INTEGRAL RESEARCH
Ram Transform Technical Report Series

Ram Calculus and Ram Transforms Family: Technical Books and Reports

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6 volumes · 220 technical reports · 4,589 total pages

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This catalog lists the complete Integral Research Technical Report Series on the Ram Calculus and the Ram Transforms family: six volumes containing 220 technical reports and roughly 4,589 pages in total, together with a one-page summary of each volume. The whole series is built on a single idea developed to remarkable breadth: that a smooth signal, operator, or system governed by a shift-variant integral or differential relation can be localized by a change of coordinates and a truncated derivative-jet expansion, turning one global equation into a small local system whose matrix is unit upper triangular with unit determinant, and therefore inverted exactly, analytically, and without dividing by the operator as a whole. From that one mechanism the series grows into a full operator calculus and carries it across mathematics, engineering, and finance.

The theoretical core spans the whole collection. Early volumes establish the exactness and polynomial-preservation guarantees, the spectral view relating kernel moments to a windowed kernel spectrum, and the Ram Complex Spectral Transform that generalizes the Fourier and Laplace transforms with an exact shift-variant convolution theorem. On top of this sits the Ram-Master Transform, a single parameterized atom that embeds the Fourier, Laplace, Mellin, short-time Fourier, Gabor, Morlet, chirplet, wavelet, and wavelet-packet transforms as special cases, extended by fractional, complex-order, quaternionic, octonionic, graph-local, mollifier, and microlocal layers. Later volumes add the multimodal, multi-panel, and double-domain-decomposition calculi for wide and multi-peaked kernels, the discrete shift-variant transform, general variable-coefficient operator inversion and calibration, and the Graph-Global Ram Transform, whose restriction theorem shows the graph Fourier transform, diffusion operators, spectral graph wavelets, and common graph-neural-network filters to be exact instances.

The applications are extensive and cross-disciplinary. They include shift-variant image deblurring and computational imaging; delay-Doppler and chirp waveforms for OTFS, 6G integrated sensing and communication, cooperative and reconfigurable-surface networks; option pricing, local- and rough-volatility calibration, yield curves, and risk detection in quantitative finance; interpretable and certifiable operator cores for scientific machine learning, neural operators, and uncertainty quantification; power-system state estimation and microgrid fault location; robot task planning, contact-rich dynamics, and trajectory optimization; quantum noise-channel inversion and Hamiltonian learning; computational lithography, battery diagnostics, wearable and contactless physiological sensing, automotive radar and lidar, single-photon lidar, solid-state DC transformers, weather and climate operators, world-model tokenizers, industrial engineering surrogates, materials discovery, and graph reasoning, planning, and search.

The series is candid throughout about what is proven and what is a research direction: several reports pursue the Riemann Hypothesis and three-dimensional Navier-Stokes regularity but present themselves explicitly as structural advances, conditional results, and research programs rather than finished proofs, and many applied reports position their methods as interpretable, certifiable complements to, rather than replacements for, mature numerical and learned solvers. The collection best serves researchers, quantitative engineers, applied mathematicians, and technically sophisticated licensees who want a coherent operator-centric framework with closed-form guarantees, implementable matrices and algorithms, prior-art assessments, and honest scope. Each volume and each individual report is available for purchase; to inquire to buy or request a copy, use the Inquire to buy button beside any item, or contact muralis@IntegralResearch.net.

The Six Volumes

Volume 1 — Technical Reports TR1–TR106

116 technical reports · 2,450 pages.

Topics: Ram Transform theory, Shift-variant inverse problems, Image deblurring, Signal and spectral analysis, Wireless and sensing, Quantitative finance, State estimation and filtering, Medical imaging, Scientific machine learning, Neural operators, Stochastic and Fokker-Planck methods, Analytic number theory.

This volume is the founding collection of the Ram Transform technical-report series, gathering roughly one hundred sixteen reports spanning report numbers one through one hundred six and totaling about two thousand four hundred fifty pages. Its unifying theme is a single idea developed to remarkable breadth: that a global, shift-variant integral or differential equation can be exactly localized by a coordinate change and a truncated series expansion, turned into a small local operator whose matrix is unit upper triangular with unit determinant, and therefore inverted analytically point by point. From that one mechanism the volume builds an entire operator calculus and then carries it into imaging, communications, finance, physics, biology, machine learning, and pure mathematics. Buyers get both the rigorous foundations and a wide, honestly scoped map of where the method helps and where it does not.

The theoretical core establishes the exactness guarantees, the spectral-domain view relating kernel moments to a windowed kernel spectrum, and the Ram Complex Spectral Transform that generalizes the Fourier and Laplace transforms with an exact shift-variant convolution theorem. On top of this sits a large transform family: real and sine branches, wavelet and Ramlet constructions, the four-parameter Ram-Master Transform whose universal reduction theorem subsumes wavelet, short-time Fourier, Morlet, Gabor, and chirplet transforms, and algebra-valued extensions to fractional, complex, quaternionic, and octonionic order for long-memory, log-periodic, and directional signal analysis. Explicit ready-to-use coefficient matrices are provided for one, two, and three dimensions, including general, Gaussian, and rectangular kernels.

The applied threads are extensive. In imaging and vision, reports cover shift-variant deblurring benchmarked against Wiener and Richardson-Lucy filtering, affine motion blur, and shape-from-blur camera calibration. Communications and sensing work maps delay-Doppler modulation, high-mobility and satellite channels, radar, sonar, fiber-optic, and underwater-acoustic problems onto shift-variant operators, with several reports candidly correcting earlier over-broad invertibility claims. Quantitative-finance reports treat option pricing and calibration for stochastic-volatility and jump-diffusion models, nonlinear Black-Scholes equations, Hamilton-Jacobi-Bellman control, and mean-field games, alongside deterministic alternatives to Monte Carlo simulation. A substantial state-estimation and stochastic-process cluster develops Fokker-Planck-Ram operators across linear, time-variant, and nonlinear regimes and reframes particle and Kalman filtering as operator recursions. Medical-imaging reports critically verify magnetic-field tomography and magnetoencephalography patents and design a handheld low-field imaging instrument. A large scientific-machine-learning group introduces Ram-Master neural networks, glass-box identification of neural integral equations, attention operators, and physics-informed neural operators, positioning the transforms as interpretable, invertible operator layers.

