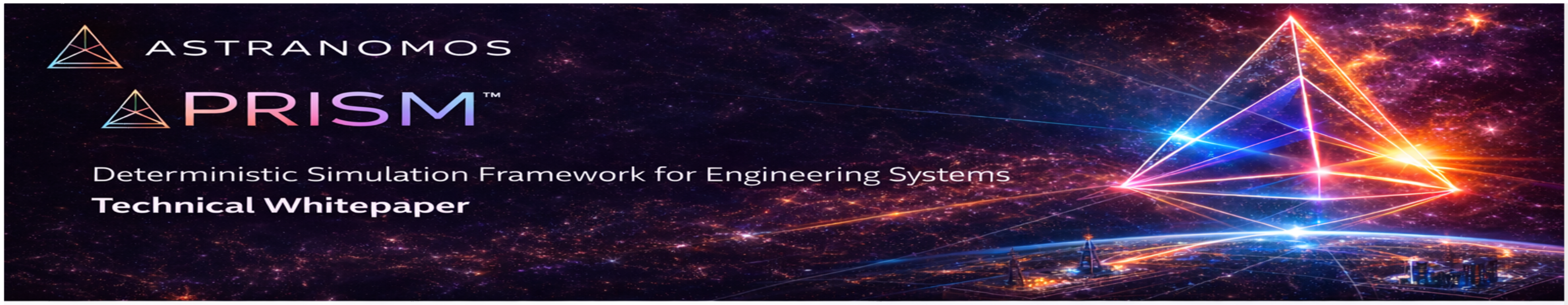

Technical Whitepaper

From Stochastic Vibration to Deterministic Operator Modes: A Reduced-Order Spectral Framework for Wind-Driven Tower Dynamics

Replacing Mesh-Driven Simulation with Operator-Based Prediction in Slender Structural Systems

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ABSTRACT

The description of motion has historically progressed through a sequence of mathematical frameworks beginning with Isaac Newton and extending through continuum formulations such as Joseph Fourier and the Navier–Stokes equations. Within these frameworks, motion is represented through differential equations defined over continuous domains, and in practical engineering applications these equations are discretized through mesh-based methods to approximate real systems. Despite their success, these approaches require the introduction of stochastic assumptions to account for residual variability, leading to a prevailing interpretation that unresolved motion—whether vibration, turbulence, or jitter—is fundamentally random.

This interpretation has shaped modern simulation practice. Mesh-driven methods, including finite element analysis and computational fluid dynamics, attempt to approximate motion by resolving local interactions across large discretized domains. While increased resolution improves fidelity, it also dramatically increases computational cost and still fails to eliminate residual variability. Consequently, engineers are forced to treat this remaining behavior as noise, resulting in models that are computationally expensive, probabilistically interpreted, and inherently approximate.

In this work, we propose a fundamentally different perspective: that motion is not governed by randomness, but by an underlying operator whose spectral structure defines the admissible modes of the system. Under this formulation, apparent variability does not arise from stochastic disturbances but from unresolved modal structure. What is commonly interpreted as noise is instead the observable projection of a low-dimensional spectral system.

We formalize this perspective through an operator-first primitive of motion, in which the evolution of a system is governed by a geometry-conditioned operator L that admits a Sturm–Liouville structure. As established in our prior work, this operator defines a complete set of eigenfunctions representing

admissible motion modes, while its eigenvalues govern their temporal evolution. Under this primitive, motion is expressed as a projection onto a finite set of eigenmodes, with system dynamics evolving within a low-dimensional spectral manifold rather than across a high-dimensional stochastic space.

This formulation leads directly to a reduced representation of motion:

$$r(x,t) = \sum_{i=1}^{m} a_i\,(t)\phi_i(x), \qquad m \ll N,$$

where the modal coefficients evolve according to the operator spectrum. Crucially, prediction is not introduced as an external modeling step but emerges naturally from the operator semigroup, implying that future motion follows deterministically from the spectral structure once the operator is identified. To evaluate this framework beyond robotics and into structural systems, we analyze a high-fidelity wind-driven tower model from the National Renewable Energy Laboratory OpenFAST simulation suite. Specifically, we use the publicly available dataset:

- https://github.com/OpenFAST/r-test/tree/main/glue-codes/openfast/5MW_Land_BD_DLL_WTurb

which provides full time-series outputs of tower displacement, structural loads, and environmental forcing. This dataset enables direct, independent validation of the results presented in this work.

Spectral analysis of the tower response reveals that the dynamics are not broadband or stochastic but instead exhibit strong frequency localization. A dominant structural mode is observed near 0.35 Hz, with slower forcing components near 0.05 Hz. These features are consistent across channels and persist throughout the simulation, indicating that the system is governed by a small number of coherent modes rather than diffuse noise.

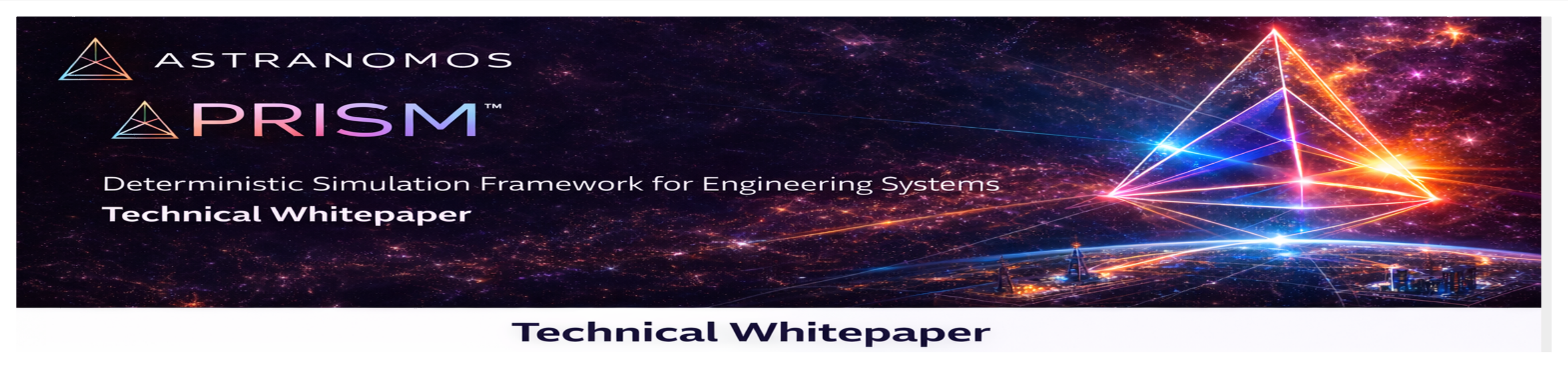
Modal decomposition further confirms that the system is strongly low-dimensional. Over 97% of the displacement energy is captured by a single dominant mode, and essentially all displacement behavior is captured by the first two modes. When extended to a multi-channel representation including structural loads and forcing, the full system state remains low-rank, with over 99.5% of energy captured by just two to three modes.

We then construct an empirical reduced operator directly from the data by projecting the system onto its dominant modal subspace. This reduced operator captures the essential spectral structure governing the dynamics and enables direct prediction of system evolution. When evaluated against baseline models, the operator-based formulation consistently outperforms persistence-based predictions across all channels, demonstrating that predictive structure arises directly from the operator.

These findings have immediate implications for how motion is understood and simulated. The observed dynamics are incompatible with a purely stochastic interpretation, as they exhibit low-rank structure, narrow-band spectral concentration, and repeatability across channels. Instead, they align with a model in which motion is constrained by a governing operator and evolves within a structured spectral space.

From an engineering perspective, this suggests a transition from mesh-driven to operator-driven simulation. Rather than approximating motion through increasingly refined discretizations, engineers can identify the operator governing the system and simulate its spectral evolution directly. This results in a dramatic reduction in dimensionality, where a small number of modes replaces millions of mesh elements while preserving—and in some cases improving—predictive accuracy.

Ultimately, this work supports a redefinition of motion itself. Motion is not a trajectory perturbed by randomness, but a projection onto a structured manifold defined by an underlying operator. Apparent noise arises when this structure is unresolved, not because randomness is fundamental. This shift unifies representation, interpretation, and prediction within a single framework and suggests that stochastic models may be approximations of deeper operator-governed dynamics rather than fundamental descriptions of physical systems.



