AP Physics 1 Kinematics is one of the most fundamental topics in the College Board AP Physics exam. This chapter covers motion in one dimension and two dimensions, including displacement, velocity, acceleration, and motion graphs.

This page provides clear and structured notes designed according to the AP Physics 1 syllabus. Students can use these notes to understand key concepts, revise important formulas, and prepare effectively for both multiple choice questions (MCQs) and free response questions (FRQs).

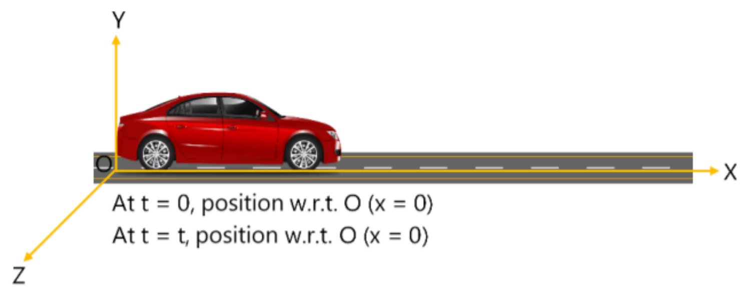
The content is designed to match the style of official AP Physics exams and helps students build a strong foundation in motion and kinematics.

Kinematics

Introduction of Kinematics

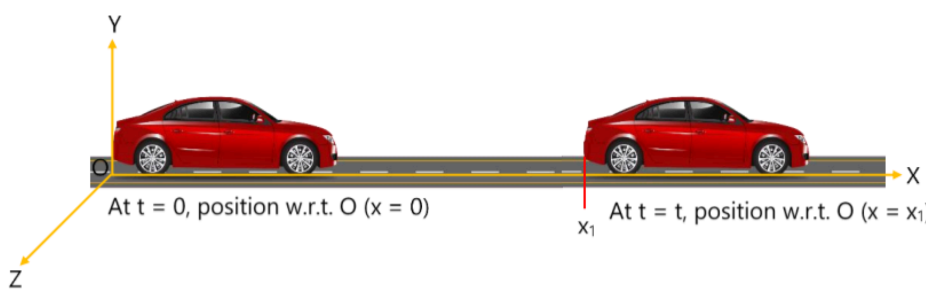
Rest

If position does not change with time, then it is at rest.



Motion

If position of particle changes with time, then it is called in motion.



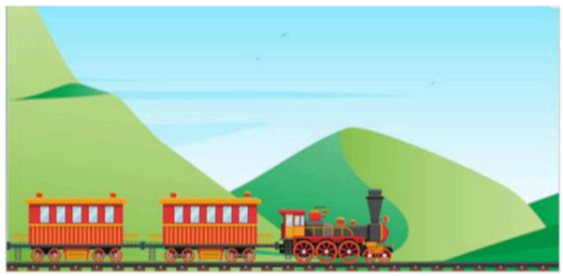
Types of Motion

1-D Motion:

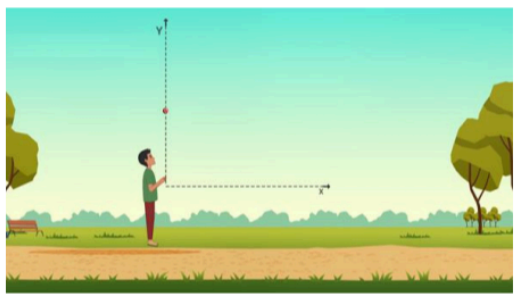
If position changes with time with respect to frame of reference along a straight line, motion is one dimensional (1-D) or straight-line motion or rectilinear motion.

Examples:

Motion of Train Along Straight Track.



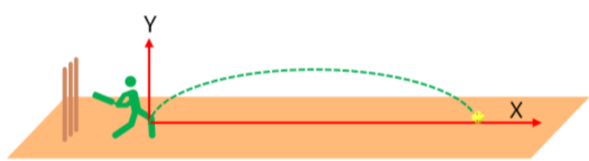
A Ball Projected Upward.



2-D Motion:

If position changes in a plane with time with respect to frame of reference then motion is 2-D.

Examples: Projectile Motion

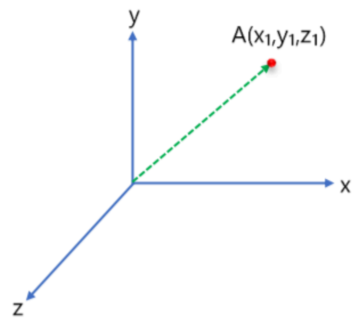


3-D Motion:

If position changes in space with time with respect to frame of reference, motion is 3-D or motion in space.

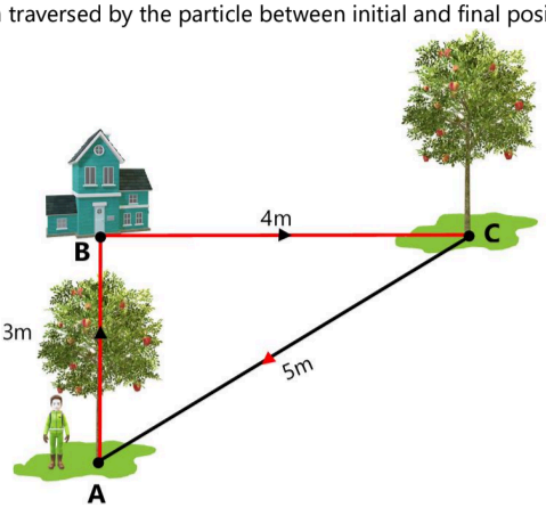
Examples:

Motions of Kite



Distance

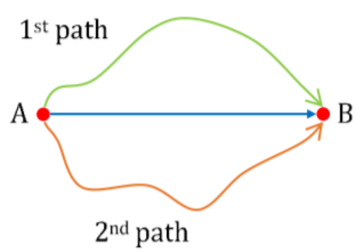
Actual length of the path traversed by the particle between initial and final position is called distance.



Distance travelled by man through path ABCA = 3 + 4 + 5 = 12m

Key Point:

- Distance always depends on path which is followed by the particle. Does not depend on initial and final position of the particle.



$(\text{Distance})_1 \neq (\text{Distance})_2$

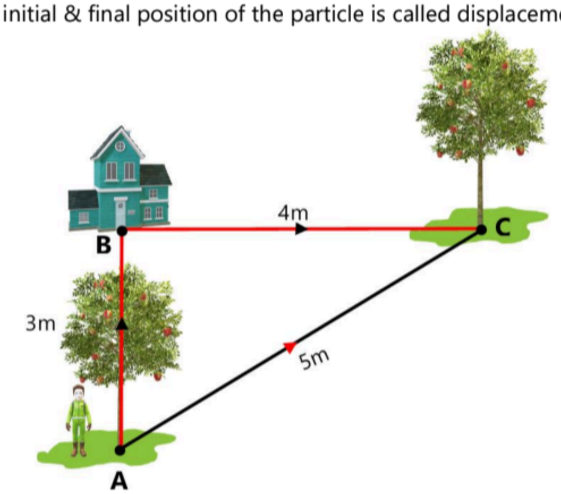
- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- It is a Scalar Quantity.

Distance

- Zero (Particle remain at rest)
- Positive (Moving)

Displacement

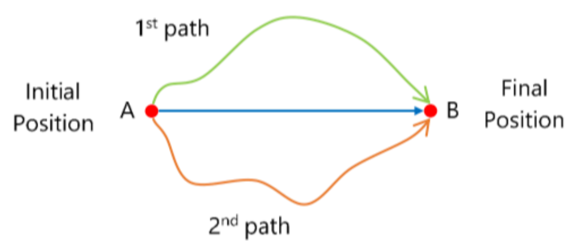
Shortest distance between initial & final position of the particle is called displacement.



Displacement of man when moving from A to B and B to C = 5m

Key Point:

- Displacement is a vector quantity, and its direction is always from initial position to final position.
- Displacement does not depend on path followed by particle. It depends only on initial & final position of the particle.