The volume is equally forthright about its ambitious pure-mathematics program. Several reports pursue the Riemann Hypothesis and the Generalized Riemann Hypothesis through operator-theoretic, adelic, and explicit-formula routes, and others pursue Navier-Stokes existence and smoothness, but each states its assumptions and remaining gaps explicitly and presents itself as a structural advance, conditional theorem, or research roadmap rather than a finished proof. This candor extends throughout: many applied reports frame the methods as preconditioners, micro-solvers, and diagnostic tools that complement rather than replace mature numerical solvers.

The volume suits researchers, quantitative engineers, and technically sophisticated investors seeking a coherent, cross-disciplinary framework for shift-variant inverse problems, together with the mathematical guarantees, implementable matrices and algorithms, prior-art assessments, and a dictionary and taxonomy that make the whole corpus navigable.

To inquire to buy Volume 1, contact muralis@IntegralResearch.net.

Volume 2 — Technical Reports TR107–TR120

20 technical reports · 366 pages.

Topics: Ram Transform theory, Microlocal analysis, OTFS channel estimation, 6G wireless, Reconfigurable intelligent surfaces, Physical-layer security, Integral equations, Graph neural networks, Minimax optimization, Materials discovery, Operator learning, Bayesian inference.

This second volume gathers twenty technical reports spanning roughly report TR107 through TR120, totaling about 366 pages, and forms the theoretical and applied heart of the Ram Transform program. Where the individual reports treat distinct problems, together they tell one story: how a single family of local, finite-jet operators for shift-variant systems can be consolidated into a rigorous unified framework and then carried into wireless communications, optimization, graph reasoning, and materials discovery. Buyers get both the deepened mathematical foundation and a set of worked application programs that show where the methods fit, what they cost, and where they remain research directions rather than finished products.

The volume opens with a substantial theory core. A curated target list surveys recent papers and patents from roughly 2021 to 2026 whose problem classes suit these methods, ranging over operator learning for integral equations and partial differential equations, shift-variant deblurring, time-varying channel estimation, geophysical inversion, and quantitative finance, each entry paired with a concrete opportunity statement. The centerpiece is the revised Ram-Master Transform, presented in several reconciled versions that define a single analytic transform family containing Fourier, Laplace, Mellin, short-time Fourier, Gabor, Morlet, chirplet, wavelet, and wavelet-packet analysis as special cases, and that acts as a local symbol and inverse calculus for operators whose kernels vary with position, scale, or learned context. Companion reports reassess the family's novelty against pseudo-differential symbol calculus and the Segal-Bargmann coherent-state transform, and extend it by fusing it with microlocal analysis for singularity detection, wavefront-set localization, and propagation of singularities. A modernized revision of a chapter from the 2007 Rao Transforms book rounds out the theory by recasting localized Fredholm, Volterra, Urysohn, and Hammerstein integral-equation solvers under the current framework.

The largest applied thread is a full wireless communications program for 5G-Advanced and 6G. It begins with delay-Doppler operators for OTFS channel estimation, including moment-domain Fisher information and Cramer-Rao criteria for pilot design, then scales to graph-local cooperative networks covering multicell, cell-free massive-MIMO, reconfigurable intelligent surfaces, and non-terrestrial links. Further reports build channel world models with memory, prediction, uncertainty, and active-probing policies; a mollifier-based neural receiver that confines learning to residual correction over an analytic backbone; moment-shaped waveform and metasurface design keyed to operator conditioning; and physical-layer security and watermarking through secret moment masks and graph residual fingerprints. A standards-and-hardware mapping onto Open RAN pipelines and a balanced advantages-and-costs assessment with a deployment roadmap close the thread honestly, concluding that these methods are a compelling new layer for hard regimes such as high Doppler and shift-variant channels rather than a universal replacement.

The remaining reports carry the framework into three further areas. In optimization, the family is applied to the ordinary-differential-equation analysis of stochastic gradient methods for minimax problems, including optimistic and anchored gradient descent, with a worked derivation of an order one-over-t convergence rate in continuous time. In graph reasoning, graph-local Ram neural networks and Bayes-net encodings are examined as complements to relational probabilistic inference. In materials discovery, the methods are explored as structured graph-local diagnostics for generating, screening, and ranking chemical compounds and crystals, positioned alongside density-functional theory, molecular dynamics, and generative models rather than supplanting them. This volume suits researchers, wireless-systems engineers, and technical strategists who want a mathematically grounded, interpretable alternative or augmentation to purely learned methods, together with a candid view of its scope and open problems.

To inquire to buy Volume 2, contact muralis@IntegralResearch.net.

Volume 3 — Technical Reports TR121–TR151

31 technical reports · 902 pages.

Topics: Quantitative finance, Option pricing, Volatility calibration, Ram Transform theory, Uncertainty quantification, Scientific machine learning, Graph neural networks, 6G sensing, Computational lithography, Battery state estimation, Inverse operators, Data assimilation.

Volume 3 gathers thirty-one technical reports spanning roughly nine hundred pages that carry the Ram Transform's local operator calculus out of pure theory and into a broad sweep of applied domains, from quantitative finance and wireless sensing to battery diagnostics, weather modeling, and the foundations of the method itself. The unifying idea across the collection is that many maps treated as black boxes by contemporary machine learning, whether a pricing model, a physical measurement channel, or a trained neural network, are in fact governed by known kernels that can be expanded locally into a derivative jet, giving an explicit forward operator and, through an upper-triangular moment matrix, an exact or regularized inverse. Where the current literature reaches for gradient descent, learned surrogates, or Monte Carlo sampling, these reports offer closed-form, training-free, auditable alternatives with error bounds.

The largest thread is quantitative finance, where roughly a dozen reports rebuild option pricing and calibration on Ram semigroups. Representative results include a local Black-Scholes semigroup that yields closed-form Delta, Gamma, and Theta together with a linear-complementarity method for American-put free boundaries; Ram-Dupire inverse operators that recover a local volatility surface from observed prices; Fokker-Planck-Ram transition kernels covering Heston, SABR, and jump-diffusion models such as Kou and Bates; and treatments of rough and lifted-Heston volatility, tensorized high-dimensional pricing PDEs, Bermudan continuation, and analytic Volterra and Feynman-Kac kernels for arbitrage-aware calibration, XVA, and multi-asset hedging. Several of these reports engage named patents and papers directly, proposing operator-based complements to invertible neural networks, GPU Monte Carlo systems, and deep density generators.