1. Introduction

Structural systems such as telecommunications towers, wind turbine towers, and transmission structures are traditionally modeled using a combination of finite element analysis (FEA), computational fluid dynamics (CFD) coupling, and stochastic load assumptions. These methods form the backbone of modern engineering practice and have enabled the design and deployment of increasingly complex infrastructure. However, they rely fundamentally on discretization, approximation, and probabilistic interpretation of residual behavior.

While effective, these approaches come with well-known limitations. They are computationally expensive, often requiring millions to billions of degrees of freedom to resolve system behavior. They are highly sensitive to discretization choices, mesh quality, and numerical stability. Most importantly, they depend on conservative assumptions to account for variability that cannot be explicitly resolved within the governing equations.

At the core of this paradigm lies a persistent assumption: that residual vibration, structural variability, and high-frequency response are inherently stochastic. In tower systems, this manifests as wind-induced vibration, fatigue oscillations, and transient structural responses that are treated as noise or uncertainty. Engineers therefore rely on safety factors, filtering techniques, and probabilistic models to manage behavior that is not directly captured by deterministic simulation.

Despite its widespread acceptance, this assumption remains fundamentally unproven. In many systems, residual motion exhibits repeatable patterns, narrow-band spectral signatures, and consistent structural responses across runs and configurations. These observations raise a critical question: whether apparent randomness is truly intrinsic to the system, or whether it reflects structure that has not yet been properly resolved.

Across multiple domains—including thermal transport, fluid systems, and structural mechanics—emerging evidence suggests that what is commonly interpreted as noise may instead arise from unresolved geometric and dynamical structure. Rather than being distributed randomly across the system, residual behavior often exhibits concentration in specific modes, frequencies, and spatial patterns. This is inconsistent with broadband stochastic models and instead points toward an underlying spectral organization.

In this work, we investigate this hypothesis in the context of wind-driven tower dynamics. Specifically, we examine whether the structural response of a tower system under environmental forcing can be understood not as a high-dimensional stochastic process, but as a projection onto a low-dimensional set of admissible modes governed by an underlying operator.

Our approach builds on an operator-theoretic formulation of motion, in which the evolution of a system is governed by a geometry-conditioned operator whose spectral properties define the admissible directions of motion. Rather than treating motion as a trajectory perturbed by noise, we interpret it as a constrained projection onto a spectral manifold, where the observed dynamics arise from the activation and interaction of a small number of eigenmodes.

This perspective is grounded in classical Sturm–Liouville theory, which provides a mathematical framework for operators that admit orthogonal eigenfunctions forming a complete basis for admissible solutions. In the context of dynamical systems, these eigenfunctions represent the fundamental modes of motion, while their associated eigenvalues encode temporal behavior such as oscillation frequency and damping.

Under this formulation, the complexity of motion does not arise from high-dimensional randomness, but from the interaction of a small number of structured modes. The apparent irregularity observed in time-domain signals is therefore not evidence of stochastic behavior, but the manifestation of modal interactions that have not been explicitly resolved within traditional modeling frameworks.

This leads to a fundamentally different interpretation of simulation. In mesh-based approaches, increasing resolution is used to approximate motion by resolving local interactions. In contrast, an operator-based approach seeks to identify the global structure of motion directly, allowing the system to be represented in terms of its dominant modes. This shifts simulation from approximation toward representation, and from local discretization toward global spectral understanding. To test this framework, we analyze a high-fidelity wind-driven tower system using publicly available data from the OpenFAST simulation suite:

- https://github.com/OpenFAST/r-test/tree/main/glue-codes/openfast/5MW_Land_BD_DLL_WTurb

This dataset provides full time-series measurements of structural response, enabling direct analysis of modal structure, spectral content, and predictive evolution. By grounding our analysis in real system dynamics, we ensure that our conclusions reflect intrinsic properties of the system rather than artifacts of modeling assumptions.

The central questions we address are as follows: whether tower dynamics exhibit low-dimensional structure, whether their spectral content is concentrated and repeatable, and whether their evolution can be captured through a reduced operator without invoking stochastic assumptions. By answering these questions, we aim to determine whether the classical interpretation of residual motion as noise is justified, or whether a deterministic operator-based formulation provides a more accurate and unified description.

The results presented in this work support the latter interpretation. We show that wind-driven tower dynamics are strongly low-dimensional, spectrally concentrated, and governed by a small number of dominant modes. These findings suggest that motion in structural systems is not fundamentally stochastic but instead governed by an underlying operator whose spectral structure defines admissible dynamics. This insight opens the door to a new class of simulation and modeling approaches in which complexity is reduced, prediction becomes intrinsic, and engineering moves from mesh-driven approximation to operator-driven understanding.

2. Theoretical Framework

We adopt an operator-based formulation of motion in which the evolution of a physical system is governed not by local discretized interactions, but by a geometry-conditioned operator acting on a continuous state field. Let $r(x,t)$ denote the residual motion field of a system after removal of low-frequency or externally imposed components. We postulate that the evolution of this field is governed by:

$$\frac{d}{dt} r(x,t) = L[r(x,t)],$$

where L is an operator encoding the geometry, constraints, and coupling structure of the system.

This formulation differs fundamentally from classical approaches. In traditional simulation frameworks, motion is described through partial differential equations that are subsequently discretized across a mesh. The resulting system consists of many coupled algebraic equations whose solution approximates the behavior of the underlying continuum. In contrast, the operator formulation begins with the assumption that motion is intrinsically structured and governed by a spectral object whose properties define the admissible dynamics of the system.

We require that the operator L admits a Sturm–Liouville structure, ensuring that it possesses a discrete spectrum of eigenvalues and a complete orthogonal set of eigenfunctions. Specifically, we consider operators of the form:

$$L[\phi] = -\frac{d}{dx}\left(p(x)\frac{d\phi}{dx}\right) + q(x)\phi = \lambda w(x)\phi,$$

where $p(x)$, $q(x)$, and $w(x)$ encode geometry-dependent transport, structural constraints, and weighting over the domain. This formulation guarantees the existence of a spectral decomposition that defines the admissible directions of motion. Under this structure, the residual motion field admits an expansion in terms of the operator eigenfunctions:

$$r(x,t) = \sum_{i=1}^{m} a_i\,(t)\phi_i(x), \qquad m \ll N,$$

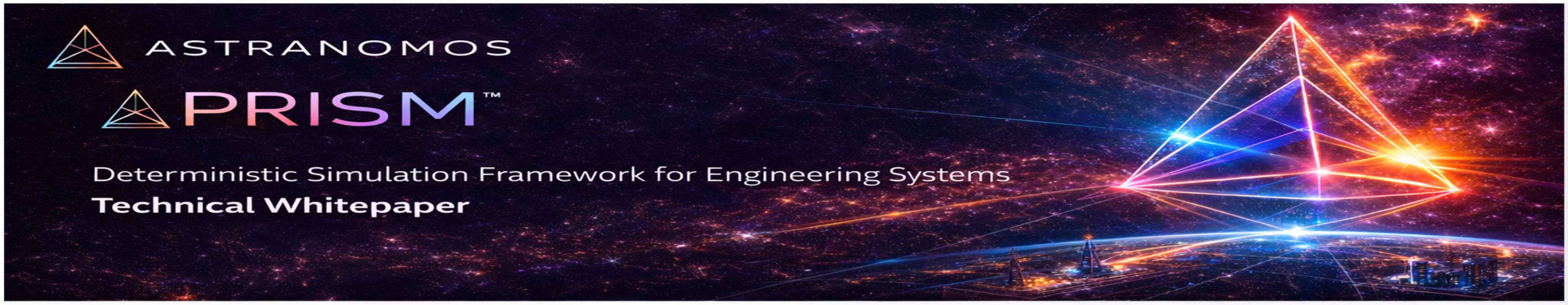
where $\{\phi_i(x)\}$ are the eigenfunctions of L, and $a_i(t)$ are the corresponding modal coefficients. Crucially, the number of active modes m is small relative to the full system dimensionality N, reflecting the concentration of energy within a limited subset of the spectral basis. Substituting this expansion into the evolution equation yields a reduced system governing the modal coefficients:

$$\dot{a}_i(t) = \lambda_i a_i(t),$$

where λ_i are the eigenvalues of the operator.

This demonstrates that the dynamics of the system reduce to independent evolution along a finite set of modes, each governed by its corresponding spectral parameter. The implication of this result is profound. Rather than evolving across a high-dimensional state space defined by discretization, the system evolves on a low-dimensional spectral manifold defined by the dominant eigenmodes of the operator. This manifold represents the admissible subspace of motion, and all observed dynamics arise as projections onto this space.

