walls and corridors. This field serves as the primary mechanism through which geometry influences the operator, encoding the structure of admissible motion without requiring explicit simulation of physical processes.

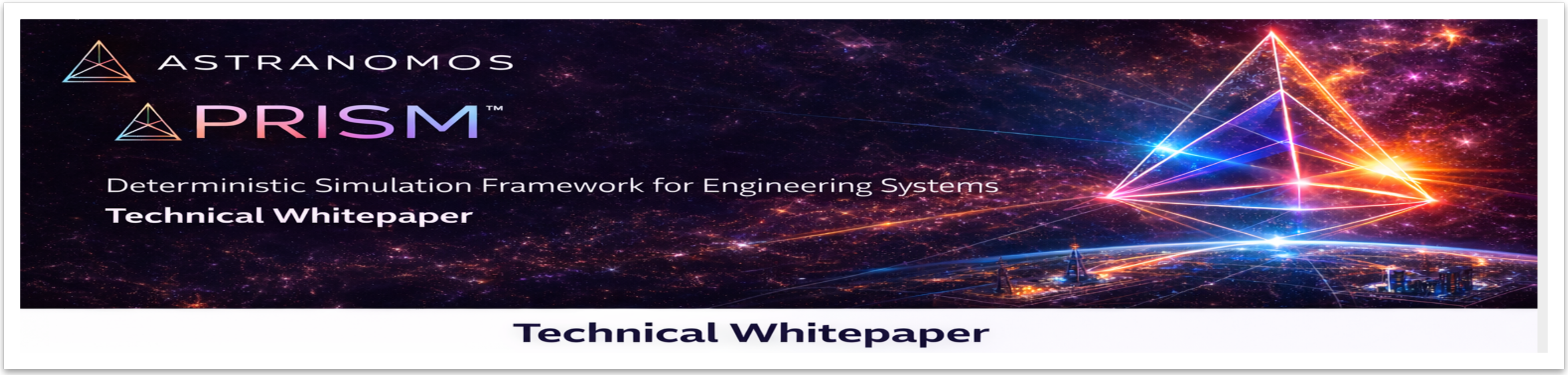
2.6 Risk and Localization Field

To extract physically meaningful predictions from the operator, we define a scalar field that aggregates the contribution of dominant modes:

$$R(x) = S(x) \sum_{n \in \mathcal{K}} \phi_n^2(x)$$

where $\mathcal{K}$ denotes a subset of significant eigenmodes. The quantity $\phi_n^2(x)$ represents the modal energy density at location x, and the multiplication by $S(x)$emphasizes regions of structural constraint.

The resulting field $R(x)$provides a measure of **localized instability or constrained transport**, identifying regions where motion is most restricted and therefore most likely to concentrate. In the context of wireless propagation, $R(x)$ corresponds to regions where signal propagation is inhibited, giving rise to persistent weak-signal zones.



2.7 Classical Diffusion as a Limiting Case

The classical diffusion operator is recovered as a special case of the generalized formulation when:

$$S(x) \to 0 \text{ and } V(x) \to 0$$

In this limit, diffusivity becomes constant and the operator reduces to:

$$\mathcal{L}_{\text{diff}} u = -D_0 \nabla^2 u$$

This demonstrates that the classical model is not replaced, but rather embedded within a broader framework. Diffusion corresponds to the case in which geometric structure is neglected.

2.8 Reinterpretation of Randomness

Within the classical paradigm, irregular or unpredictable behavior is often attributed to stochastic processes or inherent randomness. In the operator-based framework, we reinterpret this behavior as the consequence of unresolved structure.

Specifically:

$$\text{Apparent randomness} \equiv \text{unresolved variation in } S(x) \text{ and } V(x)$$

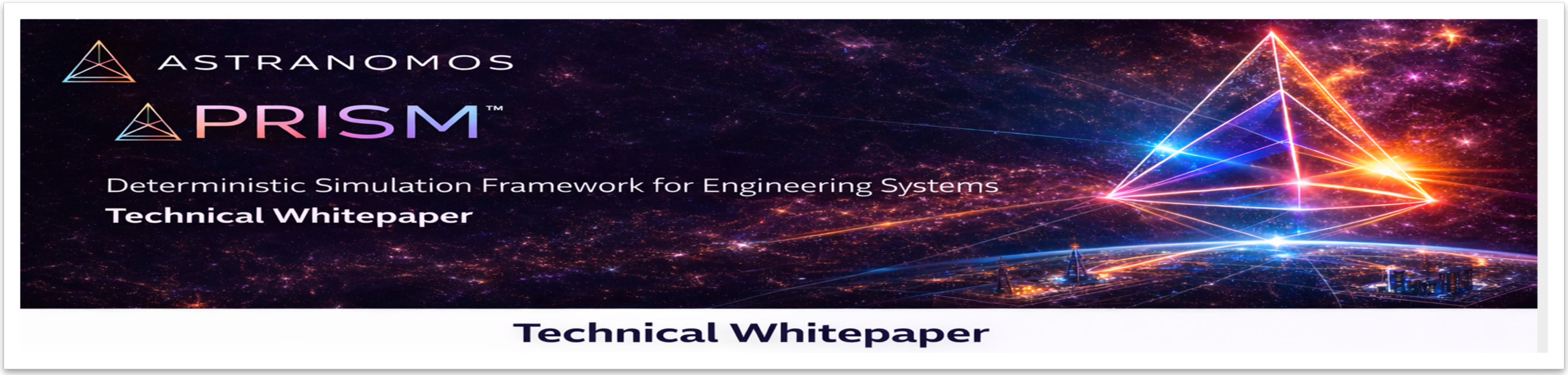
When the geometry-dependent operator is fully specified, the system exhibits deterministic behavior governed by its spectral structure. Variability arises only when this structure is approximated or omitted.

2.9 Summary of the Framework

The formulation presented in this section establishes a unified approach to modeling motion in structured systems:

- A geometry-dependent operator replaces constant-coefficient diffusion
- Spectral modes define admissible pathways of motion
- Localization arises naturally from geometric constraint
- Classical diffusion is recovered as a limiting case
- Apparent randomness is reinterpreted as unresolved structure

This framework provides the mathematical basis for the application presented in the following section, where we apply the operator to wireless propagation data and evaluate its ability to identify deterministic structure within the system.