$(\text{Displacement})_1 = (\text{Displacement})_2$

- Only single value of displacement is possible between two fixed points.
- Displacement may be positive, negative or zero.
- If motion is in straight line without change in direction then distance = |displacement| = magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance. i.e., distance $\geq$ |displacement|

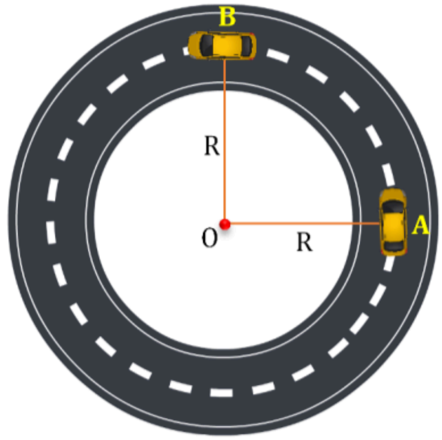
Displacement

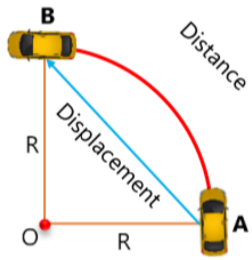
- Zero (Particle may be or may not be at rest)
- Positive/Negative (Moving)

Distance and Displacement in circular path

A car starts moving from point A and reaches at point B along a circular path then find out distance and displacement in following cases.

(1) One-fourth of the complete circle

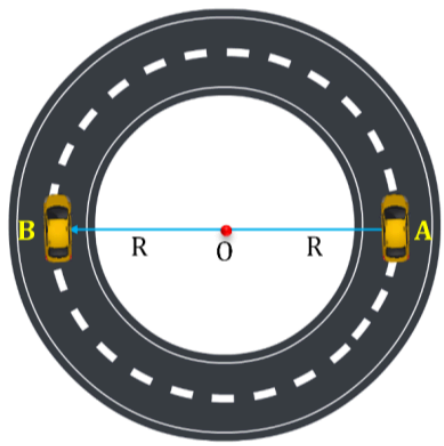


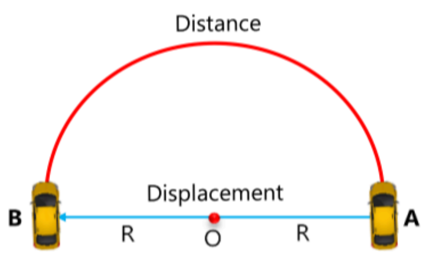


$$\text{Distance} = \frac{\pi r}{2}$$

$$\text{Displacement} = \sqrt{2r}$$

(2) Half of the complete circle

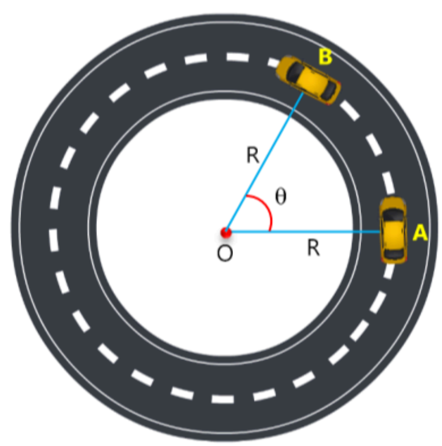


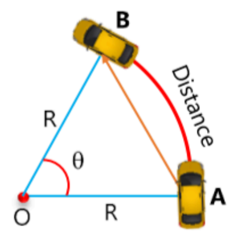


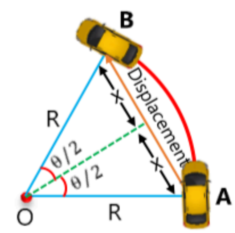
$$\text{Distance} = \pi r$$

$$\text{Displacement} = 2r$$

(3) At any arbitrary arc







Distance = $r\theta$

Displacement = 2x

$$\Rightarrow \frac{x}{r} = \sin\frac{\theta}{2}$$

$$\therefore \text{Displacement} = 2r\sin\frac{\theta}{2}$$

Illustration 4.

A cyclist moving on a circular track, completes one revolution in 10 sec. Then find out its displacement after 1 minute 5 s (radius = 1m)

Solution.

In 1 minute, 5s number of revolutions = 6.5

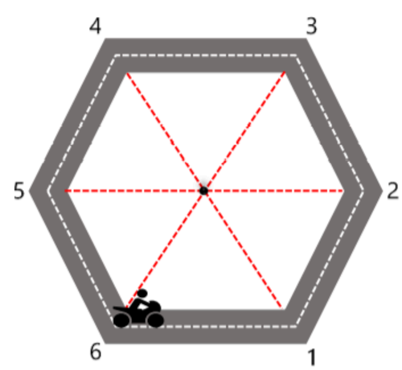
$\therefore$ Displacement = 2r = 2m

Illustration 7.

on an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Solution.

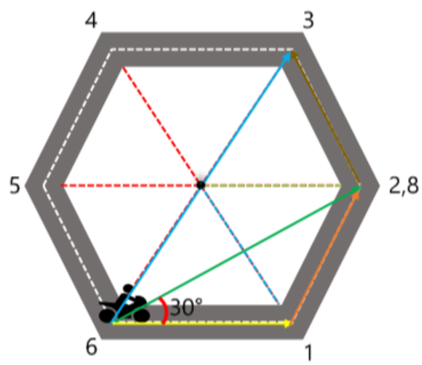
At VI turn



$\because$ initial and final positions are same so |displacement| = 0 and distance = 500 × 6 = 3000 m

$$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{0}{3000} = 0$$

At VIII turn



$$|\text{Displacement}| = 2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}\,\text{m}$$

Distance = 500 × 8 = 4000 m

$$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$$

Speed

The Rate at which distance is covered with respect to time is called speed.

Scalar Quantity

Unit : S.I. → Metre per second (m/s)

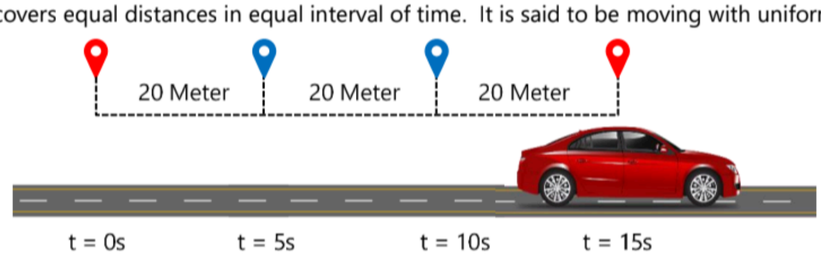
C.G.S. → Centimetre per second (cm/s)

Conversion : km/hr $\underset{\times\frac{18}{5}}{\overset{\times\frac{5}{18}}{\rightleftharpoons}}$ m/s

Types of Speed

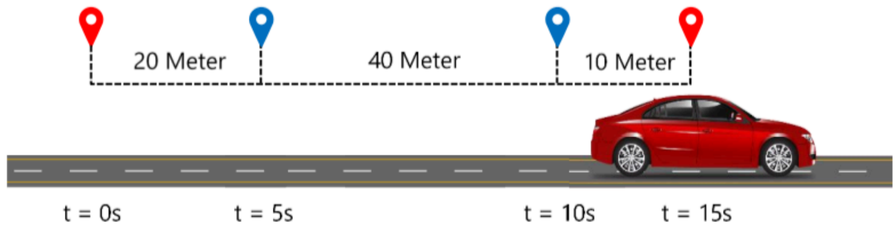
(1) Uniform Speed

Particle covers equal distances in equal interval of time. It is said to be moving with uniform speed.



(2) Non-uniform Speed (Variable Speed)

Particle covers unequal distances in equal intervals of time.



(3) Average Speed

For a given time interval is defined as the ratio of total distance travelled to total time taken.

Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$ i.e. $V_{avg} = \dfrac{\Delta s}{\Delta t}$

(4) Instantaneous speed

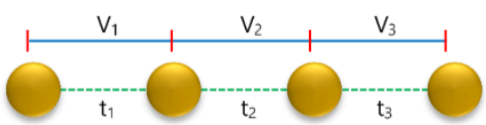
It is the speed of a particle at a particular instant of time.

$$V_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Average speed

Cases of average speed to Remember

Case I : Particle moves with different uniform speeds v_1, v_2, v_3,.........v_n in different time intervals its average speed over the total time of journey is given as –



$$V_{avg} = \frac{\text{Total Distance covered}}{\text{Total time}} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

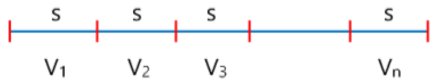
If $t_1 = t_2 = t_3 = ...= t_n$ then

$$V_{avg} = \frac{v_1 + v_2 + ... + v_n}{n}$$

For 2 equal intervals of times

$$V_{avg} = \frac{v_1 + v_2}{2}$$

Case II : Particle describes equal distances with different speeds then the average speed of particle over the total distance will be given as –



If a person moves a certain distance in n equal parts with different speeds.

$$V_{avg} = \frac{d}{\frac{d}{nv_1} + \frac{d}{nv_2} + ... \frac{d}{nv_n}}$$

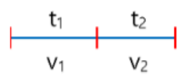
$$V_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + ... \frac{1}{v_n}}$$

For 2 equal distances.

$$V_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

Illustration 1.