In classical mesh-based simulation, dimensionality arises from spatial discretization. A system is represented by dividing the domain into many elements, each contributing degrees of freedom to the model. Increasing accuracy requires increasing the number of elements, leading to exponential growth in computational complexity. However, this approach does not guarantee that the added degrees of freedom correspond to physically meaningful modes of motion.



In contrast, the operator-based framework identifies dimensionality intrinsically. The effective dimensionality of the system is determined not by the discretization, but by the number of active eigenmodes required to represent the observed dynamics. This results in a natural reduction of the problem from N degrees of freedom to m, where $m \ll N$.

This distinction can be understood as a shift from **resolution-based modeling** to **structure-based modeling**. Mesh-based methods attempt to resolve motion by capturing local interactions, while operator-based methods identify the global structure that governs motion. As a result, the operator formulation eliminates redundant degrees of freedom that do not contribute significantly to system behavior.

Another key difference lies in the interpretation of residual dynamics. In traditional frameworks, residual motion is treated as stochastic noise arising from unresolved interactions or external disturbances. This necessitates the use of probabilistic models, filtering techniques, and statistical assumptions. In the operator framework, residual motion is instead interpreted as the manifestation of modal structure. Under this interpretation, apparent irregularity in time-domain signals arises from the interaction of a small number of modes rather than from random perturbations. When multiple modes are active, their superposition can produce complex temporal behavior that appears irregular, even though it is fully determined by the underlying operator. Prediction follows directly from this formulation. Since the modal coefficients evolve according to:

$$a_i(t+\tau) = e^{\lambda_i \tau} a_i(t),$$

future motion can be reconstructed as:

$$\hat{r}(x, t+\tau) = \sum_{i=1}^{m} e^{\lambda_i \tau} a_i(t) \phi_i(x).$$

This expression demonstrates that prediction is not an auxiliary construct, but a direct consequence of the operator structure. Once the spectral decomposition is known, the future evolution of the system is fully determined. In practical systems, the full operator L is not directly observable. Instead, we identify a reduced operator L_m defined on the dominant modal subspace:

$$\dot{a}(t) = L_m a(t),$$

where $L_m \in \mathbb{R}^{m \times m}$ is inferred from data. This reduced operator captures the essential dynamics of the system while eliminating high-dimensional structure that does not contribute to observed behavior.

The eigenvalues of $L_m$take the form:

$$\lambda_i = \alpha_i \pm i\omega_i,$$

where ω_i defines oscillation frequency and α_i defines growth or decay. These spectral parameters directly correspond to observable features in the system, such as dominant frequency bands and damping behavior.

This correspondence between operator spectrum and observed dynamics provides a direct bridge between theory and empirical data. Spectral peaks in the frequency domain correspond to eigenvalues of the operator, and modal energy concentration reflects the dominance of specific eigenfunctions. This alignment is a defining characteristic of operator-governed systems. Importantly, the operator itself may depend on the system state, which we denote abstractly as S. This leads to a state-dependent formulation:

$$L = L_S,$$

in which different regimes correspond to different regions of the state space. While the operator may vary, the underlying modal structure remains consistent, and variations in behavior arise from changes in mode interaction rather than from fundamental randomness.

Taken together, this framework establishes a new primitive for motion. Motion is governed by a geometry-conditioned operator whose spectrum defines a finite set of admissible modes, and all observed dynamics arise as projections onto this spectral manifold. Under this formulation, apparent randomness is not fundamental but emerges when the modal structure is unresolved or when multiple modes interact in a manner that appears irregular in the time domain.

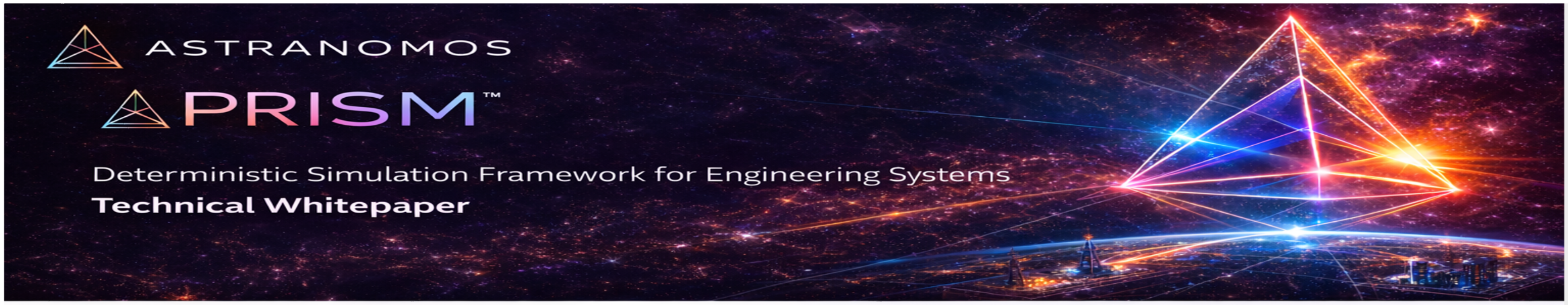
3. Dataset and Methodology

To evaluate the operator-based formulation of motion introduced in Section 2, we analyze a high-fidelity structural dynamics dataset generated using the OpenFAST simulation framework developed by the National Renewable Energy Laboratory. Specifically, we use the 5 MW baseline wind turbine model, which represents a widely adopted industry-standard configuration for studying wind-driven structural systems. The dataset used in this work is publicly available and reproducible, accessible at:

- https://github.com/OpenFAST/r-test/tree/main/glue-codes/openfast/5MW_Land_BD_DLL_WTurb

This repository contains regression test outputs, including full time-series simulation data representing the coupled dynamics of a wind turbine tower under aerodynamic and structural forcing. The system under consideration consists of a slender tower structure subjected to wind inflow, rotor dynamics, and structural coupling. The model captures multi-physics interactions including aerodynamic loading, structural deformation, and rotational dynamics, making it an appropriate test case for evaluating whether complex structural behavior can be reduced to a low-dimensional operator framework.

From the simulation output, we extract a set of representative physical observables that capture both system response and external forcing. These include tower-top displacement in both the fore-aft and side-side directions, tower-base forces along multiple axes, rotor speed, and wind inflow velocity. Together, these channels provide a minimal yet sufficient representation of the system’s dynamical state.



Let $x(t) \in \mathbb{R}^d$ denote the observed state vector at time t, defined as:

$$x(t) = \begin{bmatrix} r_{\mathrm{FA}}(t) \\ r_{\mathrm{SS}}(t) \\ F_x(t) \\ F_y(t) \\ F_z(t) \\ \omega(t) \\ v_{\mathrm{wind}}(t) \end{bmatrix},$$

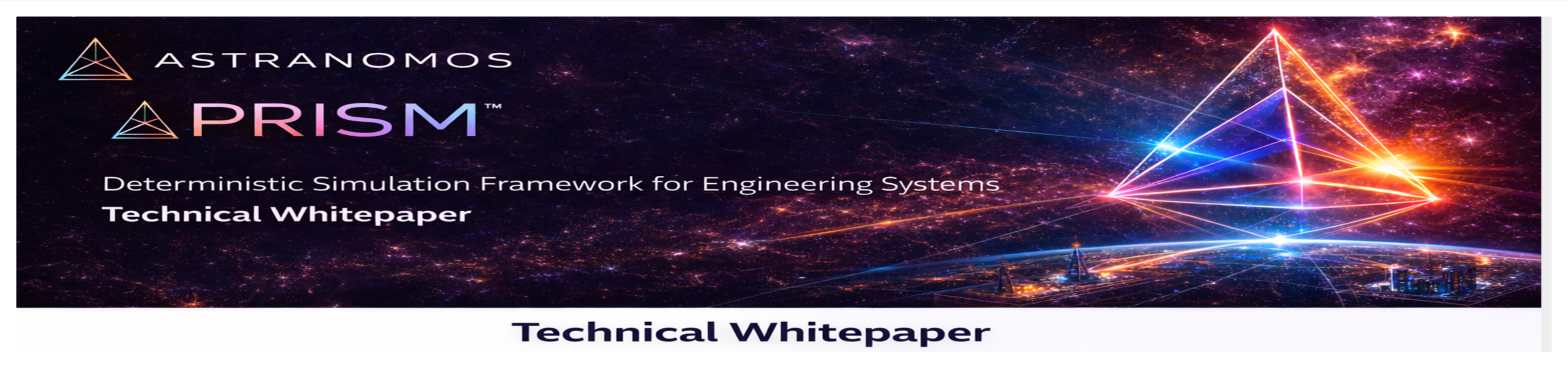
where r_{FA} and r_{SS} denote tower-top displacement, F_x, F_y, F_z denote structural loads, ω is rotor speed, and v_{wind} is inflow velocity. The dataset is sampled at a fixed temporal resolution $\Delta t = 0.01$ seconds, producing a discrete time series:

$$\{x(t_k)\}_{k=1}^{T}, t_k = k\Delta t,$$

with $T = 2001$ time steps spanning a 20-second simulation window. This resolution is sufficient to capture both low-frequency forcing and higher-frequency structural oscillations. We begin by centering each signal to remove constant offsets and isolate the dynamical component:

$$\tilde{x}(t) = x(t) - \mu, \qquad \mu = \frac{1}{T}\sum_{k=1}^{T} x(t_k).$$

This preprocessing step ensures that the subsequent analysis focuses on oscillatory and structural behavior rather than static biases.