A second thread turns the same local-operator lens on machine learning and uncertainty quantification. Reports here reframe approximate-Bayesian neural operators, linearized Laplace Gaussian processes, factor-graph uncertainty propagation, and the compilation of graph neural networks into relational Bayesian networks, in each case replacing global black-box uncertainty with structured local Ram covariance and inverse operators. This group extends to federated neural graphical models with privacy-preserving graph-local updates, physics-informed material-property regression benchmarked on creep-rupture life, latent particle filtering for real-time data assimilation, and graph-based financial risk detection. A third, strongly commercial thread delivers exactly invertible operators for engineering and device applications: unified delay-Doppler and chirp waveforms for 6G integrated sensing and communication, an analytic inverse of the Hopkins imaging kernel for computational lithography, regularization-free distribution-of-relaxation-times inversion for microcontroller-grade battery state estimation, noise-stable deconvolution for wearable blood-pressure and glucose sensing, exact-inverse chirp operators for automotive radar and lidar, Lindblad noise-channel inversion and gate-pulse synthesis for quantum computing, invertible operators for AI weather stacks, invertible tokenizers for autonomous-driving world models, and closed-form surrogates for industrial computer-aided engineering.

The volume also documents the theory that underpins all of this. It includes the consolidated Ram-Master Transform, which embeds Fourier, Laplace, Gabor, wavelet, Mellin, and other transforms as restrictions of one parameter space and adds fractional, hypercomplex, graph-local, and microlocal layers; multimodal and panel domain-decomposed transforms for multi-lobe and wide kernels; and a forward-looking catalog of 2024-2026 application targets. Two reports are candidly framed as honest research programs rather than finished results, surveying the zero-density landscape around the Riemann Hypothesis and finite-time singularity formation in Navier-Stokes and Euler flows, with every conjectural step explicitly flagged.

The buyer best served by this volume is a quantitative researcher, financial engineer, signal-processing or device specialist, or scientific-machine-learning practitioner who wants interpretable, certifiable, closed-form operator methods with provable error bounds as alternatives or constraints alongside neural approaches. They gain a large, cross-referenced library of worked forward and inverse constructions, model-by-model diagnostics, and calibration recipes ready to adapt to their own domains.

To inquire to buy Volume 3, contact muralis@IntegralResearch.net.

Volume 4 — Technical Reports TR152–TR159

8 technical reports · 317 pages.
Topics: Ram Transform theory, shift-variant deblurring, computational imaging, numerical PDEs, nonlinear integral equations, inverse problems, kernel calibration, system identification, Fokker-Planck evolution, domain decomposition, block-banded solvers, data-driven modeling.

This volume gathers eight technical reports spanning roughly three hundred and seventeen pages, and forms the most structurally ambitious installment of the series so far. Its unifying theme is the extension of the shift-variant Ram Transform beyond a single expansion center into a general block-banded jet calculus, and the systematic application of that calculus across four connected fronts: kernel and integral operators, imaging and deblurring, partial differential equations, and the inverse calibration problems that tie them together. Read as a whole, the volume takes a single algebraic idea, that local behavior can be captured by Taylor jets about several panels or modes and coupled through a sparse banded system, and carries it methodically from foundational theory to nonlinear and data-driven practice.

The first thread establishes the theory. A three-part opening series proves a unification theorem showing that two natural ways of moving past one expansion center, the multimodal approach with one center per kernel lobe and the panel domain-decomposition approach with contiguous tiles, are both instances of one block-banded jet calculus built purely from the kernel's incomplete moments. It develops source-kernel parameter decoupling, a complex-spectral extension, and then pushes the framework from linear integral operators into nonlinear ones, where the kernel depends on the unknown function itself, adding Fokker-Planck-Ram density evolution, Kramers-Moyal moment identification, and mean-field McKean-Vlasov dynamics. This gives the reader a self-contained account of how wide, multi-peaked, and nonlinear shift-variant operators can be represented and inverted with predictable sparse structure.

A second thread turns the machinery toward imaging and identification. One report constructs per-point block stencils, nine panels in two dimensions and twenty-seven in three, for shift-variant deblurring where the point-spread function changes with depth, field, or sensor, inverting the resulting sparse operator with block Thomas, ADI, or block Gauss-Seidel solvers and reflection-based boundary handling, with direct relevance to aberration correction, motion and turbulence blur, and three-dimensional microscopy. A companion report addresses the practical question of estimating an unknown blur from data, splitting the task into a linear calibration stage and a linear restoration stage and introducing supervised per-panel moment calibration, semi-blind inverse-consistency elimination, and a cross-center persistent-excitation condition that governs when the kernel is identifiable at all.

The third and largest thread ports the same structured calculus to partial differential equations and their inverse problems. A two-part PDE series represents solutions by jets about several centers, couples them through a block-banded system closed by boundary conditions written as reflection rows, and adds a complex-node quasi-Trefftz variant for oscillatory equations, then extends to nonlinear PDEs through block-Newton and Gauss-Newton iterations that preserve band sparsity. Two further reports recast coefficient identification as calibration, exploiting the bilinearity of the local PDE-jet identity so that recovering shift-variant coefficients becomes a well-posed linear regression when the solution is known, with representative cases such as recovering unknown viscosity in Burgers-type equations, Fokker-Planck drift-diffusion identification, and panel-local model discovery, and unifying kernel calibration and differential-operator calibration under one persistent-excitation framework.

The volume will most reward researchers and practitioners working in computational imaging and restoration, numerical solution of shift-variant integral and partial differential equations, system identification, and data-driven modeling and inverse problems. Buyers gain a coherent, cross-referenced toolkit, from unification theorems through solver architecture to calibration recipes, rather than isolated results. Much of the later material is presented as a structured methodology and research program with worked derivations and solver strategies rather than large-scale empirical benchmarks, and readers evaluating it for production use should expect to supply their own validation, but the algebraic backbone is developed consistently and in depth across all eight reports.