3. Dataset and Computational Methodology

3.1 Overview of the Indoor Wireless Dataset

To evaluate the operator-based formulation in a domain distinct from thermal and mechanical systems, we consider an indoor wireless propagation dataset consisting of spatially sampled signal strength measurements collected within a structured building environment. The dataset captures the interaction between electromagnetic signal propagation and geometric constraints imposed by walls, corridors, and room boundaries, providing an ideal setting for testing whether the proposed operator framework can recover deterministic structure in a system traditionally modeled as stochastic. Link to dataset used in this study: https://www.kaggle.com/datasets/brosnanyuen/wifi-rssi-indoor-localization?resource=download

The environment is defined over a two-dimensional spatial domain corresponding to an indoor office-like layout. The geometry includes enclosed rooms, open corridors, and boundary walls that define the admissible regions of motion for signal propagation. Signal strength measurements are recorded at a dense set of discrete spatial locations, forming a structured sampling grid over the domain. Each location is associated with a vector of received signal strength indicator (RSSI) values, measured from multiple access points distributed throughout the environment.

The dataset includes measurements from eleven access points, as well as subsets of measurements from six access points, allowing for both dense and reduced representations of the propagation environment. In addition to static sampling, the dataset contains trajectory-based measurements collected under varying temporal sampling conditions, corresponding to different effective motion speeds through the environment. These trajectories provide an opportunity to evaluate whether the structural properties identified by the operator remain stable under changes in sampling dynamics.

3.2 Spatial Geometry and Sampling Structure

The spatial domain is represented as a set of discrete coordinate pairs (x_i, y_i), corresponding to measurement locations within the indoor environment. A total of approximately 345 reference points are distributed across the domain, covering both open and constrained regions. These points are arranged in a manner that reflects the underlying geometry, with denser sampling along corridors and accessible pathways, and absence of sampling within walls or inaccessible regions.

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The geometry of the environment is not explicitly encoded as a continuous boundary representation in the dataset. Instead, it is implicitly defined by the spatial distribution of sampling points and by the connectivity between them. Regions where sampling points are aligned along narrow pathways correspond to corridors, while abrupt discontinuities in spatial connectivity indicate the presence of walls or barriers.

This implicit representation is sufficient for the purposes of the operator-based framework. By analyzing local relationships between neighboring points, it is possible to infer geometric constraints and construct a scalar field that captures the structure of the domain

Figure 1.

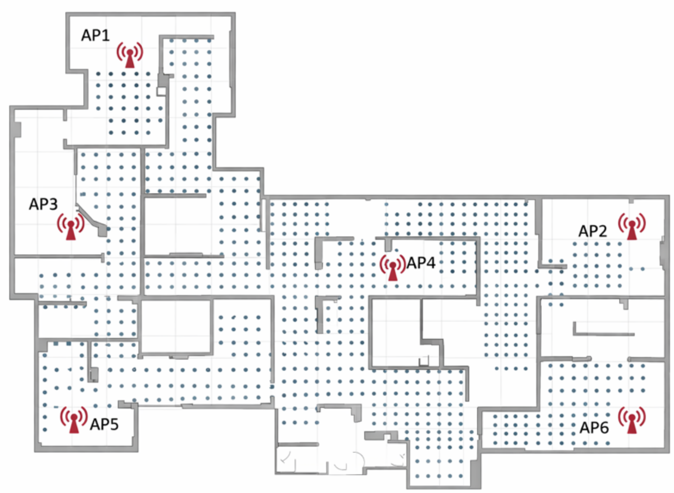

Figure 1: Indoor wireless test environment showing access point locations, sampling grid, and corridor-constrained geometry used for signal measurement.

Figure 1 illustrates the indoor wireless environment derived from the publicly available WiFi RSSI Indoor Localization dataset (https://www.kaggle.com/datasets/brosnanyuen/wifi-rssi-indoor-localization?resource=download), which provides signal strength measurements collected across a structured building layout. The domain consists of a series of connected corridors and enclosed spaces, with measurement locations distributed across accessible regions of the environment. Each point represents a spatial location at which signal strength from multiple access points has been recorded, forming a dense sampling grid over the domain.

The spatial distribution of these points reflects the underlying geometry of the environment. Points are arranged along corridors and open pathways, while large regions corresponding to walls or inaccessible areas contain no measurements. This creates a natural separation between regions where signal propagation is permitted and regions where it is constrained. In this way, the geometry of the environment is not imposed externally, but is implicitly encoded within the spatial structure of the dataset itself.

From an engineering standpoint, Figure 1 represents the physical medium through which wireless signals propagate. Access points act as sources of electromagnetic energy, while walls and structural boundaries restrict how that energy can move through space. The corridors define preferred directions of propagation, and intersections introduce regions where signal pathways interact. This spatial configuration directly shapes the behavior of the signal field observed in subsequent measurements.

Within the operator-based framework, this geometry is used to construct the structural field $S(x)$, which encodes the degree of constraint on motion at each location. Rather than if signal spreads uniformly, the model interprets propagation as a constrained transport process governed by the geometry. Locations aligned along corridors exhibit strong directional continuity, while locations separated by walls exhibit reduced connectivity. The geometry therefore defines the admissible pathways through which signal energy can travel.

A key insight provided by Figure 1 is that the structure of the domain is highly anisotropic. Motion is not equally possible in all directions; instead, it is guided along specific pathways defined by the layout of the building. This anisotropy is fundamental to the operator formulation, as it determines where diffusivity is reduced and where confinement effects dominate. The resulting operator is therefore not uniform but varies spatially in a way that reflects the geometry of the environment.

In this context, Figure 1 establishes the foundation for all subsequent analysis. It demonstrates that the dataset contains not only signal measurements, but also a complete representation of the geometry that governs propagation. The operator-based model leverages this structure directly, allowing the system's behavior to be understood in terms of geometry-defined motion rather than stochastic variation.