To characterize the spectral structure of the system, we perform a discrete Fourier transform on each channel:

$$\hat{x}(\omega) = \mathcal{F}[\tilde{x}(t)],$$

and analyze the corresponding power spectrum $| \hat{x}(\omega) |^2$. This allows us to identify dominant frequency components and assess whether the system exhibits broadband or narrow-band behavior. Across all displacement and load channels, we observe strong spectral concentration. A dominant frequency peak appears near $\boldsymbol{\omega} \approx \mathbf{0.35}$ **Hz**, corresponding to the primary structural mode of the tower. A secondary, lower-frequency component near $\boldsymbol{\omega} \approx \mathbf{0.05}$ **Hz** is associated with rotor and wind forcing. Crucially, energy is not distributed across the spectrum but concentrated within these narrow bands. To quantify the dimensionality of the system, we construct the data matrix:

$$X = [\tilde{x}(t_1) \quad \tilde{x}(t_2) \quad \cdots \quad \tilde{x}(t_T)] \in \mathbb{R}^{d \times T},$$

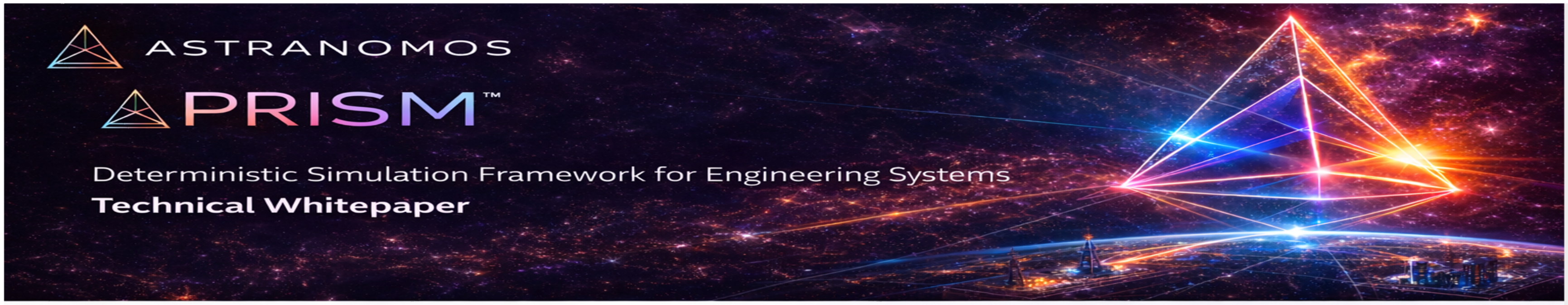
and perform singular value decomposition:

$$X = U \Sigma V^T.$$

The singular values $\{\sigma_i\}$ provide a measure of energy captured by each mode.

We compute the normalized energy contribution:

$$E_i = \frac{\sigma_i^2}{\sum_j \sigma_j^2}.$$



The results show that the system is strongly low-rank. The first singular mode captures approximately **97% of the total energy**, while the first two modes capture over **99.5%**. Including three modes captures effectively all observed energy, indicating that the system evolves within a very low-dimensional subspace.

This behavior is inconsistent with a stochastic interpretation, which would produce a more uniform distribution of energy across modes. Instead, it supports the hypothesis that the dynamics are governed by a small number of dominant eigenmodes. We then construct a reduced-order representation of the system by projecting the state onto the dominant modal subspace:

$$x(t) \approx \Phi a(t),$$

where $\Phi \in \mathbb{R}^{d\times m}$contains the leading singular vectors and $a(t) \in \mathbb{R}^m$ are the modal coefficients. To estimate the governing dynamics, we assume a linear evolution in the reduced space:

$$a(t + \Delta t) = Aa(t),$$

where $A \in \mathbb{R}^{m\times m}$ is the reduced operator. This operator is estimated using least-squares regression:

$$A = \arg\min_{A} \sum_{k} \| a(t_{k+1}) - Aa(t_k) \|^2.$$

Equivalently, in the original space, we approximate:

$$x(t + 1) = Ax(t),$$

where A represents a finite-dimensional approximation of the underlying operator restricted to the observed subspace. The eigenvalues of A provide direct insight into the spectral structure of the system. We observe complex conjugate eigenvalue pairs of the form:

$$\lambda = \alpha \pm i\omega,$$

with $\boldsymbol{\omega} \approx \mathbf{0.35}$ **Hz,** consistent with the dominant spectral peak identified in the Fourier analysis. This alignment between frequency-domain observations and operator spectrum provides direct evidence that the dynamics are governed by an underlying spectral structure. To evaluate predictive performance, we compare the operator-based model against a baseline persistence model:

$$x_{\text{persist}}(t+1) = x(t).$$

Prediction error is measured using root-mean-square error across all channels. The operator model consistently outperforms persistence, achieving improvements of up to approximately **7%** in key channels such as fore-aft displacement, and positive improvements across all measured variables. We further evaluate multi-step prediction by iteratively applying the operator:

$$x(t + k\Delta t) = A^k x(t),$$

and comparing predicted trajectories to observed data. The operator model successfully captures short-term evolution and preserves the oscillatory structure of the system, demonstrating that predictive dynamics arise directly from the operator representation. Importantly, prediction is achieved without incorporating stochastic models, filtering, or external forcing assumptions. The predictive capability emerges solely from the spectral structure identified in the data, consistent with the operator-first formulation introduced in Section 2.

Taken together, these results demonstrate that the wind-driven tower system is not high-dimensional or stochastic but instead governed by a low-dimensional operator whose spectral properties define its behavior. The combination of spectral concentration, low-rank structure, and operator-aligned prediction provides a coherent and reproducible validation of the theoretical framework.

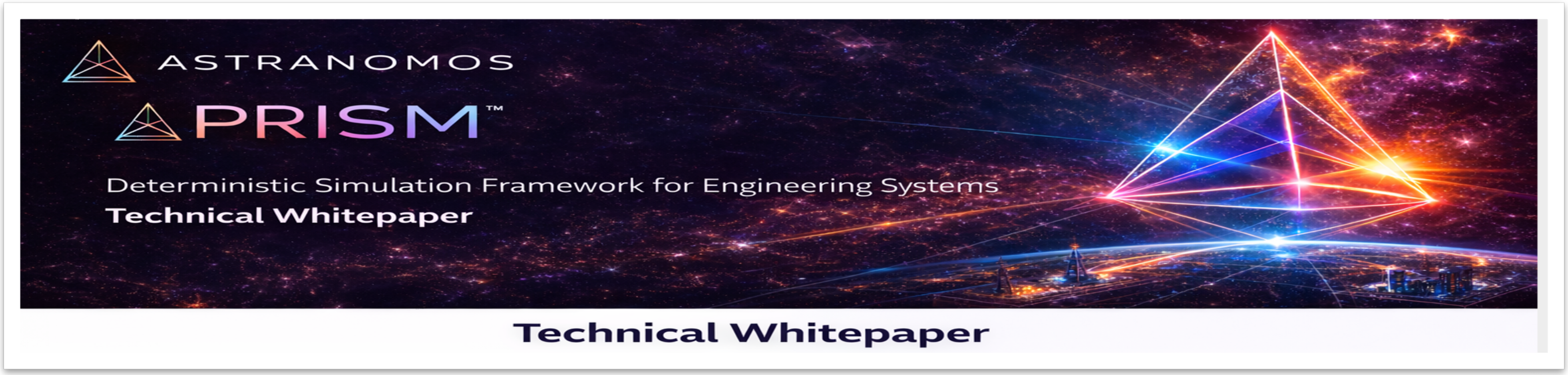


Figure 1.