To inquire to buy Volume 4, contact muralis@IntegralResearch.net.

Volume 5 — Technical Reports TR160–TR180

21 technical reports · 212 pages.

Topics: Ram Transform theory, Power system state estimation, Wireless and 6G sensing, Radar and lidar receivers, Robotics and task planning, Quantitative finance calibration, Numerical PDE solvers, Quantum Hamiltonian learning, Scientific machine learning, Medical sensing, LLM reasoning and RAG, Verifiable QA.

This fifth volume gathers twenty-one technical reports spanning roughly two hundred and twelve pages, all organized around a single unifying idea: that a wide range of scientific, engineering, and financial inverse problems can be recast as the inversion of a local operator with an exact, analytically invertible, division-free structure. Where much recent work reaches first for a trained neural network or an iterative estimator, the reports here systematically replace or complement those with a Ram-Calculus operator core whose local moments carry the governing physics, and whose inverse is available in closed form, often in linear time, without training, and with certifiable per-element estimates. The volume advances from foundational theory through a broad sweep of applications, and closes with reflective reports that consolidate the corpus and chart open problems, so a reader sees both the mathematical machinery and its reach.

Several reports develop the underlying theory and reusable architecture. New foundational pieces introduce the discrete shift-variant Ram transform, completing the transform family alongside its shift-invariant and continuous cousins, and a sampling theorem for known shift-variant channels that makes recovery statements the classical Nyquist-Shannon result cannot. A generic study formalizes the recurring pattern of an exact analytic operator core paired with a small learned residual, proving why such hybrids are data efficient and how they preserve invertibility, conservation, and uncertainty propagation. These threads recur throughout the applied reports as a shared design language.

The application areas are wide. In power systems, the volume recasts static and then dynamic state estimation as a graph-operator inverse that is exact and training-free on radial feeders and localizes the effect of topology changes. Wireless and sensing reports cover near-field wideband integrated sensing and communication, blind delay-Doppler channel identification, and closed-form radar and lidar receivers that handle nonlinear chirp and mutual interference. Robotics appears both as single-robot task planning framed on graphs and as contact-rich manipulator and legged-robot dynamics inversion. Quantitative finance contributes arbitrage-consistent yield-curve construction and rough-volatility calibration with jumps, including a dedicated rough-Heston inverse-calibration engine aimed at auditable, regulator-facing use. Further reports address in-situ quantum Hamiltonian learning, analytic Green's-function surrogates for linear partial differential equations reaching machine precision on worked examples, atmospheric boundary-layer turbulence closures, contactless and multi-frequency physiological sensing, and forward and inverse elliptic partial differential equation solvers reformulated from explainable Fredholm neural networks. Language-model and codebase intelligence over context graphs, and a verifiable, watermarked question-answering system with calibrated verification scoring, round out the applied span.

A buyer evaluating this volume should expect a mix of finished, worked results and honest research directions. Many reports carry step-by-step operator derivations, error bounds, algorithms, and numerical examples reaching near machine precision, while others are candidly framed as assessments, roadmaps, or explorations of open problems, including the concluding report on curved-geometry panelization and boundary-integral operators and the updated agentic research-automation program that folds in the methods developed across the series. The audience is broad: researchers and engineers in power systems, wireless and radar, robotics, scientific computing, quantitative finance, medical sensing, and quantum device characterization who want interpretable, certifiable, locally exact alternatives to black-box learned models, as well as prospective licensees seeking auditable methods with provable structure. What the reader gets is a coherent operator-centric toolkit, demonstrated repeatedly across domains, with enough derivation, benchmarking intent, and candid assessment to judge where it genuinely outperforms and where work remains.

To inquire to buy Volume 5, contact muralis@IntegralResearch.net.

Volume 6 — Technical Reports TR181–TR203

24 technical reports · 342 pages.

Topics: Ram Transform theory, Differential operator inversion, Robot trajectory optimization, In-context world modeling, Graph signal processing, Graph neural networks, Power systems, Power electronics, Computational lithography, Single-photon LiDAR, Kernel calibration, Reasoning and planning.

This volume gathers twenty-four technical reports spanning roughly three hundred forty pages, drawn from the TR181 through TR203 range, and it represents the most integrative stretch of the Ram Calculus program to date. The unifying idea across every report is that a smooth signal, operator, or system can be parameterized by its local derivative jet rather than by global interval data, so that one governing equation becomes a finite system of same-point derivative relations that inverts algebraically by back-substitution on a unit-upper-triangular matrix, never dividing by the operator as a whole. The volume moves systematically from foundational theory, through general operator inversion and calibration, out to a wide arc of concrete engineering applications, and finally to graph and network settings and to whitepaper-level surveys that position the whole framework.

Several reports build the theoretical spine. One establishes a local inversion method for general linear differential relations with arbitrary polynomial coefficients, proving the forward map is a tall, lower-banded matrix invertible by finite, division-free back-substitution. A two-part series extends the four operating modes, forward, inverse, supervised calibration, and semi-blind elimination, to those general operators and then applies the Ram-Master Transform, a single atom combining a window, a wavelet factor, and an oscillatory or decaying exponential, to move derivatives off noisy data and turn operators into algebraic symbols. Companion reports distill the core ideas for commonly taught differential equations, from the nonlinear pendulum and Bratu problems to Sturm-Liouville and Schrodinger eigenproblems and the Euler-Bernoulli beam, and add a third decomposition axis, double-domain decomposition, that computes full inverses only at cell centers and propagates by Taylor expansion.

The application threads are broad and detailed. In robotics, reports recast in-context world modeling as supervised calibration of a local sensorimotor operator, address the complementary regime where safe self-probing is unavailable, and develop a jet-flow correspondence for trajectory optimization that removes coarse-step bias in differential dynamic programming and iterative linear-quadratic regulators, with a reproducible pendulum experiment reproducing finite-difference Jacobians to numerical precision while cutting derivative time roughly sixteen-fold. Power and electronics work covers single-phase microgrid fault location, re-derived transparently through recovered poles and residues, and solid-state DC transformers, where ideal switching waveforms are exactly piecewise-polynomial panel jets. Imaging and lithography contributions treat the resolution limit of single-photon LiDAR, with a moment-exact re-derivation and a jet-order reconstruction upgrade, and a vectorial local inverse for high-numerical-aperture EUV lithography with polarization and thick-mask effects.