A particle moves for 20 sec such that first 10 sec it moves with 36 km/hr and for next 10 sec moves with 54 km/hr. Find average speed.



Solution.

$$v_1 = 36 \times \frac{5}{18} = 10m/s$$

$$v_2 = 54 \times \frac{5}{18} = 15m/s$$

$$v_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{s_1 + s_2}{t_1 + t_2}$$

$$= \frac{10 \times 10 + 15 \times 10}{20} = \frac{250}{20} = 12.5m/s$$

$$= 12.5 \times \frac{18}{5} = 45km/hr$$

Illustration 2.

A train travels from city A to city B with a constant speed of 10 m/s and return back to city A with a constant speed of 20 m/s. Find its average speed during its entire journey.

Solution.

Let the distance between the two cities A and B = x m.

Time taken by the train to travel from A to B = $\frac{x}{10} = t_1$

Time taken to come back from B to A = $\frac{x}{20} = t_2$

Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{x + x}{t_1 + t_2}$

$$= \frac{2x}{\frac{x}{10} + \frac{x}{20}} = \frac{40}{3} m/s$$

Illustration 3.

A car travels first half distance between two places with uniform speed of 60 km/hr. What should be its uniform speed in (m/s) for the second half of the distance so that it average speed over the entire journey become 90 km/hr.

Solution.

Let total distance be 2x and uniform speed of next half be y

Time for first half = $\frac{x}{60}$

Time for second half = $\frac{x}{y}$

Average speed = $\frac{2x}{\frac{x}{60}+\frac{x}{y}}$

$$90=\frac{2x}{\frac{xy+60x}{60y}}$$

3xy+180x=4xy

xy=180x

y=180 km/hr

Therefore, Uniform speed for next half is 180 km/hr.

Illustration 5.

A particle covers first one third distance with speed 10m/s and remaining distance with speed 20m/s. Find average speed.

Solution.

$$v_{avg} = \frac{d}{t_1 + t_2} = \frac{d}{\frac{d}{3\times 10} + \frac{2d}{3\times 20}} = \frac{3\times 10\times 20}{2\times 10 + 20} = \frac{600}{40} = 15m/s$$

Illustration 6.

If the body covers one-third distance at speed v_1, next one third at speed v_2 and last one third at speed v_3, then average speed will be

Solution.

Average speed = total distance covered / total time taken

let the total distance = 3x

time taken to cover first one third (x) = $t_1 = \frac{x}{v_1}$

time taken to cover second one third (x) = $t_2 = \frac{x}{v_2}$

time taken to cover third one third (x) = $t_3 = \frac{x}{v_3}$

average speed = $\frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3x}{x\left(\frac{v_3v_2 + v_1v_3 + v_1v_2}{v_1v_2v_3}\right)}$

average speed = $= \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_1v_3}$

Velocity

The rate of change of position with time is called velocity.

It is a Vector Quantity.

Dimension : $[M^0L^1T^{-1}]$

Unit: S.I. $\rightarrow$ m/s

C.G.S. $\rightarrow$ cm/s

Velocity can be positive, Negative or Zero.

Types of Velocity

(1) Uniform velocity (Constant velocity)

If magnitude as well as direction of its velocity remains same.

This is possible only when it moves in a straight line without reversing its direction.

(2) Non-uniform velocity (variable)

If either magnitude or direction or both varies then velocity changes.

(3) Average velocity

It is defined as the ratio of displacement to time taken by body.

$$\text{Average velocity} = \frac{\text{Net Displacement}}{\text{Time taken}}$$

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Its direction is along the displacement.

Important Points

Average speed $\geq$ |Average velocity |

- When particle moves with constant velocity then magnitude of displacement and distance covered by particle is same.
- A particle may have constant speed but variable velocity.
- A particle may have constant speed but variable velocity. Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.

Acceleration

Rate of change of velocity is called acceleration.

It is a vector quantity.

Its direction is same as that of change in velocity (not in the direction of the velocity).

Dimension: $[M^0L^1T^{-2}]$

Unit : (S.I.) $\rightarrow$ m/s^2

(C.G.S.) $\rightarrow$ cm/s^2

There are 3 ways to change a velocity (vector)

- Only magnitude change
- Only direction change
- Both direction+ magnitude change

Types of Acceleration

(1) Uniform Acceleration
(2) Non-Uniform Acceleration
(3) Average Acceleration
(4) Instantaneous acceleration

(1) Uniform Acceleration

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle .

(2) Non-Uniform Acceleration

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

(3) Average Acceleration

It is the ratio of total change in velocity to the total time taken by the particle

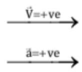
$$\vec{a}_{avg} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$= \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Direction of average acceleration is along the change in velocity

Case- I

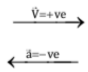
If $\vec{v}$ and $\vec{a}$ are in same direction then speed increases

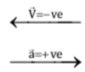
(i) 

(ii) 

Case- II

If $\vec{v}$ and $\vec{a}$ are in opposite direction then speed decreases.

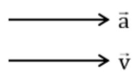
(i) 

(ii) 

Velocity and Acceleration

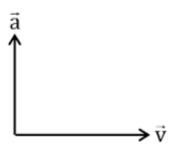
Case-I

Angle between $\vec{v}$ & $\vec{a}$ is zero or they are parallel, speed increases



Case-II

Angle between $\vec{v}$ & $\vec{a}$ is 90° or they are orthogonal, speed remains constant



Case-III

Angle between $\vec{v}$ & $\vec{a}$ is 180° or they are anti-parallel, speed increases

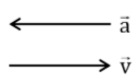


Illustration 10.

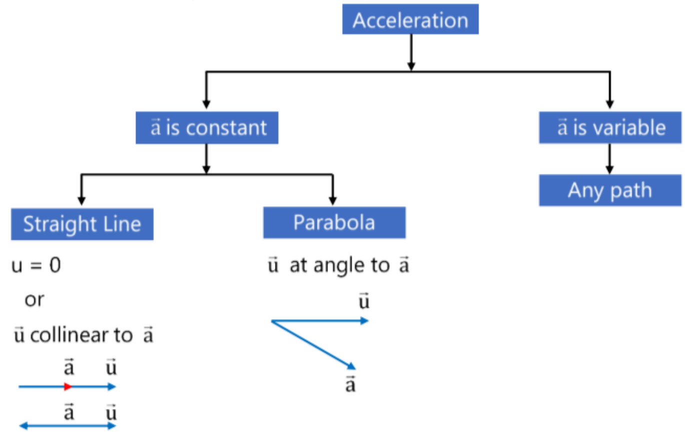
For a moving body at any instant of time. Find given statements are true or false?

(i) If the body is not moving, the acceleration is necessarily zero
(ii) If the body is slowing, the retardation is negative
(iii) If the body is slowing, the distance is negative
(iv) If displacement, velocity and acceleration at that are known, we can find the displacement at any given time in future

Solution.

(i) False
(ii) False
(iii) False
(iv) True

Path of Motion Based on Acceleration (To Remember)



Equations of Motion

S.No.	Scalar Form	Vector Form
first equation	$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
second equation	$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
third equation	$v^2 = u^2 + 2as$	$\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$

Here :

v = final velocity

s = displacement

u = initial velocity

t = time

a = acceleration = constant

These are valid only when acceleration is constant.

Some other useful equations

(i) Displacement in the n^{th} second

$$s_{n^{th}} = u + \frac{1}{2}a(2n-1)$$

(ii) $s = v_{av}t = \frac{(u+v)}{2}t$

(iii) $s = vt - \frac{1}{2}at^2$

Vector form of following equations

(i) $\vec{s}_{n^{th}} = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$

(ii) $\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$

(iii) $\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$

Steps to follow for solving numerical :

(1) Select origin. Generally we take starting point as origin if not given.

(2) Select any one direction as positive.

(3) As per direction selected in step 2, write all known variable with sign.

(4) Apply the suitable equations and put variables values with sign.

Key points

When particle starts from rest and moves with constant acceleration then ratio of distance travelled by it in successive equal intervals of time is

1 : 3 : 5 : 7........ (2n – 1)

Known as **Galileo's law of odd numbers**.

Illustration 1.

The velocity of a body moving with a uniform acceleration of $2\,m/sec^2$ is 10 m/sec. Its velocity after an interval of 4 sec is

Solution.

$$v = u + at = 10 + 2 \times 4 = 18\,m/sec$$

Illustration 2.

The initial velocity of the particle is 10 m/sec and its retardation is $2 m/sec^2$. The distance moved by the particle in 5^{th} second of its motion is

Solution.

$$S_n = u - \frac{a}{2}(2n-1) = 10 - \frac{2}{2}(2 \times 5 - 1) = 1\text{ meter}$$

Illustration 3.