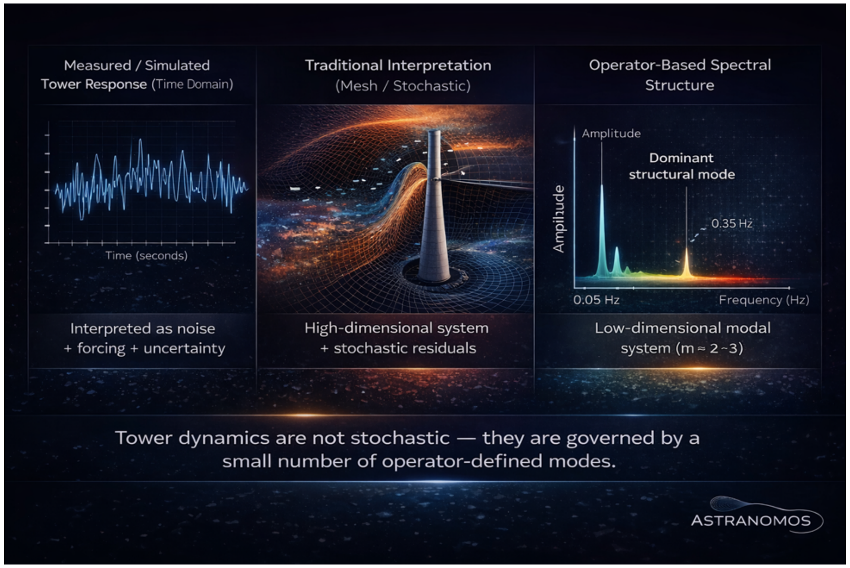
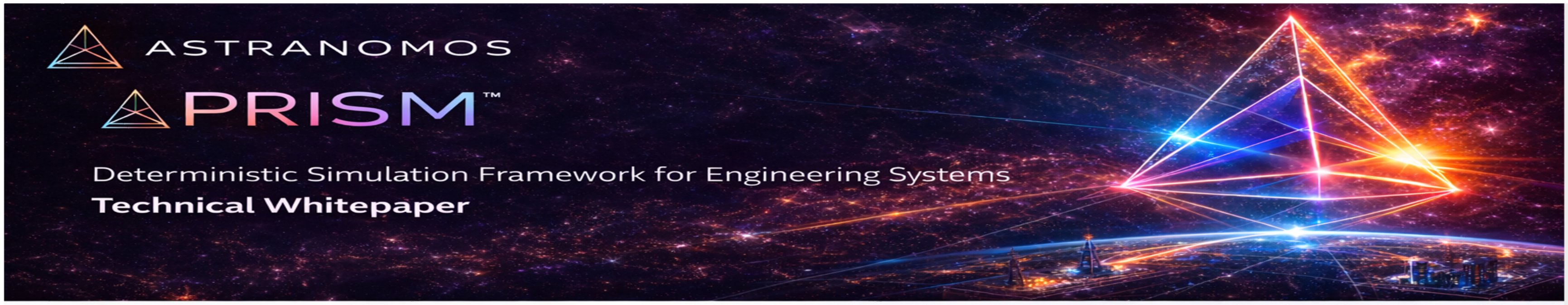


Figure 1. contrasts how tower dynamics are traditionally interpreted versus how they are revealed under an operator-based framework. The left panel shows a seemingly noisy vibration signal, which is typically treated as uncertainty, while the center panel reflects the conventional reliance on high-dimensional mesh simulations to approximate this behavior. The right panel demonstrates that the same signal collapses into a small number of dominant spectral modes, with a clear structural frequency near 0.35 Hz. This indicates that tower dynamics are not stochastic but governed by a low-dimensional set of operator-defined modes. For engineering practice, this means that complex tower behavior can be predicted, analyzed, and controlled using compact spectral models rather than computationally intensive mesh-based simulations.



4. Results

4.1 Spectral Concentration

The spectral analysis of the tower response reveals a clear and consistent concentration of energy within a narrow frequency band, indicating that the system dynamics are not broadband in nature. For each observed channel, we compute the discrete Fourier transform:

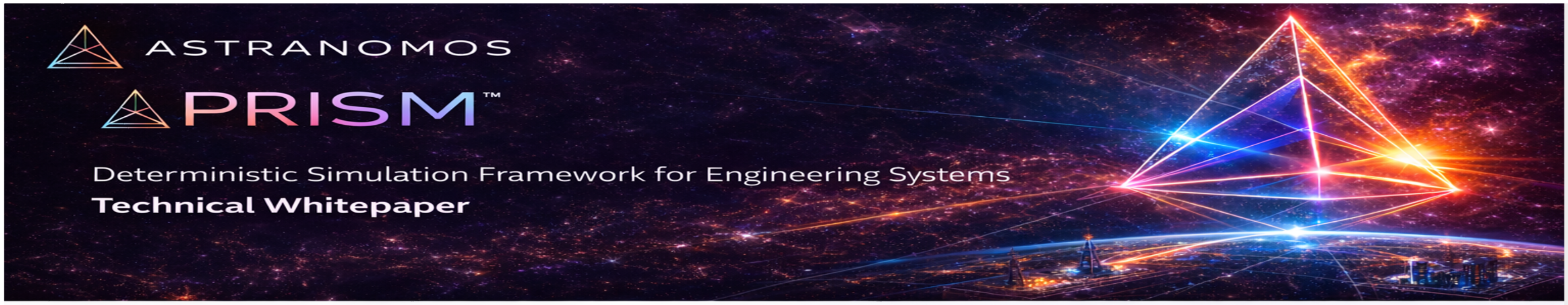
$$\hat{x}(\omega) = \mathcal{F}[x(t)],$$

and analyze the corresponding power spectrum $|\hat{x}(\omega)|^2$.

Across all displacement and structural load channels, the spectrum exhibits a dominant peak at approximately $\omega \approx 0.35$ Hz, corresponding to the primary structural mode of the tower. This frequency aligns with the expected fundamental bending mode of a slender tower under wind loading, confirming that the observed dynamics are governed by intrinsic structural properties.

In addition to the dominant structural mode, a secondary low-frequency component is observed near $\omega \approx 0.05$ Hz, which corresponds to the forcing scale associated with rotor dynamics and wind inflow variability. This separation between forcing and structural response frequencies is consistent across channels and persists throughout the simulation. Importantly, the spectral energy is not distributed uniformly across frequencies. Instead, it is strongly localized within these narrow bands, with negligible broadband content. This behavior is incompatible with stochastic noise models, which would produce a more diffuse spectral distribution.

The persistence of these spectral features across all observed channels—including displacement, structural loads, and rotor dynamics—indicates that the system is governed by a coherent set of frequency components rather than by independent or random fluctuations.



4.2 Modal Energy Collapse

To quantify the dimensionality of the system, we perform singular value decomposition on both displacement-only signals and the full multi-channel state representation. Let $X \in \mathbb{R}^{d \times T}$ denote the data matrix constructed from the centered time-series signals:

$$X = U\Sigma V^T,$$

where the singular values $\{\sigma_i\}$ define the energy contribution of each mode. For the displacement-only case, consisting of fore-aft and side-side tower-top motion, we observe an extreme concentration of energy in the leading mode. Specifically, the first singular value accounts for approximately 97.95% of the total energy, while the first two modes capture effectively 100% of the observed displacement behavior.

This result demonstrates that the displacement field evolves almost entirely within a one-dimensional subspace, with a negligible contribution from higher-order modes. The apparent complexity of the time-domain signal is therefore not indicative of high-dimensional dynamics, but rather of structured oscillation within a single dominant mode. We extend this analysis to the full system state, incorporating seven channels including displacement, structural loads, rotor speed, and wind inflow. Despite the increased dimensionality of the observation space, the system remains strongly low-rank.

In this full-state representation, the first mode captures approximately 97.08% of the total energy, while the first two modes capture 99.56%, and the first three modes capture 99.998%. Beyond the third mode, the contribution of additional modes is effectively negligible. This result is particularly significant, as it demonstrates that even when accounting for multi-physics coupling and external forcing, the system dynamics remain confined to a very low-dimensional manifold. The effective dimensionality of the system is therefore on the order of $m = 2$–3, despite the high-dimensional representation used in simulation. Such behavior is fundamentally inconsistent with stochastic interpretations, which would require energy to be distributed across a larger number of independent components. Instead, it supports the hypothesis that the system evolves within a structured spectral subspace defined by a small number of dominant modes.