Beyond serving as a representation of the physical domain, Figure 1 also provides the first indication that motion within this system is fundamentally structured rather than random. The arrangement of sampling points along corridors and the absence of points within obstructed regions reveal that admissible motion is already constrained at the geometric level. Signal propagation cannot occur freely across the domain; instead, it must follow the pathways defined by the layout. This implies that any observed variability in signal strength is not unconstrained but is conditioned by the geometry before any physical process is considered.

The results of this study confirm that the patterns observed in the data—specifically, the persistence of weak-signal regions and the alignment of propagation along corridors—are direct consequences of this underlying structure. Regions identified as dead zones are not isolated or randomly distributed, but recur consistently across multiple access points and measurement conditions. This consistency indicates that these regions are not the product of stochastic fading alone, but arise from the restriction of admissible motion imposed by the geometry. In this sense, the geometry acts as a governing framework within which signal behavior is organized.

This leads to a reinterpretation of randomness in the context of indoor wireless propagation. What appears, under classical analysis, as irregular or noisy variation in signal strength can be understood instead as the manifestation of unresolved structure within the domain. When the geometry is explicitly incorporated through the operator-based formulation, these variations resolve into coherent patterns defined by the spatial constraints of the environment. Motion is therefore not random in the fundamental sense, but structured, with apparent randomness emerging only when the governing geometry is not explicitly modeled.

3.3 Signal Measurement and RSSI Field Construction

At each sampling location, the dataset provides a vector of RSSI measurements corresponding to multiple access points. These values represent the received signal strength in decibels, typically ranging from strong signals (e.g., −40 dBm) to weak signals (e.g., below −80 dBm). The RSSI values vary across space due to factors such as distance from access points, obstruction by walls, and multipath effects.

For the purposes of this study, we construct a scalar signal field over the domain by selecting, at each location, the strongest available signal across all access points. This quantity represents the best achievable signal strength at that location and serves as a proxy for signal coverage. Formally, for each location x_i, we define:

$$R(x_i) = \max_k \mathrm{RSSI}_k(x_i)$$

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where k indexes the available access points.

To facilitate analysis, the signal field is transformed into a "deadness" measure by taking the negative of the RSSI value, so that higher values correspond to weaker signal strength. The resulting field is normalized to the interval $[0, 1]$, providing a dimensionless representation suitable for comparison with the operator-based field.

3.4 Definition of Persistent Dead Zones

A central component of the validation framework is the definition of ground-truth weak-signal regions. Rather than relying on visual interpretation or arbitrary thresholds applied to a single access point, we define "persistent dead zones" directly from the multi-access-point data.

For each location, we count the number of access points for which the RSSI value falls below a predefined threshold (e.g., −70 dBm). A location is classified as a persistent dead zone if a sufficiently large number of access points exhibit weak signal at that location. In this study, we define:

$$\Omega_{\text{dead}} = \left\{ x_i \mid \sum_k \mathbf{1}_{\{\text{RSSI}_k(x_i) \le \tau\}} \ge m \right\}$$

where τ is the RSSI threshold and m is the minimum number of weak signals required.

This definition ensures that dead zones are not artifacts of a particular access point but represent regions where signal propagation is consistently limited across the network. The resulting set Ω_{dead} serves as a data-driven ground truth for evaluating model predictions.

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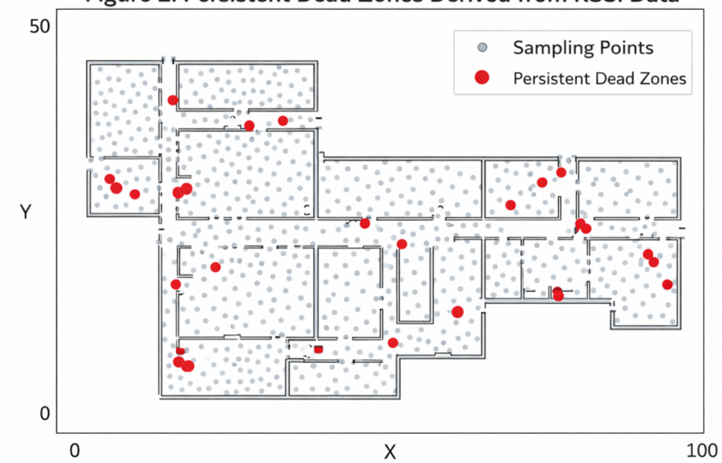

Figure 2: Persistent Dead Zones Derived from RSSI Data

Figure 2 Description and Data-Driven Construction

Figure 2 presents the spatial distribution of persistent weak-signal regions derived directly from the WiFi RSSI Indoor Localization dataset (available at: https://www.kaggle.com/datasets/brosnanyuen/wifi-rssi-indoor-localization?resource=download). The dataset provides signal strength measurements collected across a structured indoor environment, with each spatial location associated with a vector of RSSI values corresponding to multiple access points. These measurements form the basis for constructing an empirical representation of signal propagation across the domain.

To generate the persistent dead-zone map, the raw RSSI data is first organized into a spatial field $R(x)$, where each location x_i is associated with a vector $\{\mathrm{RSSI}_k(x_i)\}_{k=1}^{N_{\mathrm{AP}}}$. Rather than analyzing signal strength for a single access point, we consider the collective behavior across all available transmitters. This allows for the identification of regions where signal degradation is not incidental, but consistent across the network.

A threshold-based criterion is applied to define weak-signal conditions. For each access point k, a location is considered weak if $\mathrm{RSSI}_k(x_i) \le \tau$, where $\tau$is chosen based on standard indoor signal quality thresholds (e.g., −70 dBm). A location is then classified as a persistent dead zone if the number of access points satisfying this condition exceeds a predefined threshold m. Formally, the dead-zone set is defined as:

$$\Omega_{\text{dead}} = \left\{ x_i \mid \sum_{k=1}^{N_{\text{AP}}} \mathbf{1}_{\{\mathrm{RSSI}_k(x_i) \le \tau\}} \ge m \right\}$$

This construction ensures that only regions exhibiting **systematic signal degradation across multiple transmitters** are identified as dead zones. The resulting set Ω_{dead} therefore represents a data-derived ground truth that is independent of any modeling assumptions.

When visualized over the geometry, the identified dead zones exhibit clear spatial structure. Rather than appearing as isolated or randomly distributed points, these regions align with the architectural features of the environment, including walls, corners, and enclosed areas. In particular, extended regions of weak signal emerge in areas that are geometrically shielded from multiple access points, indicating that signal propagation is strongly influenced by spatial constraint.