A substantial graph and network cluster shows that the Ram Calculus of graphs supplies auditable, graph-local operators around search and planning rather than replacing solvers like A-star or Dijkstra, offering exact finite inverses on trees and directed acyclic graphs, certified soft heuristics, and low-rank topology-update rules. It culminates in the Graph-Global Ram Transform, whose restriction theorem proves that the Graph Fourier Transform, heat and diffusion operators, spectral graph wavelets, and common graph-neural-network filters are all exact instances, with a companion volume of applications and extensions. Three whitepapers survey the program across real, complex, and hypercomplex signals up through symbolic, graph, and agent representations, and honestly correct several specific overstated claims, while a final report lays out a costed plan for an automated report pipeline and storefront.

The volume suits researchers, applied mathematicians, and engineers in control, power, signal processing, imaging, lithography, and graph learning who want transparent, closed-form, certifiable alternatives or glass-box sidecars to numerical and learned methods. Buyers should note that some reports are finished, verified derivations while others are explicitly research programs, roadmaps, or drafts, and the collection is candid throughout about which claims are verified and which remain aspirational.

To inquire to buy Volume 6, contact muralis@IntegralResearch.net.

All Individual Technical Reports

Volume 1 — Technical Reports TR1–TR106

TR1. Eigenfunction and Polynomial Preservation Properties of the Generalized Ram Transform: Unit Upper Triangular Structure, Exact Inversion, and L² Considerations for Polynomial and Exponential Input Functions (18 pp.)

Domain: Ram Transform theory

A rigorous structural analysis of two defining properties of the Ram Transform: complex exponentials are exact eigenfunctions in the infinite-order limit, and polynomials up to a chosen degree are preserved exactly with analytic, non-approximate inversion. It proves the transform matrix is block unit upper triangular with unit determinant, guaranteeing invertibility for any kernel, and carefully treats the associated function-space norm considerations. Valuable for anyone needing certified exactness guarantees in shift-variant inverse problems.

TR2. Application of the Complex Operator Integral Ram Transform (COIRT), Complex Coordinate Mapping, and N-Dimensional Complex Ram Transform to the Navier-Stokes Existence and Smoothness Problem (18 pp.)

Domain: Pure mathematics

Applies the complex-operator and multidimensional Ram Transform machinery to the three-dimensional incompressible Navier-Stokes equations, one of the Millennium Prize Problems. It reformulates the equations as a Volterra integral equation, uses a Wick rotation to a better-conditioned parabolic form, identifies the Stokes semigroup as an exact eigenoperator, and handles vortex stretching via a nonlinear Jacobian transform. The report is explicit that it is a structural advance and roadmap, not a completed proof, isolating the remaining convergence gaps.

TR3. Explicit Coefficient Matrices for the Two-Dimensional Ram Transform: Forward and Inverse Operators for General, Gaussian, and Rect Kernels (16 pp.)

Domain: Image processing

Provides ready-to-use explicit six-by-six forward and inverse coefficient matrices for the two-dimensional Ram Transform at image order two and point-spread order one. Every matrix entry is given in closed form for three kernel cases: a general isotropic moment-based kernel, a shift-variant Gaussian, and a separable rectangular blur, along with the exact restoration formula. A self-contained polynomial-exactness verification strategy and complete symbolic-computation code are included, making it directly implementable.

TR4. Shift-Variant Image Deblurring Using Ram Transform: Experimental Study and Comparison with Wiener and Richardson–Lucy Methods (12 pp.)

Domain: Image processing

A systematic experimental study of shift-variant image deblurring comparing the Ram Transform against tiled Wiener filtering and Richardson-Lucy deconvolution on a common test image with a spatially varying Gaussian blur. Five strategies are evaluated, with signal-to-noise, peak signal-to-noise, cost, and output images tabulated. The Ram methods outperform both Wiener variants, and a hybrid that seeds Richardson-Lucy from the Ram estimate gives the best result overall, useful evidence for practitioners choosing a deblurring pipeline.

TR5. One-Dimensional Ram Transform: Derivation, Explicit Coefficient Matrices, and Program Implementation (23 pp.)

Domain: Signal processing

A self-contained derivation and complete implementation guide for the one-dimensional Ram Transform, covering the localizing change of variable, truncated Taylor expansions, and division-free back-substitution that recovers the input signal from local derivatives of the blurred output. Explicit closed-form forward and inverse coefficient matrices are given for general symmetric, shift-variant Gaussian, and rectangular kernels, together with a polynomial-exactness correctness check and full working code with symbolic and numerical tests.

TR6. Explicit Coefficient Matrices for the Three-Dimensional Ram Transform: Forward and Inverse Operators for the Gaussian Kernel (12 pp.)

Domain: Image processing

Delivers explicit ten-by-ten forward and inverse coefficient matrices for the three-dimensional Ram Transform at volume order two and point-spread order one, specialized to a shift-variant isotropic Gaussian blur with spatially varying width. Every entry is written in closed form, and the exact scalar restoration formula is expressed in terms of the volume derivatives and the local blur width and its spatial gradients. Includes implementation notes for applying both operators in software, ready for volumetric deblurring work.

TR7. Ram Transforms: Topics for Future Research and Benchmark Studies (32 pp.)

Domain: Ram Transform theory

A comprehensive survey organizing the research and development topics arising from sixteen provisional patent applications on the Ram Transform family and its complex-order, quaternionic, and octonionic extensions. It catalogs each problem, summarizes the current state of the art, and lays out directions for algorithm development, error analysis, convergence proofs, and benchmarking across quantitative finance, physics, fluid dynamics, communications, machine learning, medical imaging, drug discovery, structural mechanics, and pure mathematics. A planning document for prioritizing follow-on work by commercial impact.

TR8. Ram Transform and Spectral Domain Analysis: Fourier and Laplace Representations of the Ram Kernel and Their Consequences (16 pp.)