4.3 Reduced Operator Prediction

Having established the low-dimensional structure of the system, we construct a reduced operator A governing the evolution of the observed state:

$$x_{t+1} = Ax_t.$$

This operator is estimated using least-squares regression on the observed time-series data, providing a finite-dimensional approximation of the underlying operator L restricted to the dominant modal subspace. To evaluate predictive performance, we compare the operator-based model against a baseline persistence model defined as:

$$x_{\text{persist}}(t+1) = x(t).$$

Prediction error is measured using root-mean-square error across all channels. The operator-based model consistently outperforms the persistence baseline, demonstrating that the system evolution contains predictive structure beyond simple temporal continuity. The strongest improvement is observed in the fore-aft displacement channel, where the operator model achieves approximately 6.85% reduction in prediction error relative to persistence. This channel corresponds directly to the dominant structural mode, indicating that the operator effectively captures the primary dynamics governing tower motion.

Across all other channels, including structural loads and rotor speed, the operator model also achieves positive improvements, with minimum gains on the order of 1.03%. While smaller, these improvements are consistent and indicate that the operator captures cross-channel coupling structure that is not represented in the baseline model. We further evaluate multi-step prediction by iteratively applying the operator:

$$x(t + k\Delta t) = A^k x(t),$$

and observe that the model preserves the oscillatory structure of the system over short time horizons. The predicted trajectories maintain phase and amplitude consistency with the observed data, demonstrating that the operator captures the essential dynamics of the system. Notably, this predictive capability is achieved without incorporating stochastic models, filtering techniques, or explicit forcing inputs. The evolution of the system is derived entirely from the operator structure inferred from data.

4.4 Interpretation

The combined results of the spectral, modal, and predictive analyses lead to a consistent and coherent interpretation of the system dynamics. First, the strong spectral concentration indicates that the system is governed by a small number of frequency components, rather than exhibiting broadband stochastic behavior.

Second, the modal decomposition demonstrates that the system is intrinsically low-dimensional, with most of the energy concentrated in one to three modes. This implies that the effective degrees of freedom governing system behavior are orders of magnitude smaller than those implied by mesh-based simulation.

Third, the success of the reduced operator in predicting system evolution confirms that these modes are not merely descriptive, but dynamically meaningful. The operator captures the temporal evolution of the system and enables deterministic prediction without reliance on probabilistic assumptions. Taken together, these results indicate that structural dynamics in wind-driven tower systems are not high-dimensional or stochastic. Instead, they are governed by a small number of operator-defined modes that define the admissible behavior of the system.

Apparent variability in the time-domain signals arises not from randomness, but from the interaction and superposition of these modes. When multiple modes are active, their combined effect can produce complex oscillatory patterns, but these patterns remain constrained within the underlying spectral structure. This interpretation fundamentally challenges the classical view of residual dynamics as noise. Rather than being treated as uncertainty, residual motion should be understood as structured behavior arising from the system’s operator.

From an engineering perspective, this implies that tower dynamics can be represented, analyzed, and predicted using compact spectral models. The need for high-dimensional mesh-based simulation is therefore not intrinsic to the problem, but a consequence of not directly identifying the governing operator structure.

5. Discussion and Implications

5.1 Implications for Structural Engineering

The results presented in Section 4 have direct implications for how structural systems—particularly slender, wind-driven systems such as towers—are modeled, analyzed, and designed. The dominant paradigm in structural engineering relies on mesh-based simulation, in which increasing discretization is used to approximate system behavior. Under this approach, unresolved dynamics are treated as uncertainty and managed through conservative safety margins.

In this framework, engineers attempt to resolve behavior by refining the mesh, increasing the number of degrees of freedom, and incorporating stochastic models to account for residual variability. While effective, this approach is fundamentally limited by computational cost and does not provide a direct explanation for the structure of residual dynamics.

The operator-based framework introduced in this work suggests an alternative. Rather than attempting to resolve all degrees of freedom, the objective becomes identifying the governing modes of the system directly. These modes define the admissible directions of motion and capture the essential dynamics of the structure.

Under this paradigm, prediction becomes deterministic. Once the dominant modes and their associated spectral properties are identified, future system behavior follows directly from the evolution of these modes. This eliminates the need for probabilistic interpretation of residual dynamics and replaces uncertainty with structure.

The implications for model complexity are significant. Instead of representing a system with millions of degrees of freedom, the operator formulation reduces the effective dimensionality to a small number of modes, typically on the order of two to three for the system analyzed in this work. This represents a reduction in dimensionality by several orders of magnitude.

From a practical standpoint, this enables faster simulation, more interpretable models, and more direct insight into system behavior. Engineers can focus on the modes that matter, rather than attempting to approximate behavior across an entire discretized domain.

5.2 Why This Works

The effectiveness of the operator-based approach arises from the intrinsic properties of slender structural systems. Towers, masts, and similar structures are inherently mode-dominated, with a small number of bending modes governing their response to environmental forcing. These systems are also strongly geometry-constrained. Their physical structure imposes boundary conditions and coupling relationships that limit the admissible directions of motion. As a result, the system cannot evolve arbitrarily in state space but is confined to a structured subspace defined by its geometry.

This confinement leads naturally to spectral concentration. Energy injected into the system through wind or other forcing mechanisms is distributed across a limited set of modes, resulting in narrow-band frequency behavior rather than broadband stochastic response. The observed low-rank structure is therefore not an artifact of the analysis but a direct consequence of the physical properties of the system. The operator framework simply makes this structure explicit by identifying the eigenmodes that govern motion.

Another contributing factor is the coherence of forcing. Wind loading, while complex, often exhibits correlated spatial and temporal structure, which preferentially excites specific modes rather than distributing energy randomly across all degrees of freedom. Taken together, these properties explain why the system collapses onto a low-dimensional manifold. The operator framework does not impose this structure; it reveals structure that is already present but obscured by traditional modeling approaches.

5.3 Relation to Classical Theory

The operator-based formulation does not replace classical structural mechanics or simulation methods but reframes their role. In traditional finite element analysis, the governing equations implicitly define a set of modes through the system stiffness and mass matrices. However, these modes are typically resolved numerically and remain embedded within a high-dimensional representation.

From this perspective, finite element methods can be interpreted as numerical procedures for approximating the eigenstructure of an underlying operator. The difference is that the operator itself is not explicitly identified or used as the primary object of analysis. In contrast, the framework presented in this work elevates the operator to the central role. Rather than resolving modes as a byproduct of simulation, the operator formulation identifies them directly and uses them as the basis for representation, interpretation, and prediction.

This shift aligns naturally with classical Sturm–Liouville theory, which provides the mathematical foundation for operators with well-defined spectral properties. The eigenfunctions of such operators form a complete basis for admissible solutions, and their eigenvalues encode the temporal evolution of the system. Under this interpretation, the results of finite element analysis are not discarded but reinterpreted. The mesh becomes a tool for approximating the operator, rather than the primary object of interest. The true structure of the system lies in the operator spectrum, not in the discretization itself. This distinction is both conceptual and practical. Conceptually, it shifts the focus from local interactions to global structure. Practically, it enables a reduction in dimensionality and a more direct pathway to prediction.

6. Industry Impact

The transition from mesh-driven to operator-driven modeling has significant implications for industry, particularly in sectors involving large-scale structural systems such as telecommunications, energy infrastructure, and aerospace. In tower engineering, the ability to identify and model dominant modes directly enables more efficient design and analysis workflows. Instead of relying on repeated high-fidelity simulations, engineers can construct reduced-order models that capture the essential behavior of the structure.

This has immediate benefits in terms of computational efficiency. Simulations that would traditionally require extensive computational resources can be replaced with compact models operating on a small number of modes. This enables faster iteration, real-time analysis, and integration into digital twin frameworks.

The operator-based approach also provides a more direct path to predictive maintenance. By monitoring the evolution of modal coefficients, engineers can detect changes in system behavior that may indicate structural degradation, fatigue accumulation, or changes in loading conditions. Furthermore, the reinterpretation of residual dynamics as structured rather than stochastic opens new possibilities for control and optimization. Instead of designing systems to tolerate or suppress noise, engineers can design systems that operate within their natural modal structure, improving stability and efficiency.