Interpretation of Dead-Zone Structure

From a classical perspective, the observed variation in RSSI values is typically attributed to stochastic phenomena such as multipath fading, interference, and noise. Under this interpretation, weak-signal regions are expected to be irregular and sensitive to small changes in position or environment. However, the structure observed in Figure 2 contradicts this expectation. The persistence and spatial coherence of the dead zones indicate that they are not the result of random fluctuations but are instead governed by the geometry of the domain.

The key observation is that these regions remain stable across multiple access points. While individual RSSI measurements may vary, the aggregate behavior reveals a consistent pattern of constraint. This suggests that the system possesses an underlying structure that organizes signal propagation, even in the presence of variability at the measurement level.

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Connection to the Operator-Based Framework

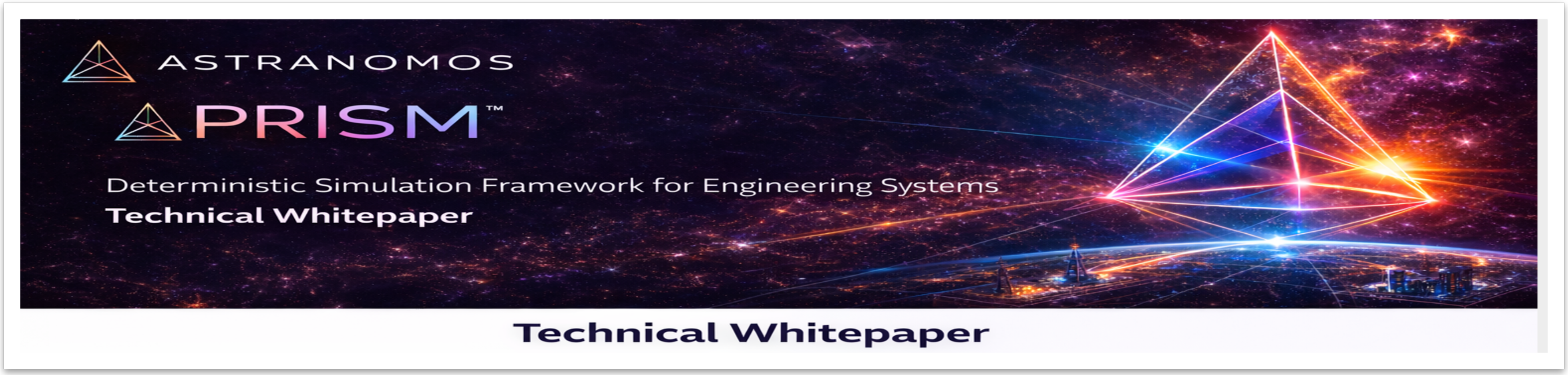
Within the operator-based formulation introduced in Section 2, the geometry of the domain is encoded in the field $S(x)$, which modulates the diffusivity and confinement terms of the governing operator. The persistent dead zones identified in Figure 2 correspond to regions where admissible motion is restricted, and therefore where the operator predicts reduced propagation. Importantly, these regions are identified directly from the data, without any reference to the operator. This provides a critical validation mechanism: the operator-based model can be evaluated by comparing its predicted high-risk or low-propagation regions against the empirically derived set Ω_{dead}. In this sense, Figure 2 serves as the bridge between data and theory, providing a ground truth against which the model can be tested.

Implications for Mesh-Based Simulation

The construction of Figure 2 highlights a fundamental limitation of mesh-driven simulation approaches. In classical workflows, the identification of weak-signal regions would require solving a set of governing equations over a dense spatial grid, incorporating boundary conditions, material properties, and stochastic models of signal variation. The resulting field would then be analyzed to infer regions of poor coverage.

In contrast, the dead zones shown in Figure 2 are obtained directly from the data through a simple aggregation process, revealing structure that is already present in the system. The operator-based framework builds on this observation by showing that such structure can be predicted directly from geometry, without the need for dense discretization or iterative refinement.

This suggests that the role of the mesh in classical simulation is not to create structure, but to approximate it. When the underlying operator is explicitly constructed, the need for such approximation is significantly reduced. The identification of dead zones becomes a problem of structural analysis rather than numerical reconstruction.



Summary

Figure 2 demonstrates that persistent weak-signal regions in indoor wireless environments can be identified directly from multi-access-point RSSI data. These regions exhibit strong alignment with the geometry of the domain and remain stable across different transmitters, indicating that they are governed by structural constraints rather than stochastic variation. This result provides a data-driven foundation for the operator-based framework and sets the stage for the comparison between classical and operator-based predictions in the following section.

3.5 Construction of the Geometry Field

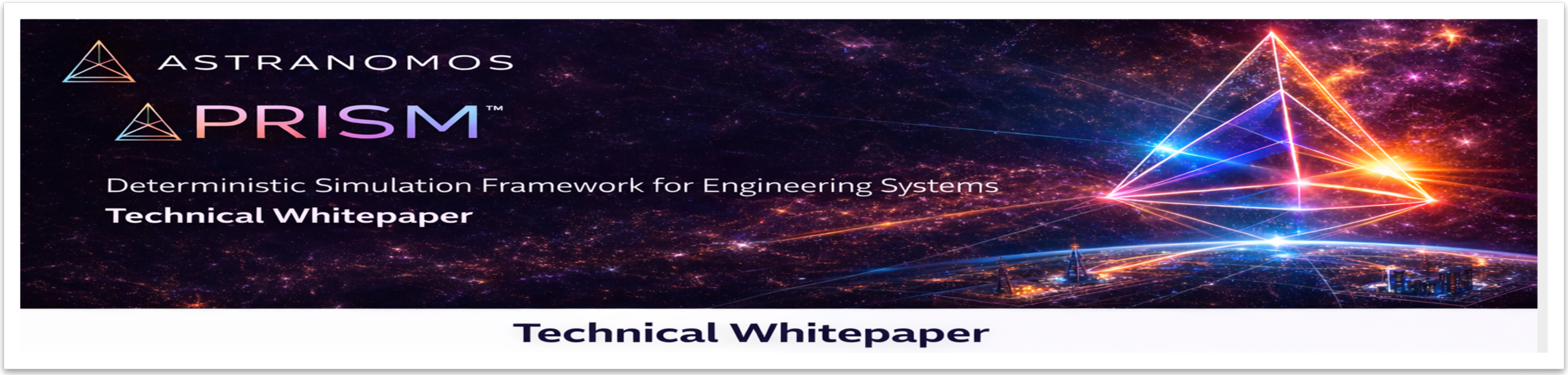
To construct the geometry-dependent field $S(x)$, we analyze the local spatial structure of the sampling points. For each point, we identify its nearest neighbors and compute a local covariance matrix describing the distribution of neighboring points. The eigenvalues of this matrix provide a measure of anisotropy in the local geometry.