If a car at rest accelerates uniformly to a speed of 144 km/h in 20 s. Then it covers a distance of

Solution.

Here

v=144 km/h = 40 m/s

$$v = u + at \Rightarrow 40 = 0 + 20 \times a \Rightarrow a = 2\,m/s^2$$

$$\therefore s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (20)^2 = 400\,m$$

Illustration 4.

A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second

Solution.

$$S_n = u + \frac{a}{2}(2n-1) = \frac{a}{2}(2n-1)$$

because u = 0

Hence $\frac{S_4}{S_3} = \frac{7}{5}$

Illustration 5.

A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds, then

Solution.

If particle starts from rest and moves with constant acceleration then in successive equal interval of time the ratio of distance covered by it will be

1:3:5:7 ....(2n-1)

i.e. ratio of x and y will be a : 3 i.e. $\frac{x}{y} = \frac{1}{3} \Rightarrow y = 3x$

Problems based on Uniformly Accelerated Motion

Illustration 1.

A particle having initial velocity 10 m/s, moves with uniform acceleration. After 2 seconds its velocity becomes 32 m/s. Find distance covered in this duration.

Solution.

u = 10 m/s $\quad s=\left(\frac{u+v}{2}\right)t$

a = constant $\quad s=\left(\frac{10+32}{2}\right)\times 2=42m$

t = 2s

v = 32 m/s

AVERAGE VELOCITY FOR UNIFORMLY ACCELERATED MOTION

Illustration 2.

A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

Solution.

$\because s=\frac{1}{2}at^2$

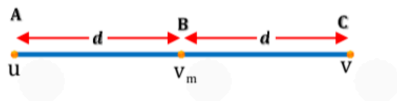
$\therefore \frac{s^2}{t^2}=\frac{1}{2}as$

Average velocity = $\frac{s}{t}=\sqrt{\frac{as}{2}}$

Illustration 3.

A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is?

Solution.



Here, u = 30 km/hr, v = 40 km/hr

Between points A and B

$v_m^2 = 30^2 + 2a(d)$

$2ad = v_m^2 - 900$ ....(i)

Between points B and C

$40^2 = v_m^2 + 2a(d)$

$2ad = 1600 - v_m^2$ ....(ii)

From eq. (1) and (2)

$v_m^2 - 900 = 1600 - v_m^2$

$2v_m^2 = 2500$

$v_m = \sqrt{\dfrac{2500}{2}}$ km/hr

Illustration 4.

A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec

Solution.

Let initial (t=0) velocity of particle = u

For first 5 sec motion s_5 = 10 m

$s = ut + \frac{1}{2}at^2 \Rightarrow 10 = 5u + \frac{1}{2}a(5)^2$

2u+5a=4 ...(i)

For first 8 sec of motion s_8 = 20 m

$20 = 8u + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 5$ ...(ii)

By solving $u = \frac{7}{6}$ m/s and $a = \frac{1}{3}$ m/s^2

Now distance travelled by particle in Total 10 sec.

$s_{10} = u \times 10 + \frac{1}{2}a(10)^2$

By substituting the value of u and a we will get s_{10} = 28.3 m

so the distance in last 2 sec = $s_{10} - s_8$

= 28.3 – 20 = 8.3m

Stopping Distance

For Vehicles,

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$a = -a_0$ $u = u_0$

$v = 0$ $s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = u_0^2 - 2a_0 s$

$\Rightarrow 2a_0 s = u_0^2$

$\Rightarrow s = \frac{u_0^2}{2a_0}$ [since a is constant]

$\Rightarrow s \propto u_0^2$

So, we can say that if u becomes n times then s becomes n^2 times that of previous value.

Stopping Time

$a = -a_0$ $u = u_0$

$v = 0$ $t = ?$

$\Rightarrow v = u + at$

$\Rightarrow 0 = u_0 - a_0 t$

$\Rightarrow a_0 t = u_0$

$\Rightarrow t = \frac{u_0}{a_0}$ [since a is constant]

$\Rightarrow t \propto u_0$

So we can say that if u becomes n times then t becomes n times that of previous value.

Reaction Time

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Illustration 5.

A motor car moving with a uniform speed of 20m/sec comes to stop on the application of brakes after travelling a distance of 10m Its acceleration is

Solution.

From $v^2 = u^2 + 2aS \Rightarrow 0 = u^2 + 2aS$

$\Rightarrow a = \frac{-u^2}{2S} = \frac{-(20)^2}{2 \times 10} = -20 m/s^2$

Illustration 7.

A driver of car which is going with 20 m/s and was at x = 0 at t = 0 applies brake on seeing a red signal. Signal is at x = 100 m. Find the distance of car after 6 sec. if brakes produced retardation of 4 m/s^2?

Solution.

0 to 6 sec

u = 20 m/s

a = –4 m/s^2

v = u + at

$\Rightarrow$ 0 = 20 + (-4t)

$\Rightarrow$ 4t = 20

$\Rightarrow$ t = 5 sec

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \; s = 20\times5+\frac{1}{2}\times(-4)\times5\times5$

$\Rightarrow$ s = 100 – 50

$\Rightarrow$ s = 50 m

Illustration 8.

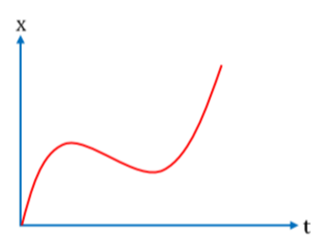
A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

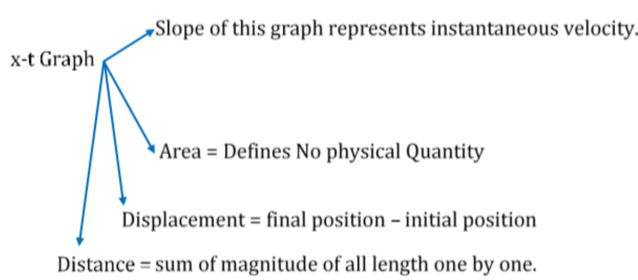
Solution.

$S \propto u^2$ If u becomes 3 times then S will become 9 times

i.e. 9×20 = 180 m

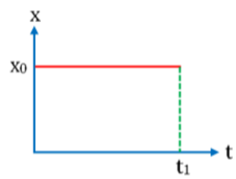
Position-time graph





$\because \tan\theta = \frac{\text{displacement}}{\text{time}} = \text{velocity}$

Case-I



Slope = instantaneous velocity

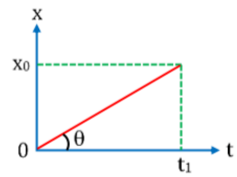
$\theta = 0$

$\tan\theta = \tan 0 = 0$

velocity = 0

i.e. body is at rest.

Case-II



Slope = instantaneous velocity

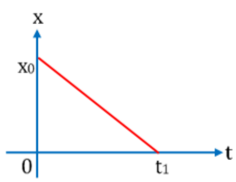
θ = constant

tan θ = constant

velocity = constant

i.e. the body is in uniform motion

Case-III



Slope = instantaneous velocity

θ > 90

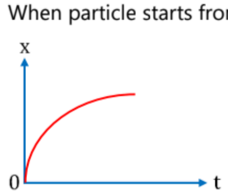
tan θ = –ve

velocity = –ve but constant

i.e. uniform motion

Case-IV

When particle starts from rest and moves with constant acceleration

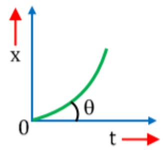


θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ velocity is decreasing with time

i.e. non uniform motion



θ is increasing with time

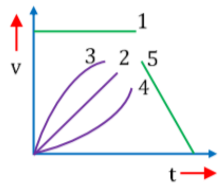
$\therefore \tan\theta$ is increasing with time

$\therefore$ velocity is increasing with time

i.e. non uniform motion

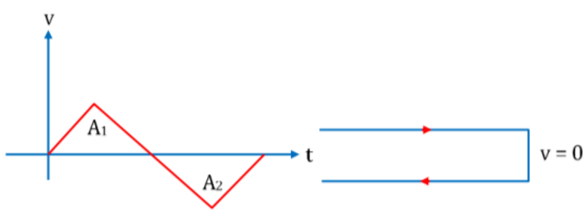
- Area of x-t graph = $\int x\,dt$ = No physical significance

Velocity-time graph



Slope of this graph represents acceleration.

$$\because \tan\theta = \frac{\text{velocity}}{\text{time}} = \text{acceleration}$$