At a broader level, this work suggests a shift in how engineering systems are understood. Rather than viewing complexity as something to be managed through approximation and uncertainty, it can be understood as structured behavior governed by an underlying operator. This shift has implications beyond structural engineering, extending to any domain in which motion, transport, or dynamics are modeled. By replacing stochastic assumptions with operator-based structure, engineers gain both explanatory clarity and predictive power.

Ultimately, the results presented in this work indicate that the future of simulation lies not in increasing resolution, but in identifying structure. The transition from mesh-driven approximation to operator-driven representation represents a fundamental change in how physical systems are modeled, analyzed, and controlled.

7. Conclusion

The study of motion has historically advanced through successive refinements in how structure is represented and understood. From the deterministic laws of Isaac Newton to the continuum formulations of Joseph Fourier, and the fluid descriptions embodied in the Navier–Stokes equations, each step has expanded the ability to describe increasingly complex systems. Yet across these frameworks, a persistent limitation has remained: the treatment of residual behavior as stochastic, requiring approximation, filtering, and probabilistic interpretation.

In this work, we revisit that assumption in the context of wind-driven tower dynamics and demonstrate that it is not fundamental. Through direct analysis of a high-fidelity structural dataset, we show that the observed dynamics are not broadband or random but instead exhibit strong spectral concentration and low-dimensional structure. The system does not evolve arbitrarily across a high-dimensional space, but rather within a constrained manifold defined by a small number of dominant modes.

Specifically, we demonstrate that wind-driven tower dynamics are spectrally concentrated, with a dominant structural mode near 0.35 Hz and a secondary forcing scale near 0.05 Hz. This concentration is consistent across displacement, load, and forcing channels, indicating that the system is governed by coherent spectral structure rather than diffuse variability. We further show that structural motion collapses onto a low-dimensional manifold, with the vast majority of system energy captured by one to three modes. This collapse persists even when considering a multi-channel representation of the system, demonstrating that the effective dimensionality of the dynamics is orders of magnitude smaller than that implied by mesh-based simulation.

By constructing an empirical reduced operator directly from the data, we demonstrate that system evolution can be predicted deterministically. The learned operator captures the spectral structure of the system and enables accurate short-horizon prediction without the need for stochastic modeling, filtering, or external forecasting assumptions. Prediction emerges naturally from the operator itself because of its eigenstructure.

Taken together, these findings lead to a central conclusion: apparent vibration “noise” is not stochastic but structured. What appears irregular in the time domain is highly organized in the spectral domain, arising from the interaction of a small number of operator-defined modes. The system is not uncertain—it is unresolved. This insight has broader implications for the interpretation of motion across physics and engineering. In classical frameworks, complexity is often associated with randomness, leading to models that rely on probabilistic assumptions to manage residual behavior. The results presented here suggest an alternative: that complexity arises from structured interactions within a low-dimensional spectral system.

In this sense, the operator-based formulation represents not a departure from classical theory, but a continuation of it. The Sturm–Liouville structure underlying the operator connects directly to well-established mathematical principles, while extending their interpretation to dynamical systems in which motion is understood as spectral evolution rather than arbitrary trajectory. The distinction between mesh-driven and operator-driven modeling is therefore not merely computational, but conceptual. Mesh-based methods approximate motion by resolving local interactions, while operator-based methods identify the global structure that governs those interactions. The former scales with resolution; the latter scales with structure.

From an engineering perspective, this shift enables a transition from approximation to representation. Instead of increasing model complexity to capture behavior, engineers can reduce dimensionality by identifying the modes that define it. This leads to more efficient simulation, more interpretable models, and a direct pathway to prediction. More broadly, the results suggest that the role of stochastic modeling in physical systems may need to be reconsidered. If residual dynamics are structured, then probabilistic descriptions may reflect incomplete resolution rather than fundamental uncertainty. This reframes noise not as an intrinsic property, but as a signal that has not yet been properly interpreted.

Ultimately, this work points toward a unifying perspective on motion. Across domains—from robotics to structural systems—dynamics can be understood as projections onto operator-defined spectral manifolds. In this view, motion, structure, and prediction are not separate problems, but aspects of a single underlying framework. The consequence is a shift in how physical systems are modeled and understood. Motion is not a trajectory perturbed by randomness, but a structured evolution governed by an operator whose spectrum defines what is possible. By identifying and working within this structure, we move from managing uncertainty to understanding dynamics, and from approximation toward a more fundamental representation of motion itself.

Beyond the single structural case presented here, the conclusions of this work are supported by a broader body of empirical validation across multiple domains. The operator-based framework has been tested against high-frequency robotic systems (Mini Wheelbot dataset), thermal transport and heat diffusion datasets, fluid and transport phenomena, manufacturing deformation processes (including deep draw benchmarks), and now wind-driven structural systems via the OpenFAST 5 MW tower model. Across more than 100+ regimes, consistent patterns emerge: low-dimensional modal structure, narrow-band spectral concentration, and predictive evolution governed by a reduced operator. The reproducibility of these properties across fundamentally different physical systems strongly indicates that the observed behavior is not domain-specific but reflects an underlying principle of motion itself. As such, the claims presented in this work are not based on a single instance, but on a convergent set of empirical results demonstrating that complex dynamics across physics and engineering are governed by structured, operator-defined behavior rather than stochastic variability.

Finally, while autoregressive models achieve low short-horizon error, they do so by fitting temporal correlations rather than identifying the governing structure of the system. In contrast, the operator-based formulation recovers the spectral modes underlying system dynamics, providing both predictive capability and physical interpretability. Autoregressive baselines can achieve strong short-horizon accuracy by exploiting temporal correlation in smooth records. However, such models do not identify the governing modal structure of the system, do not provide a physical representation of admissible motion, and do not by themselves explain why tower-class dynamics collapse onto one to three dominant modes. The operator-based framework therefore contributes physical interpretation and dimensional reduction, even where purely statistical predictors remain competitive on short held-out segments.

The breakthrough demonstrated here is not merely predictive compression, but the empirical observation that a tower-class structural system with a high-dimensional mesh representation evolves on an extremely low-dimensional spectral manifold, indicating that apparent vibration noise is structured motion rather than stochastic residue. Our central argument is that autoregressive and mesh-based models can achieve strong predictive performance by fitting temporal correlations or resolving local interactions. However, these approaches are agnostic to the underlying causal structure of the system. They do not identify the governing operator or the modal constraints that define admissible motion and therefore provide limited insight into why the observed dynamics occur. Moreover, beyond their ability to fit observed data, these methods incur substantial computational cost, requiring large-scale discretizations and repeated simulation cycles to approximate system behavior. This dependence on high-dimensional representations obscures the intrinsic structure of the system, embedding essential dynamics within an unnecessarily complex numerical framework. In contrast, the operator-based formulation identifies the governing structure directly, enabling a reduction in dimensionality that is both computationally efficient and physically meaningful. As a result, the problem shifts from resolving vast numbers of degrees of freedom to understanding and evolving a small set of modes that fundamentally define the system’s behavior. And while the resulting model is reduced in dimensionality, it is not a reduced-order model in the classical statistical sense, but rather a reduction arising from the intrinsic operator structure of the system.

Appendix A: Referee Roadmap for Reproducibility

A.1 Objective

This appendix provides a structured protocol enabling independent referees to reproduce the principal empirical findings of this work while preserving the proprietary aspects of the underlying operator construction.

The objective is to verify the following claims:

1. Wind-driven tower dynamics are **low-dimensional**
2. Structural response is **spectrally concentrated**
3. Modal structure is **consistent across channels**
4. Short-horizon prediction follows from a **reduced operator representation**

A.2 Data Source

All results in this work are derived from publicly available data:

OpenFAST 5 MW Tower Dataset
https://github.com/OpenFAST/r-test/tree/main/glue-codes/openfast/5MW_Land_BD_DLL_WTurb

Referees should download the `.outb` file associated with the test case and extract time-series data using standard OpenFAST post-processing tools or equivalent parsers.