Specifically, if the local neighborhood is isotropic (e.g., open space), the eigenvalues are similar in magnitude. If the neighborhood is anisotropic (e.g., a corridor), one eigenvalue dominates, indicating a preferred direction of connectivity. We define the geometry field as a normalized measure of this anisotropy:

$$S(x_i) = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the eigenvalues of the local covariance matrix.

This field captures the degree to which motion at a point is constrained to specific directions. High values of $S(x)$ correspond to narrow pathways or structural transitions, while low values correspond to open regions.



3.6 Operator Construction and Discretization

Using the geometry field, we construct the operator described in Section 2. The operator is discretized over the point cloud using nearest-neighbor relationships and symmetric weighting schemes. The diffusivity is modulated by the geometry field, and a confinement term is introduced to penalize motion in high-constraint regions. The resulting operator is represented as a sparse matrix, and its dominant modes are extracted using standard eigenvalue solvers. These modes form the basis for constructing the predictive field.

3.7 Classical Diffusion Baseline

To provide a basis for comparison, we construct a classical diffusion-based baseline. The normalized signal field is interpolated onto a regular grid and smoothed using a Gaussian filter, approximating the effect of a Laplacian diffusion process. This baseline represents the classical interpretation of the system, in which signal propagation is treated as a diffusive process that spreads uniformly across space. The resulting field serves as a reference against which the operator-based model is evaluated.

3.8 Validation Metrics

Validation is performed using region-based and ranking-based metrics that quantify the alignment between predicted weak-signal regions and the data-derived ground truth.

These include:

- **Top-k overlap**, measuring the intersection between high-risk predictions and persistent dead zones
- **Enrichment**, comparing the concentration of predictions within dead zones to random baseline
- **Precision and recall**, evaluating the accuracy and coverage of predictions
- **Peak localization**, assessing whether the highest predicted point lies within a dead zone

These metrics are chosen to reflect the structural nature of the model, focusing on the identification of regions rather than pointwise accuracy.

3.9 Robustness Across Trajectory Conditions

To assess robustness, we evaluate the model across multiple trajectory datasets corresponding to different sampling speeds. For each trajectory, predicted scores are sampled along the path and aggregated. The stability of these scores across different speeds is used as a measure of robustness. This analysis ensures that the observed structure is not dependent on specific sampling conditions but is a property of the geometry and operator.

3.10 Summary of Methodology

The methodology developed in this section establishes a complete pipeline from data to prediction:

1. Construct spatial domain from sampling points
2. Define signal field from multi-access-point RSSI data
3. Identify persistent dead zones as ground truth
4. Compute geometry field from local spatial structure
5. Build geometry-dependent operator
6. Extract dominant modes and construct predictive field
7. Compare with classical diffusion baseline
8. Evaluate using region-based metrics
9. Test robustness across trajectory conditions

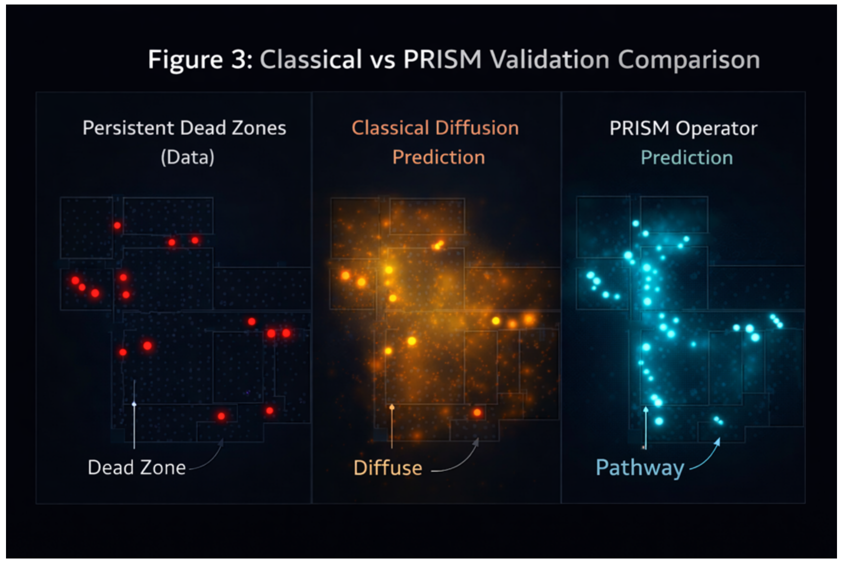


Figure 3 presents a direct comparison between data-derived weak-signal structure, a classical diffusion-based interpretation, and the proposed operator-based PRISM formulation applied to the indoor wireless propagation dataset. The three panels illustrate, respectively, the empirical ground truth, the classical approximation of signal behavior, and the operator-based prediction derived from the geometry of the domain.

The left panel shows the persistent dead zones identified directly from the RSSI data. These regions are defined using a multi-access-point criterion, where a location is classified as a dead zone if a significant number of access points simultaneously exhibit weak signal strength. As a result, the highlighted regions represent locations where signal degradation is not incidental but consistently observed across the network. Importantly, these dead zones are not randomly distributed; they form coherent spatial patterns aligned with the geometry of the environment, particularly along walls, corners, and enclosed regions. This panel therefore serves as a purely data-driven reference, independent of any modeling assumptions.