Area under the v-t curve gives

|displacement| = $A_1 - A_2$

Distance = $A_1 + A_2$

Case-I

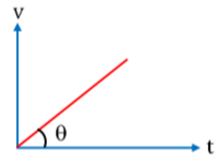
$\theta = 0°$

$\tan\theta = \tan 0° = 0$

acceleration = 0

i.e. v = constant or uniform motion

Case-II



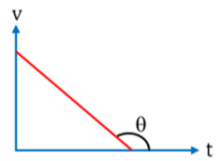
θ = constant

tan θ = constant

acceleration = constant

i.e. uniformly accelerated motion

Case-III



$\theta > 90°$

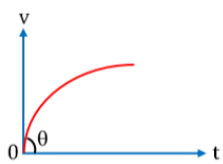
$\tan\theta$ = –ve

acceleration = –ve but constant

i.e. constant or uniform retardation

is acting on the body

Case-IV

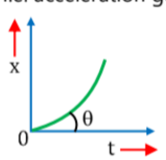


θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ acceleration is decreasing with time

i.e. acceleration goes on decreasing with time but it is not retardation



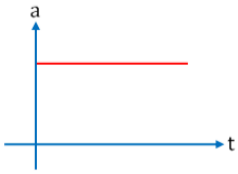
θ is increasing with time

$\therefore \tan\theta$ is increasing with time

$\therefore$ acceleration is increasing with time

i.e. acceleration goes on increasing with time

Acceleration-time graph



Slope of a-t graph defines nothing

But Area of a-t graph gives change in velocity

$\therefore$ Area of a–t graph = $\int a\,dt = \int dv = v_2 - v_1$ = change in velocity

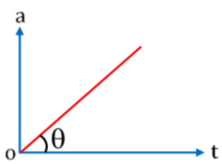
- Area ≠ final velocity

Case-I

$a \propto t^0$

i.e. uniform or constant acceleration

Case-II

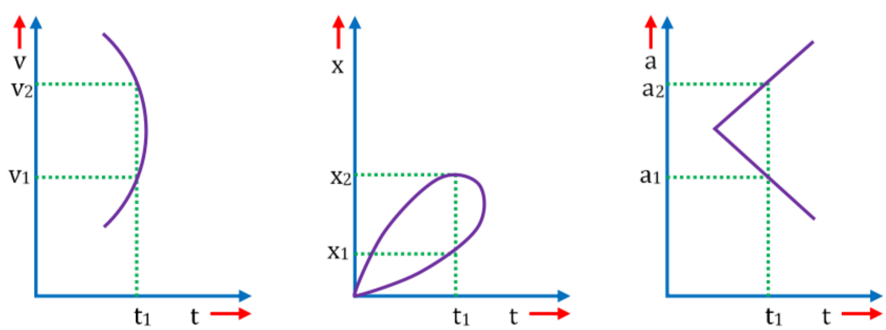


i.e. uniformly increasing acceleration.

Key Points

Following graphs do not exist in practice :

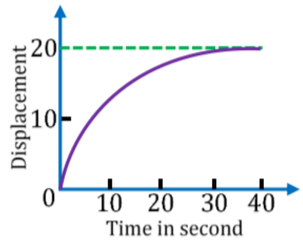
Case-I



Explanation : In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

Illustration 1.

The displacement of a particle as a function of time is shown in the figure. The figure shows that

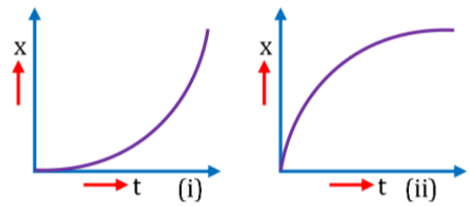


Solution.

The slope of displacement-time graph goes on decreasing, it means the velocity is decreasing i.e. It's motion is retarded and finally slope becomes zero i.e. particle stops.

Illustration 2.

Figures (i) and (ii) below show the displacement-time graphs of two particles moving along the x-axis. We can say that

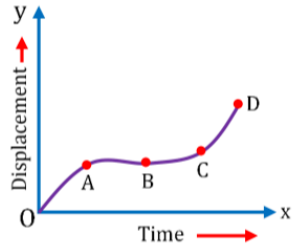


Solution.

Particle (i) is having a uniformly accelerated motion while particle (ii) is having a uniformly retarded motion.

Illustration 3.

The graph between the displacement x and time t for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is



Solution.

Region OA shows that graph bending toward time axis i.e. acceleration is negative.

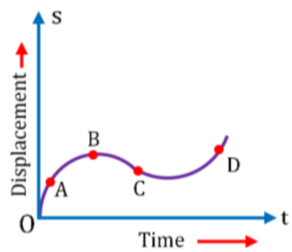
Region AB shows that graph is parallel to time axis i.e. velocity is zero. Hence acceleration is zero.

Region BC shows that graph is bending towards displacement axis i.e. acceleration is positive.

Region CD shows that graph having constant slope i.e. velocity is constant. Hence acceleration is zero.

Illustration 4.

The displacement-time graph of moving particle is shown below



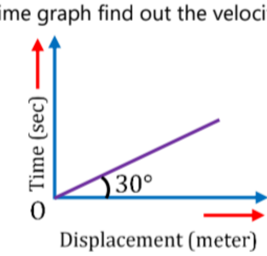
The instantaneous velocity of the particle is negative at the point

Solution.

Slope of displacement time graph is negative only at point C.

Illustration 5.

From the following displacement-time graph find out the velocity of a moving body



Solution.

Slope of the x-t graph gives instantaneous velocity but here t-x graph,

$\upsilon = \tan 30^\circ = \frac{1}{\sqrt{3}}$ m/s.

But it is wrong because formula $v = \tan\theta$ is valid when angle is measured with time axis.

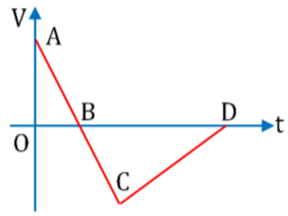
Here angle is taken from displacement axis. So angle from time axis = 90° - 30° = 60°

Now $\upsilon = \tan 60^\circ = \sqrt{3}$

Illustration 3.

For a given velocity time graph, find out :

(i) In which duration acceleration is Positive, Negative and Zero.

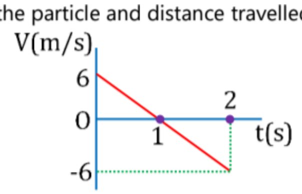


Solution.

AB $\rightarrow$ Negative
BC $\rightarrow$ Negative
CD $\rightarrow$ Positive

Illustration 4.

Velocity-time graph for a particle moving in a straight line is given
Calculate the displacement of the particle and distance travelled in first two sec.



Solution.

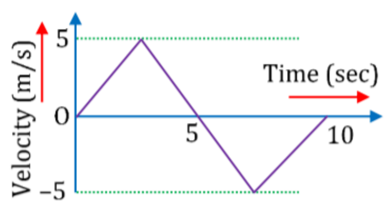
(i) Displacement = area

$$= \left(\frac{1}{2}\times 6\times 1\right)+\left[\frac{1}{2}\times 1\times(-6)\right]=0$$

(ii) Distance $= \left(\frac{1}{2}\times 6\times 1\right)+\left(\frac{1}{2}\times 6\times 1\right)=6\text{m}$

Illustration 5.

The v-t plot of a moving object is shown in the figure. The average velocity of the object during the first 10 seconds is

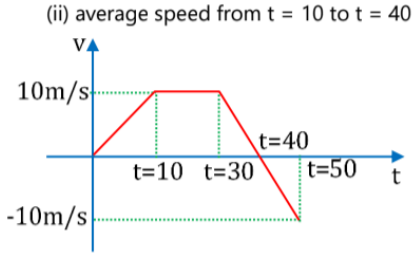


Solution.

Since total displacement is zero, hence average velocity is also zero.

Illustration 6.

Find (i) average velocity from t = 0 to t = 30
(ii) average speed from t = 10 to t = 40



Solution.

Average velocity from t = 0 to t = 30

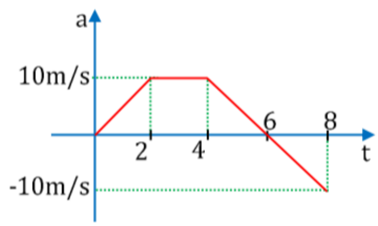
$$<v> = \frac{\Delta x}{\Delta t} = \frac{50+200}{30} = \frac{25}{3} m/s$$

Average speed from t = 10 to t = 40

$$\frac{\text{distance}}{\text{time}} = \frac{200+50}{30} = \frac{25}{3} m/s$$

because there is no turning point.

Illustration 7.



Find (i) Velocity at t = 2 sec, If at t = 0, velocity = 5 m/s

(ii) Maximum velocity from t = 0 to t = 8 sec

Solution.