Domain: Signal processing

Develops the spectral-domain view of the one-dimensional Ram Transform by substituting the Fourier or Laplace representation of the unknown signal into the shift-variant integral equation. This yields a fundamental identity in which the output equals an inverse Fourier transform of the input spectrum times a windowed kernel spectrum, whose Taylor coefficients are exactly the kernel moments, unifying the moment-based and frequency-domain descriptions. For shift-variant kernels it opens new inversion strategies, spectral factorization, and a link to the complex Ram Transform.

TR9. FFT Applicability Analysis for the Shift-Variant Ram Transform: Detailed Analysis and Special Cases (14 pp.)

Domain: Signal processing

A rigorous analysis of when the Fast Fourier Transform can and cannot accelerate the shift-variant Ram Transform. It shows why the standard Cooley-Tukey butterfly breaks down when the kernel spectrum depends jointly on position and frequency, leaving quadratic cost in the fully general case. It then identifies the structural conditions enabling sub-quadratic algorithms and details three practical routes: locally shift-invariant block methods, low-rank separable decomposition, and non-uniform FFT for specific phase structures. Practical guidance for efficient implementations.

TR10. The Ram Complex Spectral Transform (RamCS Transform): Eigenfunctions, Transform Pairs, and Applications to the Ram Transform Integral Equation (21 pp.)

Domain: Ram Transform theory

Introduces the Ram Complex Spectral Transform, a new transform pair built on the exponential eigenfunctions of the Ram Transform, defined by integrating the signal against those eigenfunctions and inverting over a complex frequency domain. It is shown to strictly generalize both the multidimensional Fourier transform and the bilateral Laplace transform, and a generalized convolution theorem is proved that diagonalizes the shift-invariant case exactly as Fourier does. Foundational for spectral analysis of shift-variant operators.

TR11. Historical Precedents and Novelty of the Ram Complex Spectral (RamCS) Transform – A Survey of the Closest Existing Frameworks and an Assessment of What is Genuinely New (17 pp.)

Domain: Ram Transform theory *Source:* Abu-Ghuwaleh, Mathematics 2025

A scholarly novelty assessment situating the Ram Complex Spectral Transform against the closest existing mathematical frameworks. It finds that no prior transform simultaneously has all five defining features (a complex spectral variable on arbitrary contours, an exact exponential eigenfunction kernel, exact analytic local inversion via unit-determinant triangular structure, built-in polynomial preservation, and an explicit shift-variant convolution theorem). It analyzes the nearest antecedents, including pseudodifferential operator symbol calculus and the Segal-Bargmann coherent-state transform, clarifying what is genuinely new.

TR12. The Master Integral Transform (MIT) and the Ram Complex Spectral (RamCS) Transform: Global Integral Form, Relationship, Generalizations, and Mutual Extensions (15 pp.)

Domain: Ram Transform theory *Source:* Abu-Ghuwaleh, Mathematics 2025, 13, 3431

Analyzes the recently published Master Integral Transform in relation to the Ram Complex Spectral and Ram Transforms. It writes the Master Integral Transform in global integral form, proves it is not a special case of the complex spectral transform but rather a weighted superposition of imaginary-axis evaluations on a modified signal, and shows where the Fourier and Laplace cases coincide. It further defines a shift-variant Master Integral Transform that falls squarely within the Ram inversion framework, and identifies concrete ways the two theories reinforce each other.

TR13. Ram Transforms and RamCS Transforms: A Unified Framework for Modelling, Solving, and Inverting Linear and Nonlinear Integral and Partial Differential Equations (29 pp.)

Domain: Applied mathematics

A comprehensive unified treatment of the Ram Transform family and the Ram Complex Spectral Transform as tools for modeling, solving, and inverting linear and nonlinear integral and partial differential equations. It gives the full four-step derivation from a shift-variant integral equation to a local differential operator, the triangular system for the output derivatives, and exact division-free back-substitution inversion. It verifies the framework across polynomials, square-integrable analytic functions like Gaussians, and decaying oscillatory signals, serving as a consolidated reference for the whole approach.

TR14. Application of Ram Transforms to Problems in Orthogonal Time Frequency Space Modulation (28 pp.)

Domain: Wireless / 6G *Source:* Hadani et al., OTFS (2018)

A technical analysis mapping seven core problem domains from a foundational orthogonal time-frequency space (OTFS) modulation paper onto the Ram Transform and Ram Complex Spectral framework. For each domain, from time-varying channel modeling and equalization to nonlinear effects and MIMO extensions, it derives an alternative methodology, its expected results, and a head-to-head comparison of the advantages and disadvantages versus the conventional OTFS approach. Useful for engineers evaluating alternative channel-inversion methods for high-mobility communications.

TR15. Moment Constraints and Operator Duality for the N-Dimensional Complex Ram Transform: Derivation, Bounds, Theoretical Significance, and Applications to Navier–Stokes Equations (22 pp.)

Domain: Ram Transform theory

Derives and analyzes moment-constraint and operator-duality identities for the N-dimensional complex Ram Transform, generalizing earlier one-dimensional scalar results. It establishes a fundamental moment identity linking kernel moments to the spectral kernel, an infinite moment-constraint hierarchy, an operator-duality theorem equating spectral differentiation with spatial moment computation, plus coefficient sum bounds, truncation-error bounds, and spectral-radius bounds, then applies these to Navier-Stokes equations. Valuable for researchers needing rigorous theoretical foundations and error control.

TR16. The Wavelet–Ram Transform: Extension of the N-Dimensional Complex Ram Transform to Incorporate Wavelet Analysis for Multiresolution Modelling and Inversion (25 pp.)

Domain: Ram Transform theory

Introduces the Wavelet-Ram Transform, extending the N-dimensional complex Ram Transform with wavelet analysis for multiresolution modeling and inversion of shift-variant integral equations. It develops continuous and discrete versions using orthogonal wavelet bases, extends the construction to complex domains with analytic wavelets, and establishes a wavelet spectral variant, deriving the connection coefficients that link wavelet-domain coefficients across scales and translations. Useful for practitioners tackling multi-scale inverse problems.

TR17. Fractional Order N-Dimensional Complex Ram Transforms: Theory, Mittag-Leffler Eigenvalues, Operator Duality, and Applications to Anomalous Diffusion, Fractional PDEs, and Multi-Scale Physical Systems (24 pp.)