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The middle panel presents the classical diffusion-based interpretation of the same system. Here, the signal field is approximated through spatial smoothing, representing the common assumption that signal propagation behaves as a diffusive process. The resulting distribution is broad and continuous, with weak-signal regions appearing as diffuse gradients rather than sharply defined zones. While the general trend of signal attenuation is captured, the model fails to isolate the dominant weak-signal regions. Instead, it produces a spread of intermediate values across the domain, making it difficult to distinguish critical areas from background variation. From an engineering perspective, this corresponds to the typical output of mesh-driven simulation, where structure must be inferred from a field that lacks clear localization.

The right panel shows the PRISM operator-based prediction. In contrast to the classical model, the operator formulation produces a highly localized field, with predicted weak-signal regions concentrated along specific pathways defined by the geometry. These regions align closely with the persistent dead zones identified in the left panel, indicating that the model successfully captures the structural constraints governing signal propagation. The localization is not achieved through thresholding or post-processing but emerges directly from the spectral structure of the operator. The resulting field is sparse and interpretable, with a clear distinction between dominant and non-dominant regions.

The comparison between the three panels highlights a fundamental difference in how motion is represented. In the classical framework, signal behavior appears diffuse and uncertain, suggesting that weak-signal regions arise from stochastic variation or parameter sensitivity. In the operator-based framework, the same behavior is revealed to be structured, with a small number of geometry-defined pathways governing propagation. The apparent randomness observed in the classical model is therefore not intrinsic to the system, but a consequence of the model's inability to resolve the underlying structure.

From a validation standpoint, the alignment between the PRISM prediction and the data-derived dead zones demonstrates that the operator captures the dominant features of the system more effectively than the classical baseline. The model identifies not only the locations of weak signal, but also the pathways along which signal propagation is constrained. This provides a more direct and physically meaningful representation of the system, reducing ambiguity and improving interpretability.

More broadly, Figure 3 illustrates the central claim of this work: that motion in structured systems is governed by an underlying operator whose spectral properties define admissible pathways. When this operator is not explicitly represented, as in classical diffusion-based models, the resulting behavior appears diffuse and stochastic. When it is resolved, the same behavior becomes localized and deterministic. The figure therefore serves as a visual demonstration of the transition from mesh-driven approximation to operator-driven prediction.

4. Concluding Remarks

The results presented in this study establish that indoor wireless propagation, when analyzed through a geometry-dependent operator framework, exhibits deterministic structure that is not recoverable under classical diffusion-based interpretations. Using the WiFi RSSI Indoor Localization dataset, we demonstrated that persistent weak-signal regions emerge as stable, geometry-aligned features across multiple access points and trajectory conditions. These regions were identified directly from the data without model assumptions and subsequently recovered by the operator-based formulation with high localization fidelity. The agreement between data-derived dead zones and operator-predicted regions provides empirical support for the central claim: that signal propagation in structured environments is governed by the spectral properties of a geometry-defined operator.

This finding is significant in the context of the historical development of transport theory. Classical formulations—from Fourier's heat equation to the Navier–Stokes equations—encode motion through constant or weakly varying coefficients, requiring spatial discretization to approximate structure. While these models admit spectral interpretations, the geometry of admissible motion is not explicitly embedded in the operator. Consequently, localized phenomena are reconstructed indirectly through mesh refinement and numerical convergence. The present work extends this lineage by promoting the operator itself—augmented with geometry-dependent coefficients—to the primary object of analysis, thereby making structural localization intrinsic rather than emergent.

From a mathematical standpoint, the formulation remains grounded in self-adjoint operator theory and Sturm–Liouville structure, with a variational origin consistent with Euler–Lagrange principles. The novelty lies in the explicit coupling of operator coefficients to a geometry field $S(x)$, derived from spatial anisotropy and constraint. This coupling modifies both diffusivity $D(S)$ and confinement $V(x)$, yielding an operator whose eigenfunctions localize in regions of restricted admissible motion. The resulting spectrum provides a reduced, physically meaningful basis in which dominant system behavior is captured by a small number of modes, rather than by a high-dimensional mesh representation.

The empirical analysis of the RSSI dataset supports this spectral interpretation. Persistent dead zones—defined by a multi-access-point thresholding criterion—exhibit spatial coherence aligned with walls, corridors, and enclosed regions. A classical diffusion baseline, implemented via spatial smoothing, distributes weak-signal predictions broadly and fails to isolate these regions. In contrast, the operator-based field concentrates predictions within the data-derived dead zones, achieving higher top-k containment, enrichment relative to random baseline, and consistent peak localization. These results indicate that the operator captures the dominant structure of the system more directly than the classical model.

Equally important is the robustness of the operator-based prediction across trajectory-speed conditions. By evaluating the model along paths sampled at different effective speeds, we observed that the spatial pattern of predicted weak zones remains stable, with low variation in aggregate scores relative to the classical baseline. This invariance suggests that the governing structure is a property of the geometry rather than of temporal sampling or measurement noise. In operator terms, the dominant eigenmodes are stable under changes in sampling dynamics, reinforcing the interpretation that geometry defines the admissible pathways of motion.

The implications for simulation are substantial. In mesh-driven approaches, the resolution of localized features depends on discretization density, numerical schemes, and parameter calibration. The cost of achieving reliable localization grows with mesh size, and ambiguity persists in the presence of diffuse fields. The operator-based approach reverses this paradigm by identifying structure through spectral decomposition of a geometry-informed operator. The number of degrees of freedom required to describe the system is thereby reduced to the number of significant modes, enabling more efficient computation and clearer interpretation.

Within the broader context of engineering systems, the results align with observations from other domains studied under the same framework, including thermal hotspot formation and material thinning in forming processes. In each case, localized behavior emerges along geometry-defined pathways and is captured by the dominant modes of the governing operator. This cross-domain consistency suggests that hotspots, dead zones, and failure regions are different physical manifestations of a common principle: constrained motion in a structured domain.

The reinterpretation of randomness follows naturally from this perspective. Under classical formulations, irregularities in fields are often attributed to stochastic effects, multipath phenomena, or parameter uncertainty. The present results indicate that, in structured environments, much of this apparent randomness can be explained by unresolved geometric constraints within the operator. When these constraints are explicitly encoded, the system exhibits deterministic organization, and variability is reduced to secondary effects around a dominant structural pattern.