(i) v at t = 2 sec.

$(\text{Area})_{0 \text{ to } 2} = v_2 - v_0$

$10 = v_2 - 5$

$v_2 = 15$ m/s

(ii) from graph we can say maximum velocity at t = 6 sec

$(\text{Area})_{4 \text{ to } 6} = v_6 - v_4$

$10 = v_6 - 35$

$v_6 = 45$ m/s

Motion under Gravity-Vertical Projection from Ground

Motion Under Gravity

Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of $g = 9.8$ m/s^2

or $g = 980$ cm/s^2

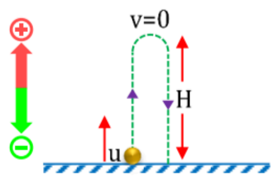
or $g = 32$ ft/s^2

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

Sign Conventions

Negative and positive sign are matter of our choice, so we can select any direction as positive and opposite side as negative.

Vertical Projection from ground (Body is Projected Vertically Upward)



Positive / Negative directions are a matter of choice. You may take another choice.

Equations of motion : Taking initial position as origin and direction of motion (i.e. vertically up) as positive and (vertical downward) as negative.

u (initial velocity) = +u

a (acceleration) = -g

at time t,

Final velocity = v

height above ground = h

$\therefore$ v = u – gt ...(i)

$h = ut - \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 - 2gh$ ...(iii)

$h_{n^{th}} = u - \frac{g}{2}(2n-1)$

Maximum height (H)

Final velocity (v) = 0 so,

From above equation (iii)

$0 = u^2 - 2gH$

$H = \frac{u^2}{2g}$

Total time of flight(T)

at maximum height v = 0

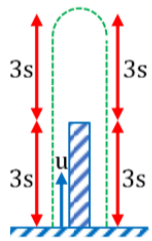
So from above equation (i) u = g t

it is called time of ascent (t_1) = u/g

Illustration 2.

A ball is projected upwards from the foot of a tower. The ball crosses the top of the tower twice after an interval of 6s and the ball reaches the ground after 12s. The height of the tower is ($g=10m/s^2$):

Solution.



$T=\frac{2u}{g}=12$

2u = 120

u = 60 m/s

$h = 60\times3-\frac{1}{2}\times10\times9$

=180-45=135 m

Illustration 3.

A body A is projected upwards with a velocity of 98m/s. The second body B is projected upwards with the same initial velocity but after 4 sec. Both the bodies will meet after

Solution.

Let t be the time of flight of the first body after meeting, then (t-4)sec will be the time of flight of the second body. Since $h_1 = h_2$

$\therefore 98t-\frac{1}{2}gt^2=98(t-4)-\frac{1}{2}g(t-4)^2$

On solving, we get t = 12seconds

Illustration 4.

When a ball is thrown up vertically with velocity V_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity

Solution.

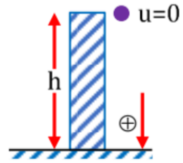
$H_{max}\propto u^2$

$\therefore u\propto\sqrt{H_{max}}$

i.e. to triple the maximum height, ball should be thrown with velocity $\sqrt{3}u$

Motion under Gravity-Dropping from height

Body dropped from some height (initial velocity zero)



Equations of Motion

Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

u = 0 [As body starts from rest]

a = +g [As acceleration is in the direction of motion]

at time t,

final velocity = v

height below dropping point = h

$v = gt$ ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

$v^2 = 2gh$ ...(iii)

Results

from equations (i) (ii) and (iii)

- Time to reach bottom(T)

  $T = \sqrt{\frac{2H}{g}}$

- Maximum speed(at bottom)

  $v = \sqrt{2gH}$

Key Points

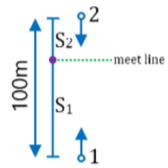
- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of consecutive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integers i.e.
$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots$
- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.
- The distance covered in the n^{th} second, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in 1^{st}, 2^{nd}, 3^{rd} second, etc., will be in the ratio of 1 : 3 : 5, i.e., odd integers only.

Illustration 2.

A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the height where two particle will meet? (take g = 10 m/s²)

Solution.



$S_1 = ut - \frac{1}{2}gt^2$ ....(i)

$S_2 = \frac{1}{2}gt^2$ .....(ii) $S_1 + S_2 = 100$ m

from eq. (i) and (ii) $S_2 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 2^2 = 19.62\text{m}$ (distance from top)

We get $S_1 + S_2 = ut$ Height (s_1) = 100-19.62=80.38m

ut = 100

50t = 100

t = 2s

Illustration 3.

A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of

Solution.

$\frac{1}{2}g(3)^2 = \frac{g}{2}(2n-1) \Rightarrow n = 5\,s$

Illustration 4.

A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec

Solution.

Let the body after time t/2 be at x from the top, then

$$x = \frac{1}{2}g\frac{t^2}{4} = \frac{gt^2}{8} \qquad .....(i)$$

$$h = \frac{1}{2}gt^2 \qquad .....(ii)$$

Eliminate t from (i) and (ii), we get $x = \frac{h}{4}$

$\therefore$ Height of the body from the ground $= h - \frac{h}{4} = \frac{3h}{4}$

Illustration 5.

A particle is dropped under gravity from rest from a height $h(g = 9.8 m/sec^2)$ and it travels a distance 9h/25 in the last second, the height h is

Solution.

Let h distance is covered in n sec

$$\Rightarrow h = \frac{1}{2}gn^2 \qquad ...(i)$$

Distance covered in n^{th} sec = $= \frac{1}{2}g(2n-1)$

$$\Rightarrow \frac{9h}{25} = \frac{g}{2}(2n-1) \qquad ...(ii)$$

From (i) and (ii), h=122.5 m

Illustration 6.

A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

Solution.

The separation between the two bodies, two seconds after the release of second body

$$= \frac{1}{2} \times 9.8[(3)^2 - (2)^2] = 24.5\,m$$

Illustration 7.

A body falls from a height h = 200m. The ratio of distance travelled in each 2 sec during t = 0 to t = 6 second of the journey is

Solution.

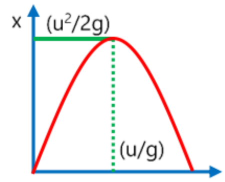
$S_n \propto (2n-1)$ In equal time interval of 2 seconds

Ratio of distance = 1 : 3 : 5

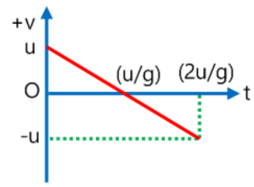
Graphical Problems in Vertical Motion

1. A body is projected vertically upwards then

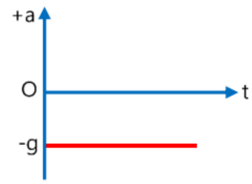
(a) Graphs of displacement with respect to time :



(b) Graph of velocity with respect to time

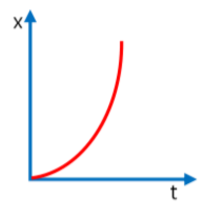


(c) Graph of acceleration with respect to time

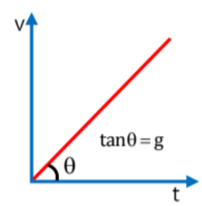


2. A body dropped from some height

(a) Graphs of displacement with respect to time:



(b) Graph of velocity with respect to time



(c) Graph of acceleration with respect to time

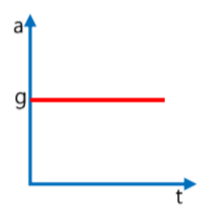


Illustration 1.

A particle is thrown vertically upward with 30 m/s, reaches on maximum height and come on ground after some time. Consider vertically upward direction positive and point of projection as origin. (if g=10 m/sec^2)

(i) Acceleration time graph

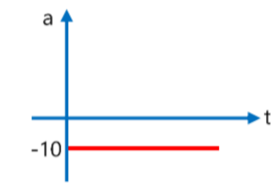
(ii) Velocity time graph

(iii) Speed time graph

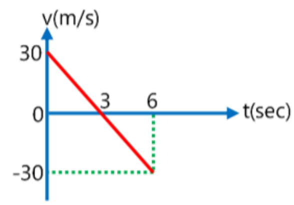
(iv) Displacement time graph

Solution.