Domain: Ram Transform theory

Extends the fractional-order Ram Transform from real one-dimensional functions to the full N-dimensional complex setting by replacing integer-order Taylor expansions and derivatives with fractional-order ones across several definitions. Key results include fractional kernel moments, a forward fractional differential operator, a fractional eigenvalue theorem yielding multivariate Mittag-Leffler eigenvalues, and a structured fractional system matrix with determinant one. Aimed at researchers modeling anomalous diffusion, fractional partial differential equations, and multi-scale physical systems.

TR18. Complex-Order N-Dimensional Complex Ram Transforms: Theory of Complex-Order Integral and Differential Operators, Log-Periodic Mittag-Leffler Eigenvalues, the Doubly-Complex Ram Framework, and Applications to Log-Periodic Systems, Viscoelasticity, and Complex Resonances (24 pp.)

Domain: Ram Transform theory

Extends the complex-order Ram Transform to the full N-dimensional complex variable domain, establishing a doubly-complex framework in which both variables and integral or derivative orders are complex. The imaginary parts of the orders introduce log-periodic oscillatory factors into the kernel moments and yield complex-order Mittag-Leffler eigenvalues with a distinctive log-oscillatory envelope. It derives the relevant complex-order derivatives, Taylor series with remainder bounds, and kernel moments, targeting log-periodic systems, viscoelasticity, and complex resonances.

TR19-A. A Grand Unified Framework Based on Ram Transforms and RamCS Transforms – Reformulating, Re-Solving, and Re-Engineering Solutions to Integral and Partial Differential Equations in Science, Engineering, and Applied Mathematics (33 pp.)

Domain: Applied mathematics

Proposes a grand unified framework built on the Ram Transform and its spectral variant for systematically reformulating, re-solving, and re-engineering integral equations, partial differential equations, and systems of them. It rests on a four-step paradigm of equation formulation, kernel localization into a differential operator, forward and inverse solution via the transform series and its convergence-accelerated spectral variant, and practical verification, then extends this from real and complex variables to quaternionic and octonionic algebras and from integer to fractional and complex operator orders.

TR19-B. Updated Grand Unified Framework Based on Ram Transforms, RamCS Transforms, and Ram-Master Transforms – Post-TR19 Extensions, Application Categories, Literature-Search Targets, and Practical Pros/Cons (20 pp.)

Domain: Applied mathematics

An updated synthesis of the grand unified Ram Transform program, incorporating a broad range of newer developments including Ram-Master Transforms, wavelet and cross-Ramlet constructions, neural-network variants, graph-local operators, density-propagation and filtering methods, and many application areas. It organizes everything around a central local derivative-jet relation in which a kernel-moment, Green-kernel, transition-kernel, graph-kernel, or learned operator matrix links the jets of observed and unknown functions. Serves as a roadmap across the expanded corpus and its application categories.

TR20. A Comprehensive Operator-Theoretic Approach to the Riemann Hypothesis via the Extended Ram Transform Framework: Incorporating Fractional-Order, Complex-Order, Doubly-Complex, N-Dimensional, Moment Constraint, and Operator Duality Extensions (26 pp.)

Domain: Pure mathematics

A comprehensive, revised operator-theoretic attempt at the Riemann Hypothesis using the full extended family of Ram Transforms, including fractional-order, complex-order, doubly-complex, and N-dimensional variants plus moment-constraint and operator-duality tools. The strategy maps the zeta function to a self-adjoint operator whose eigenvalues coincide with the imaginary parts of the nontrivial zeros, develops seven principal steps, and re-examines four previously identified gaps in light of the new extensions. Presented as a proof attempt, not a completed proof.

TR21. Application of Ram Transform Variants to Particle Filtering: Theory, Methods, and Comparative Analysis (17 pp.)

Domain: State estimation

A comprehensive analysis of applying the Ram Transform framework and its variants, including generalized, nonlinear-localization, complex-spectral, and Fokker-Planck-Ram operators, to particle filtering for nonlinear state estimation. It connects the localized operator-inversion methodology to the prediction, update, and resampling stages, proposes a deterministic density-propagation alternative to stochastic particle propagation, and introduces a neural-Ram hybrid filter, with a detailed comparison against state-of-the-art particle filters on complexity, accuracy, and parallelizability.

TR22. Application of Ram Transform Variants to Kalman Filtering: Theory, Algorithms, and Comparative Analysis (17 pp.)

Domain: State estimation

A comprehensive analysis of applying the Ram Transform framework and its variants, including a higher-algebraic Ram-Kalman construction, to Kalman filtering and its nonlinear extensions. It develops foundations for replacing the costly global matrix inversions of the Kalman update with localized operators, introduces a filter for high-dimensional partial-differential-equation-governed fields, and proposes preconditioned variants of the extended and ensemble Kalman filters, with a systematic comparison of complexity, accuracy, parallelizability, and applicability.

TR23. Gap Analysis Update for the Riemann Hypothesis Proof Attempt via Ram Transforms – Assessment of How Additional Reference Documents Narrow the Gaps and Weaken the Assumptions in Technical Report TR-2026-04-14-0020 (11 pp.)

Domain: Pure mathematics

A gap-analysis update assessing whether additional documents in the Ram Transform corpus narrow the four identified gaps and weaken the four axioms underpinning the earlier conditional Riemann Hypothesis proof attempt. It identifies seven specific new contributions, including a spectral-spatial duality theorem, a kernel spectrum identity unifying Fourier and moment descriptions, factorial convergence bounds, and windowed error control, then concludes that two gaps are further narrowed, one gains structural support, and the critical remaining gap gains new tools.

TR24. Application of Ram Transform Variants to Hamilton–Jacobi–Bellman Equations for Portfolio Optimization and Stochastic Control – Theory, Algorithms, and Comparative Analysis (21 pp.)

Domain: Quantitative finance

A comprehensive treatment of the Hamilton-Jacobi-Bellman equation for portfolio optimization and stochastic control using the Ram Transform framework and its extensions. It formulates the Merton portfolio problem and general HJB equation, maps them to localized form, derives closed-form local inverse operators, and presents a policy-iteration algorithm with preconditioning, residual refinement, and multiresolution continuation, backed by convergence and stability results and compared against finite differences, deep-learning solvers, and Gaussian process regression.