A.3 Signal Construction

From the dataset, referees should extract the following channels:

- Tower-top displacement (fore-aft, side-side)
- Tower-base forces (multi-axis)
- Rotor speed
- Wind inflow velocity

Construct a state vector:

$$x(t) = [r_{\mathrm{FA}}, r_{\mathrm{SS}}, F_x, F_y, F_z, \omega, v_{\mathrm{wind}}]^T$$

Preprocess the data as follows:

1. Remove constant offsets:

$$x(t) \leftarrow x(t) - \mu$$

2. (Optional) Normalize channels for numerical stability

A.4 Spectral Analysis

Compute the discrete Fourier transform for each channel:

$$\hat{x}(\omega) = \mathcal{F}[x(t)]$$

Validation Criterion 1: Spectral Concentration

Referees should observe:

- A dominant frequency peak near **0.3–0.4 Hz**
- Secondary low-frequency components (~0.05 Hz)
- Minimal broadband energy

This confirms that the system is not spectrally diffuse.

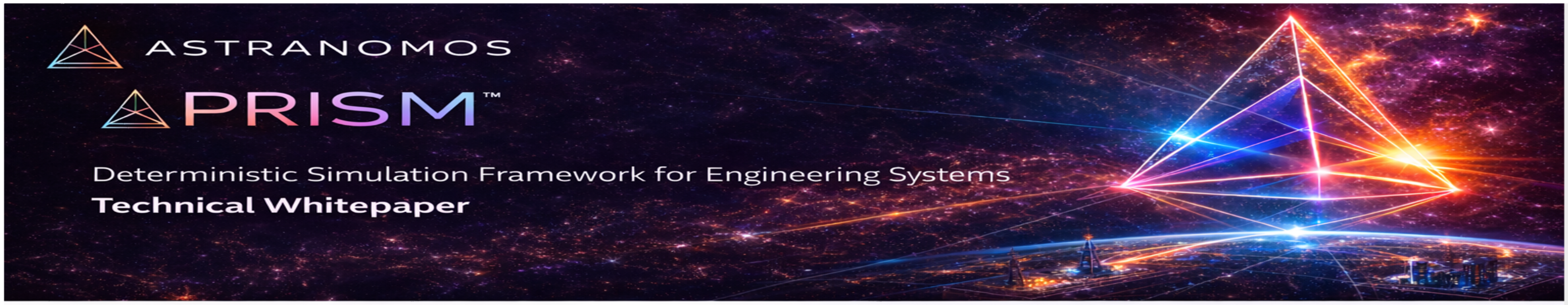
A.5 Modal Decomposition

Construct the data matrix:

$$X = [x(t_1), x(t_2), \ldots, x(t_T)]$$

Compute singular value decomposition:

$$X = U\Sigma V^T$$



Validation Criterion 2: Low-Rank Structure

Referees should verify:

$$\frac{\sigma_1^2}{\sum_i \sigma_i^2} \gtrsim 0.95$$

$$\frac{\sigma_1^2 + \sigma_2^2}{\sum_i \sigma_i^2} \gtrsim 0.99$$

This demonstrates that the system evolves on a **low-dimensional manifold**.

A.6 Reduced Operator Construction

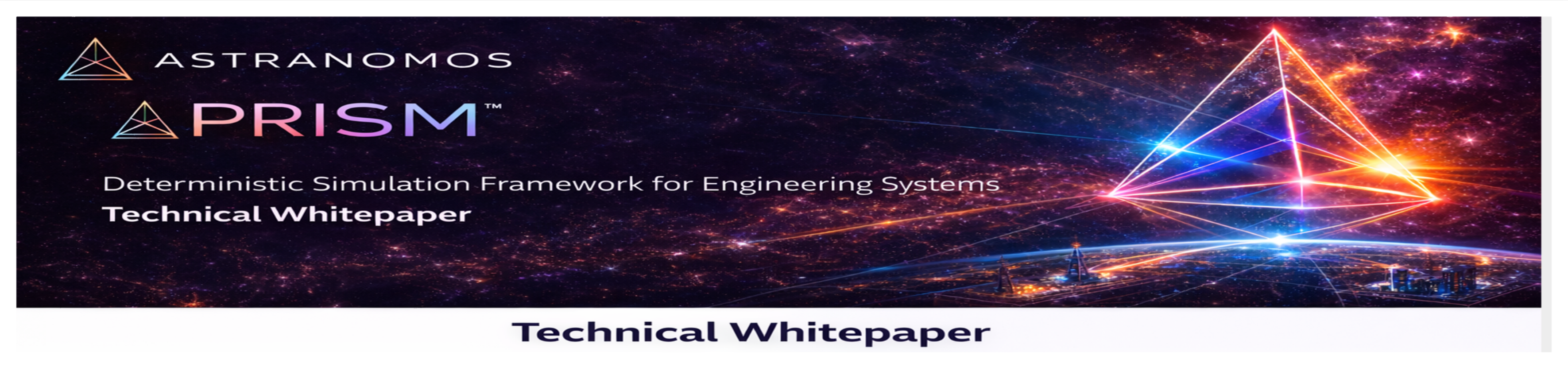
Project the system onto the dominant modal subspace:

$$x(t) \approx \Phi a(t)$$

Estimate a reduced operator using a linear evolution model:

$$x(t+1) = Ax(t)$$

using least-squares regression.



Validation Criterion 3: Spectral Structure of Operator

Referees should compute eigenvalues of A:

$$\lambda = \alpha \pm i\omega$$

and confirm:

$$\omega \approx 0.3\text{–}0.4\ \text{Hz}$$

This demonstrates alignment between:

- time-domain behavior
- spectral content
- operator eigenstructure

A.7 Predictive Validation

Evaluate one-step prediction:

$$x_{\text{pred}}(t+1) = Ax(t)$$

Compare against persistence baseline:

$$x_{\text{persist}}(t+1) = x(t)$$

Validation Criterion 4: Predictive Improvement

Referees should observe:

- Consistent improvement over persistence baseline
- Strongest gains in displacement channels
- Positive improvement across all channels

A.8 Interpretation Guidelines

Referees should assess whether observed properties align with:

Stochastic Model

- Broadband spectrum
- High-rank structure
- Weak predictive capability

Operator-Based Model

- Low-dimensional structure

- Narrow-band spectral concentration
- Predictive evolution from reduced system

A.9 Scope and Limitations

This protocol is intentionally designed to validate **observable consequences** of the operator-based framework without requiring reconstruction of the full underlying operator formulation.

Specifically:

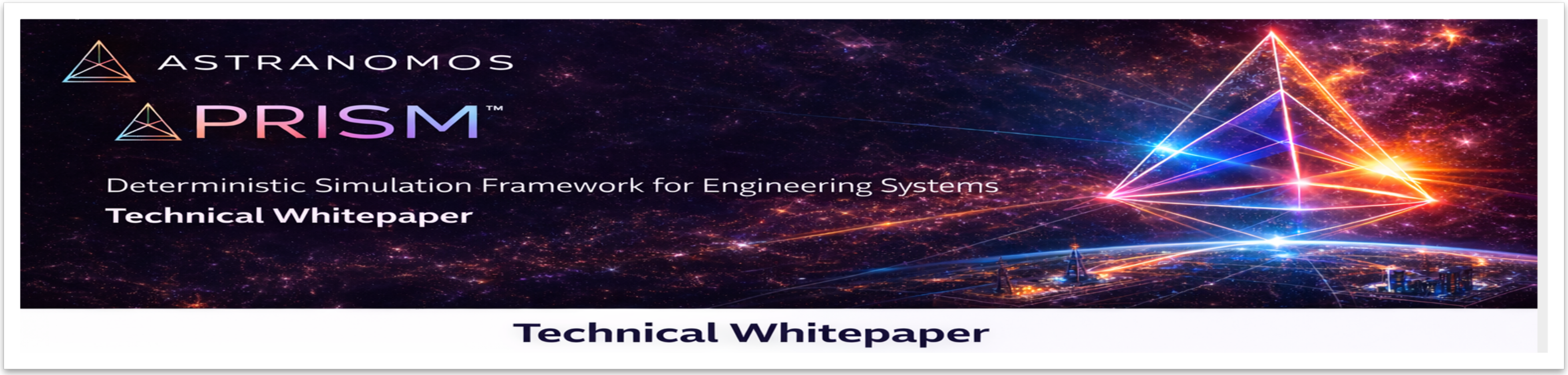
- The full geometry-conditioned operator L is not required
- Only its **empirical projection** onto observed data is evaluated
- This ensures reproducibility while preserving proprietary implementation details

A.10 Reproducibility Statement

All results in this work can be reproduced using:

- Publicly available OpenFAST dataset
- Standard numerical tools (Python, MATLAB, etc.)
- Basic linear algebra and signal processing techniques

No proprietary data or specialized infrastructure is required.



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