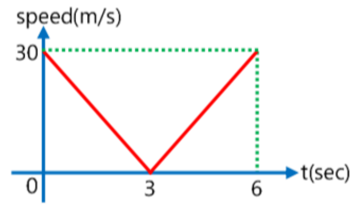
(i) Acceleration time graph



(ii) Velocity time graph



(iii) Speed time graph



(iv) Displacement time graph

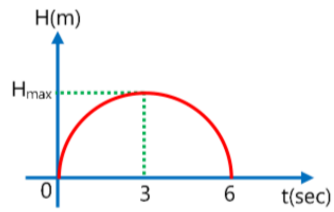
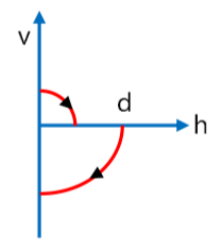


Illustration 2.

A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity υ varies with the height h above the ground is

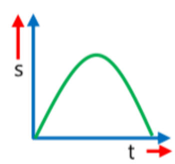
Solution.



For the given condition initial height h = d and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground & just after the collision it becomes half and in opposite direction. As the ball moves upward its velocity again decreases and becomes zero at height d/2.

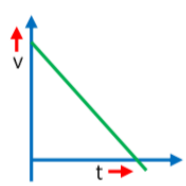
Illustration 3.

The graph of displacement v/s time is



Its corresponding velocity-time graph will be

Solution.



We know that the velocity of body is given by the slope of displacement – time graph. So it is clear that initially slope of the graph is positive and after some time it becomes zero (corresponding to the peak of graph) and then it will becomes negative

Ground to Ground Projectile Motion

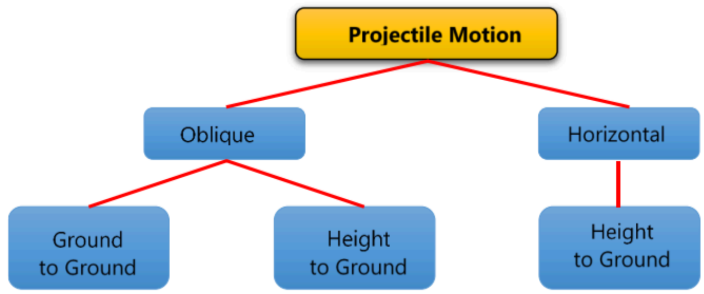
2D Motion:

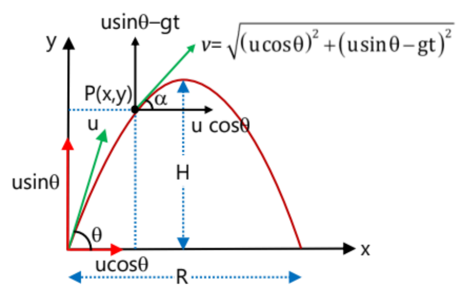
- When the motion of an object is restricted within a plane, it is said to undergo a motion in 2D.
- 2D motion can be studied as two independent 1D motions. (One along x-axis and the other along y-axis).

  Example: Motion of a carrom coin, Projectile Motion, Circular Motion.

Projectile Motion

When a body moves with constant acceleration such that its initial velocity and acceleration are non-collinear then its path is parabola and motion is known as projectile motion.





Projectile Motion	=	Horizontal Motion	+	Vertical Motion
		$u_x = u\cos\theta$		$u_y = u\sin\theta$
		$a_x = 0$		$a_y = -g$
		$v_x = u_x = u\cos\theta$		$v_y = u_y - gt$
		$x = (u\cos\theta)t$		$y = (u\sin\theta)t - \frac{1}{2}gt^2$

Horizontal Motion

- Initial velocity in horizontal direction = $u\cos\theta = u_x$
- Acceleration along horizontal direction = a_x = 0. (Neglect air resistance)
- Therefore, Horizontal velocity remains unchanged.
- At any instant horizontal velocity $v_x = u\cos\theta$
- In time t, displacement along horizontal direction or x co-ordinate is

$$x = u_x t \quad \text{or} \quad x = (u\cos\theta)t$$

Vertical Motion

- It is motion under the effect of gravity so that as particle moves upwards the magnitude of its vertical velocity decreases.
- Initial velocity in vertical direction = $u\sin\theta = u_y$
- Acceleration along vertical direction = $a_y = -g$ (Neglect air resistance)
- At any instant, vertical speed $v_y = u_y - gt = u\sin\theta - gt$
- In time t, displacement in vertical direction or "height" of the particle above the ground

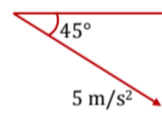
$$y = u_y t - \frac{1}{2}gt^2 = u\sin\theta t - \frac{1}{2}gt^2$$

- Equations of motion should be applied separately for x and y directions.

x-axis	y-axis
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$\Delta x = u_x t + \frac{1}{2}a_x t^2$	$\Delta y = u_y t + \frac{1}{2}a_y t^2$
$v_x^2 = u_x^2 + 2a_x \Delta x$	$v_y^2 = u_y^2 + 2a_y \Delta y$

Illustration 2:

A roller coaster goes down along 45° incline with an acceleration of 5.0 ms^{-2}. (Starts from rest)



(a) How far will the roller coaster travel in 10 seconds horizontally?
(b) How far will the roller coaster travel in 10 seconds vertically?

Solution:

The motion of the roller coaster can be considered as a 1D motion along the direction of acceleration. Taking the direction of acceleration as r-direction.

$$x = x_0 + ut + \frac{1}{2}at^2$$

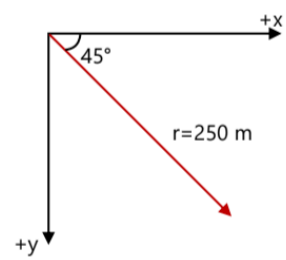
In r-direction,

$$r = r_0 + ut + \frac{1}{2}at^2$$

$$= 0 + 0 \times t + \frac{1}{2}(5)(10)^2$$

= 250 m

Taking right direction as positive x axis and downwards as positive y axis,



Displacement along y-direction

$$\Delta y = r\sin\theta$$

=250 sin 45°

$$= \frac{250}{\sqrt{2}} m$$

Displacement along x-direction

$$\Delta x = r\cos\theta$$

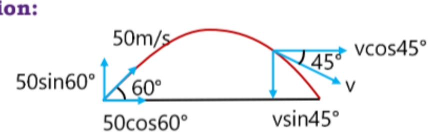
= 250 cos 45°

$$= \frac{250}{\sqrt{2}} m$$

Illustration 3:

A body is projected from ground with speed 50m/s at an angle 60° from horizontal. Find out time after which velocity vector makes 45° angle from horizontal.

Solution:



Horizontal component 50cos60° = vcos45°

v = $25\sqrt{2}$ m/s

v_y = vsin45° = 25 m/s

$a_y = -g$

Now applying equation of motion in y direction

$v_y = u_y + a_{yt}$

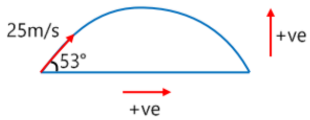
$-25 = 25\sqrt{3} - 10t$

$$t = \frac{25(\sqrt{3}-1)}{10} = 1.83s$$

Illustration 5:

A particle is projected with initial velocity u = 25m/s and θ = 53° with horizontal? Find height of projectile when horizontal distance is 45 m?

Solution:



x-direction

u_x = 25 cos 53°

= 15 m/s

a_x = 0

s_x = 45

y-direction

u_y = 25 sin 53°

20 m/s

a_y = −10

t = 3 sec.

$$s_y = 20\times3-\frac{1}{2}\times10\times3^2$$

s_y = 60 – 45

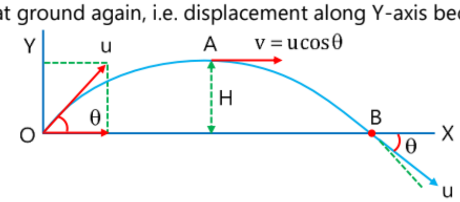
s_y = 15 m

Standard Results of Ground to Ground Projectile Motion

Time of flight (T)

Time of flight is the time for which projectile remains in air.

At time T particle will be at ground again, i.e. displacement along Y-axis becomes zero.



$\because\ y = u_y t - \frac{1}{2}gt^2$ $\qquad\therefore 0 = u_y T - \frac{1}{2}gT^2$

or $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Time of ascent = Time of descent = $\frac{T}{2} = \frac{u_y}{g} = \frac{u\sin\theta}{g}$

At time $\frac{T}{2}$ particle attains maximum height of its trajectory.

Maximum height (H)

At maximum height vertical component of velocity becomes zero. At this instant y coordinate is, its maximum height.

$\because\ v_y^2 = u_y^2 - 2gy \qquad \therefore\ 0 = u_y^2 - 2gH\ \{\because v_y = 0, y = H\}$

$$H = \frac{u_y^2}{2g} = \frac{u^2 \sin^2\theta}{2g}$$

Horizontal range or Range (R)

It is the displacement of particle along X-direction during its complete flight.