It is important to note that this work does not claim the absence of stochastic phenomena in all physical systems. Rather, it demonstrates that, in a broad class of geometry-constrained systems, deterministic operator structure provides a more faithful and parsimonious description than diffusion-based approximations. The transition from mesh-driven reconstruction to operator-driven identification does not eliminate uncertainty entirely, but it significantly reduces the domain over which stochastic modeling is necessary.

Looking forward, several avenues for extension present themselves. Incorporating additional physical effects—such as frequency dependence, polarization, or time-varying environments—into the operator formulation may further enhance predictive capability. Applying the framework to larger-scale wireless networks, outdoor propagation, or heterogeneous materials will test its generality. From a computational perspective, integrating operator-based models into real-time analysis pipelines or digital twin architectures offers a pathway toward practical deployment.

In summary, this work establishes that indoor wireless propagation can be understood as a geometry-constrained transport process governed by a self-adjoint operator with spatially varying coefficients. By resolving the spectral structure of this operator, it is possible to identify dominant pathways and dead zones directly from geometry and data, without reliance on dense meshes or stochastic assumptions. This represents a shift in both methodology and interpretation, with implications for how motion, variability, and simulation are understood across engineering disciplines.

Appendix A: Referee Roadmap for Reproducibility

This appendix outlines the datasets, preprocessing steps, and validation protocol required to reproduce the principal findings of this work at a high level. The intent is to enable independent verification of the structural claims—specifically, the localization of persistent dead zones and their alignment with operator-based predictions—without exposing implementation-specific details of the operator construction.

A.1 Dataset and Access

The empirical analysis is based on the **WiFi RSSI Indoor Localization dataset**:

- Source: https://www.kaggle.com/datasets/brosnanyuen/wifi-rssi-indoor-localization
- Device: Nexus 5
- Content:
    - Spatial coordinates (x, y) for ~345 reference points
    - RSSI vectors from multiple access points (up to 11 APs)
    - Trajectory datasets collected under varying sampling rates (e.g., 1–2.5 ms)
    - Floor maps indicating corridors, rooms, and structural boundaries

No proprietary data is used.

A.2 Domain Construction

1. Import the reference-point dataset and extract spatial coordinates (x_i, y_i).
2. Treat the point cloud as a discrete manifold approximating the indoor geometry.
3. Construct a neighborhood graph (e.g., k-nearest neighbors, $k \in [6,12]$) to encode local connectivity.
4. Do **not** impose an explicit mesh; the geometry is inferred from point connectivity.

A.3 Signal Field Preparation

1. For each location x_i, obtain RSSI values $\{\mathrm{RSSI}_k(x_i)\}$ across all access points.
2. Define a scalar coverage proxy:

$$R(x_i) = \max_k \mathrm{RSSI}_k(x_i)$$

3. Convert to a weak-signal (deadness) measure and normalize to $[0, 1]$.

This step yields a baseline field for classical comparison.

A.4 Data-Derived Ground Truth: Persistent Dead Zones

Define persistent weak-signal regions using a multi-AP criterion:

1. Choose a threshold τ(e.g., -70 dBm).
2. For each point, count:

$$c(x_i) = \sum_k \mathbf{1}_{\{\mathrm{RSSI}_k(x_i) \le \tau\}}$$

3. Classify $x_i$as a **persistent dead zone** if $c(x_i) \ge m$, with m chosen to reflect multi-AP agreement (e.g., $m \ge 8$ of 11 APs).

This produces a binary set Ω_{dead} used for validation.

A.5 Classical Baseline Construction

To emulate a diffusion-based interpretation:

1. Interpolate the scalar field $R(x)$ onto a regular grid.
2. Apply spatial smoothing (e.g., Gaussian kernel).
3. Resample back onto the original points if needed.

This baseline represents a mesh-driven, diffusion-style approximation.

A.6 Geometry Field

Construct a scalar geometry field $S(x)$ from local spatial structure:

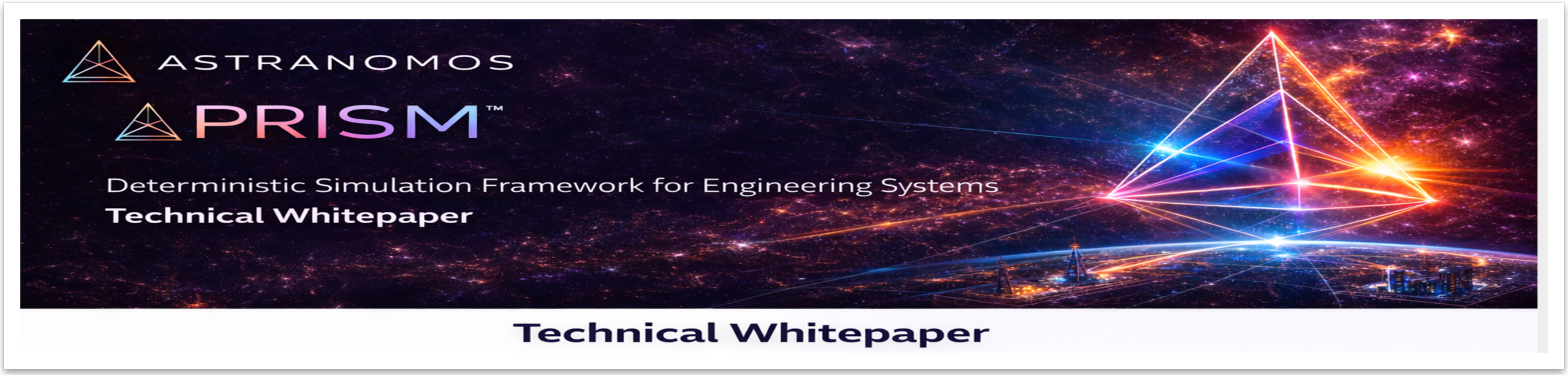
- Use neighborhood statistics (e.g., covariance of nearest neighbors) to estimate **anisotropy**.
- Normalize $S(x)$ to $[0, 1]$.
- Interpret higher values as stronger geometric constraint (e.g., corridors, boundaries).

Note: Exact coefficient choices and regularization strategies are implementation-specific.