$\because x = u_x t \qquad \therefore R = u_x T = u_x \frac{2u_y}{g}; \qquad R = \frac{2u_x u_y}{g}$

$$R = \frac{2(u\cos\theta)(u\sin\theta)}{g} \Rightarrow R = \frac{u^2 \sin 2\theta}{g} \qquad (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

Maximum horizontal range (R_{max})

If value of θ is increased from θ = 0° to 90°, then range increases from θ = 0° to 45° but it decreases beyond 45°. Thus range is maximum at θ = 45°

For maximum range, $\theta = 45°$ and $R_{max} = \frac{u^2 \sin 2(45^\circ)}{g} = \frac{u^2 \sin 90°}{g}$

$$\Rightarrow R_{max} = \frac{u^2}{g}$$

Equation of Trajectory

Along horizontal direction $x = u_x t$ or $x = u\cos\theta t$

Along vertical direction $y = u_y t - \frac{1}{2}gt^2$ or $y = u\sin\theta t - \frac{1}{2}gt^2$

On eliminating t from these two equations

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \quad y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again $y = x\tan\theta\left[1 - \frac{gx}{2u^2\sin\theta\cos\theta}\right] = x\tan\theta\left[1 - \frac{x}{R}\right]$

Kinetic Energy of a Projectile

Kinetic energy = $\frac{1}{2}\times$ Mass × (Speed)^2

Let a body is projected with velocity u at an angle θ.

Thus, initial kinetic energy of projectile, $K_0 = \frac{1}{2}mu^2$

Since velocity of projectile at maximum height is $u\cos\theta$.

Kinetic energy at highest point, $K = \frac{1}{2}m(u\cos\theta)^2 = K_0\cos^2\theta$

Which is the minimum kinetic energy during whole motion.

Key Points

- At maximum height, $v_y = 0$ and $v_x = u_x = u\cos\theta$ so that at maximum height $v = \sqrt{v_x^2 + v_y^2} = u\cos\theta$
- At maximum height angle between velocity and acceleration is 90°.
- Magnitude of velocity at height 'h'.

$v_y^2 = u_y^2 - 2gh$

$v_y^2 = (u\sin\theta)^2 - 2gh$

$v_x = u\cos\theta$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta)^2 - 2gh}$

$|\vec{v}| = \sqrt{u^2 - 2gh}$

Illustration 1:

A projectile is thrown with speed u making angle θ with horizontal at t =0. It just crosses two points of equal height, at time t = 1s and t = 3s respectively. Calculate the maximum height attained by it? ($g=10 m/s^2$)

Solution:

Displacement in y direction $y = u_y \times 1 - \frac{1}{2} g \times (1)^2 = u_y \times 3 - \frac{1}{2} g(3)^2 \Rightarrow u_y = 2g = 20$ m/s

Maximum height attained $h_{max} = \frac{u_y^2}{2g} = 20$m

Illustration 2:

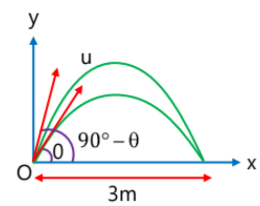
A stone is to be thrown so as to cover a horizontal distance of 3m. If the velocity of the projectile is 7 m/s, find:

(a) the angle at which is must be thrown.

(b) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

Solution:

(a) Range $R = \frac{u^2}{g}\sin 2\theta$



$\Rightarrow \sin 2\theta = \frac{gR}{u^2} = \frac{9.8 \times 3}{(7)^2} = 0.6 = \sin 37° \Rightarrow 2\theta = 37° \Rightarrow \theta = 18.5°$

angle of projection may also be = $90° - \theta = 90° - 18.5° = 71.5°$

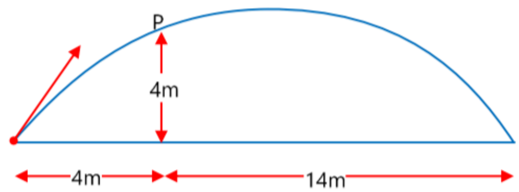
(b) For largest horizontal displacement $\theta = 45°$

Maximum range $R_{max.} = \frac{u^2}{g} = \frac{(7)^2}{9.8} = \frac{49}{98} \times 10 = 5$ m

Illustration 4:

A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of initial velocity of the ball figure is given below.

Solution:



The ball passes through the point P(4, 4). Also, range = 4 + 14 = 18 m.

The trajectory of the ball is, $y = x\tan\theta\left(1-\frac{x}{R}\right)$

Now x = 4m, y = 4m and R = 18 m

$\therefore 4 = 4\tan\theta\left[1-\frac{4}{18}\right] = 4\tan\theta\left(\frac{7}{9}\right)$ or $\tan\theta = \frac{9}{7} \Rightarrow \theta = \tan^{-1}\frac{9}{7}$

And $R = \frac{2u^2\sin\theta\cos\theta}{g}$, or $18 = \frac{2}{9.8} \times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}} \Rightarrow u = \sqrt{182}$

Illustration 5:

A ground to ground projectile has equation of trajectory, $y = x - \frac{x^2}{40}$. Find angle of projection and speed of projection.

Solution:

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ (General equation) ....(i)

$y = x - \frac{x^2}{40}$ (Given equation) ....(ii)

By comparing equation (i) and (ii)

$\tan\theta = 1; \theta = 45°$

$\frac{g}{2u^2\cos^2\theta} = \frac{1}{40}$

$\therefore u = 20\ m/s$

$\left(y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}\right)$ is valid when we take starting point as origin and $\xrightarrow{+ve} \uparrow +ve$

Illustration 6:

The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle 45° to the horizontal.

Solution:

$R = \frac{u^2\sin(2\times15°)}{g} = 1.5$ or $\frac{u^2}{g}\times\frac{1}{2} = 1.5$ or $\frac{u^2}{g} = 3\,km$

Horizontal range for angle of projection 45° is

$R' = \frac{u^2}{g}\times\sin(2\times45°) = \frac{u^2}{g}\sin90° = \frac{u^2}{g} = 3km$

Illustration 1:

A projectile is fired horizontally with a speed of 98 ms^{-1} from the top of a hill 490 m high. Find

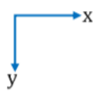
(i) the time taken to reach the ground

(ii) the distance of the target from the hill and

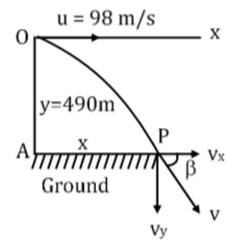
(iii) the velocity with which the projectile hits the ground. (take g = 9.8 m/s^2)

Solution:

(i) The projectile is fired from the top O of a hill with speed u = 98 ms^{-1} along the horizontal as shown as OX. It reaches the target P at vertical depth



OA, in the coordinate system as shown,



OA = y = 490 m

As, $y = \frac{1}{2}gt^2$

$\therefore\ 490 = \frac{1}{2} \times 9.8\,t$

or $t = \sqrt{100} = 10$ s.

(ii) Distance of the target from the hill is given by, AP = x = Horizontal velocity × time = 98 × 10 = 980 m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$v_x = u = 98\ ms^{-1}$

$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ ms^{-1}$

$V = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2}\ ms^{-1}$

Now if the resultant velocity v makes an angle β with the horizontal, then

$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \qquad \therefore \beta = 45°$

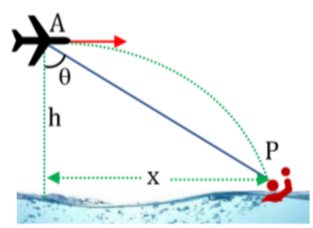
Illustration 3.

A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? (g = 9.8m/s^2)

Solution.

Plane is flying at a speed = $600 \times \frac{5}{18} = \frac{500}{3}$ m/s horizontally (at a height 1960m)

Time taken by the kit to reach the ground $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20s$



In this time the kit will move horizontally by x = ut= $\frac{500}{3} \times 20 = \frac{10{,}000}{3}$m

So, $\tan\theta = \frac{x}{h} = \frac{10{,}000}{3 \times 1960} = \frac{10}{5.88} = 1.7 \simeq \sqrt{3}$ or $\theta = 60°$

About Kinematics in AP Physics 1

The Kinematics unit in AP Physics 1 focuses on describing motion without considering the forces causing it. Students learn how to analyze displacement, velocity, acceleration, and motion graphs in different physical situations.

This topic is frequently tested in the AP Physics exam through conceptual questions, numerical problems, and graph-based analysis. Questions often involve interpreting position-time and velocity-time graphs and applying kinematic equations.

A strong understanding of kinematics is essential for solving advanced topics such as forces, energy, and momentum.

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