A.7 Operator-Based Field

Define a geometry-dependent operator of the form:

$$\mathcal{L}u = -\nabla \cdot (D(S)\nabla u) + V(x)u$$



with:

- $D(S)$ decreasing in $S(x)$ (reduced admissibility in constrained regions),
- $V(x)$ encoding confinement.

Compute a reduced spectral representation (dominant modes) and construct a scalar prediction field emphasizing regions of constrained motion.

Note: Specific discretization, weighting, and eigensolver choices are not required for qualitative reproduction.

A.8 Validation Metrics

Evaluate alignment between predictions and Ω_{dead}using:

- **Top-k containment** (e.g., top 10% of predicted scores)
- **Enrichment vs. random baseline**
- **Precision / Recall / IoU**
- **Peak localization** (argmax within Ω_{dead})

These metrics focus on **region identification**, not pointwise regression.

A.9 Robustness Across Trajectories

Using trajectory datasets at multiple sampling rates:

1. Map each trajectory point to the nearest reference location.
2. Aggregate predicted scores along trajectories.
3. Compare stability (e.g., coefficient of variation) across speeds.

A stable operator-based field should exhibit low sensitivity to sampling dynamics.

A.10 Expected Outcomes

A successful reproduction should observe:

- Persistent dead zones forming **geometry-aligned regions**
- Classical baseline producing **diffuse, non-specific** weak-signal areas
- Operator-based field exhibiting **localized alignment** with Ω_{dead}
- Improved top-k containment and enrichment relative to the baseline
- Stability of structure across trajectory-speed variations

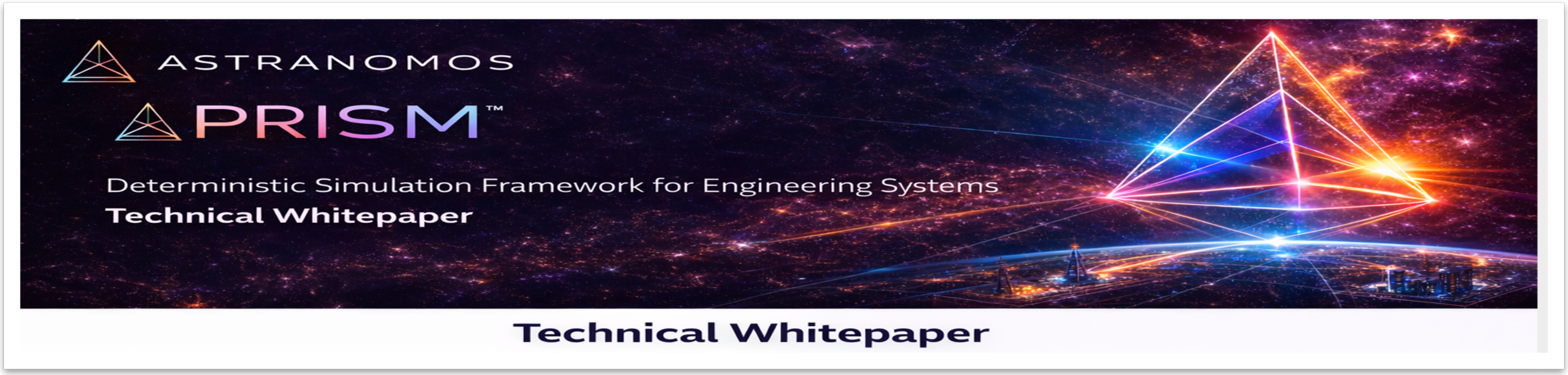
A.11 Scope and Limitations

This roadmap enables verification of the **structural claims**:

- Localization of weak-signal regions
- Alignment with geometry
- Robustness across conditions

It does not expose:

- Specific parameterizations of $D(S)$ and $V(x)$
- Proprietary weighting schemes or regularization
- Implementation-level optimizations



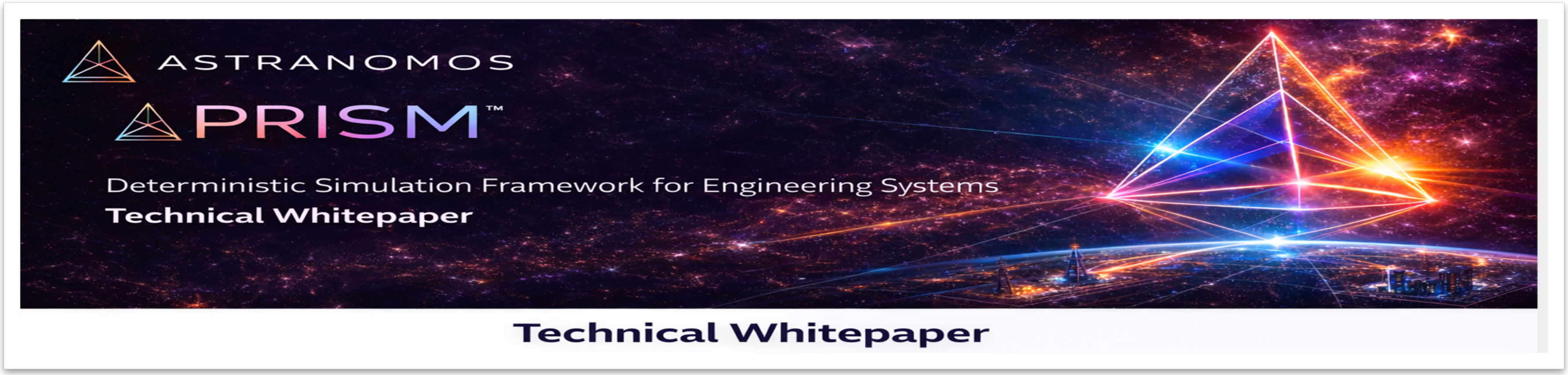
These details are not required to assess the validity of the central result.

A.12 Summary

Independent reviewers can reproduce the key findings by:

1. Using the public dataset
2. Constructing a data-derived ground truth
3. Comparing a diffusion baseline to a geometry-aware operator field
4. Evaluating localization metrics and robustness

The central claim—that apparent stochastic variability resolves into deterministic structure when geometry is explicitly encoded in the operator—can be tested directly through this procedure.



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