TR25. Application of Ram Transform Variants to Mean-Field Games for Systemic Risk and Stochastic Control – Theory, Algorithms, and Comparative Analysis for Coupled HJB–Fokker–Planck Systems (23 pp.)

Domain: Quantitative finance

A comprehensive treatment of mean-field game systems, the coupled forward-backward Hamilton-Jacobi-Bellman and Fokker-Planck partial-differential-equation systems describing Nash equilibria of large interacting populations, using the Ram Transform framework and its extensions. It maps both equations to localized form via semigroup reformulations, derives block-structured local inverse operators, develops a fixed-point algorithm with preconditioning, analyzes convergence under a monotonicity condition, and compares against classical and deep-learning solvers. Applications include systemic risk in interbank networks and crowd dynamics.

TR26-A. Ram Transform Family Methods for Stochastic Volatility and Jump-Diffusion Models – Local Ram-PDOs, Analytic Micro-Inverses, Jump-Kernel Moment Splitting, Rough-Volatility Extensions, and Fast Calibration (31 pp.)

Domain: Quantitative finance

A Ram Transform framework for pricing and calibrating stochastic volatility and jump-diffusion models, covering Heston, Bates, Merton, Kou, SABR, local-stochastic, and rough-volatility cases. It writes the short-time risk-neutral transition kernel as a shift-variant integral operator, localizes it via log-price and Lamperti coordinate maps, and inverts it with small unit-upper-triangular block matrices at each grid point, reproducing the drift, variance, and jump terms of the generator for fast calibration.

TR26-B. Addendum to TR26: Ram Transform Filtering, Monte Carlo, and Particle Methods for Stochastic Volatility and Jump-Diffusion Models (16 pp.)

Domain: Quantitative finance

An addendum extending the stochastic-volatility and jump-diffusion framework of TR26 to the filtering, Monte Carlo, and particle-method side of pricing and calibration. It shows that Ram Transform particle-filtering, Kalman-filtering, and spectral reports apply to latent volatility and jump-state estimation, parameter tracking, variance reduction, and proposal design, exploiting the shared kernel structure between the Chapman-Kolmogorov prediction step and the Feynman-Kac pricing semigroup for deterministic density propagation.

TR27. Ram Transform Family Methods for Nonlinear Black–Scholes Equations (28 pp.)

Domain: Quantitative finance

A Ram Transform framework for nonlinear Black-Scholes equations where the diffusion coefficient depends on the option price, Delta, Gamma, or local curvature, covering transaction-cost, market-impact, illiquidity, uncertain-volatility, and risk-adjusted hedging models. It rewrites the nonlinear equation as a short-time shift-variant integral equation and converts the kernel integral into a small nonlinear algebraic system in local Ram moments and the option-value jet, handling Leland, Barles-Soner, Black-Scholes-Barenblatt, and RAPM type models.

TR28. Ram Transform Family Based Deterministic Alternatives and Enhancements for Monte Carlo Simulation Problems – Real-Time Derivative Pricing, Path-Dependent Options, Greeks, Particle Filtering, Sequential Bayesian Estimation, and General Simulation (31 pp.)

Domain: Quantitative finance

A framework for replacing or accelerating Monte Carlo simulation in real-time derivative pricing with deterministic Ram Transform methods, spanning path-dependent options, Greeks, particle filtering, and sequential Bayesian estimation. It recasts sampling tasks as evaluations of shift-variant integral operators and posterior-density propagators, substituting random sampling with local moment extraction, jet expansions, and small-block inversion that can serve as standalone solvers or as variance-reduction and proposal modules inside existing Monte Carlo pipelines.

TR29. Ram Transform Family Based Methods for Drug Discovery, Molecular Dynamics, Docking, Binding Free Energy, and Molecular Design – A Comprehensive Technical Report Addressing Section 8.1 of the Ram Transform Future-Research Report (22 pp.)

Domain: Drug discovery

A technical program applying the Ram Transform family to structure-based drug discovery, treating molecular dynamics, docking, and binding free-energy estimation as inversions of high-dimensional integral operators, nonlinear PDEs, and Boltzmann-weighted partition functions. It formulates Ram-based solutions for molecular force micro-solvers, generalized Born implicit solvent, protein-ligand pose kernels, alchemical free-energy acceleration, and rare-event sampling, offering a unified local-moment framework to accelerate, precondition, and interpret these computations.

TR30. Ram Transform Family Based Methods for Drug Release, Pharmacokinetics, Pharmacodynamics, Physiologically Based Modeling, and Personalized Dosing – A Comprehensive Technical Report Addressing Section 8.2 of the Ram Transform Future-Research Report (25 pp.)

Domain: Pharmacokinetics

A Ram Transform program for drug release, pharmacokinetics, pharmacodynamics, and physiologically based modeling, addressing controlled release governed by nonlinear diffusion, dissolution, swelling, erosion, and moving-boundary mechanisms coupled to compartmental, PBPK, PBBM, and PD systems. It casts these as integral, integro-differential, PDE, stochastic, and inverse problems solved with local kernel moments and jets, positioning Ram methods as deterministic operator modules that accelerate mechanistic release simulation, biorelevant dissolution, and personalized dosing.

TR31. Ram Transform Family Based Methods for Enzyme Kinetics, Biochemical Reaction Networks, Systems Biology, and Multi-Scale Cellular Modeling – A Comprehensive Technical Report Addressing Section 8.3 of the Ram Transform Future-Research Report (24 pp.)

Domain: Systems biology

A Ram Transform solution program for enzyme kinetics, biochemical reaction networks, systems biology, and multi-scale cellular modeling, covering Michaelis-Menten and Hill kinetics, gene regulatory networks, and cell signaling governed by stiff nonlinear ODEs, reaction-diffusion PDEs, and chemical master equations. It focuses on three targets: localization transforms for stiff signaling cascades, parameter identification from fluorescence time-series, and multi-scale links from single-cell stochastic reactions to tissue-level PDEs, with comparisons to current methods.

TR32. Graph-Local Ram Operators, Brain Graphs, and Graph Neural Network Learning (23 pp.)

Domain: Graph neural networks

An investigation of whether the Graph-Local Ram Operator and its inverse yield new theory for graph neural networks, biological brain-graph models, and local credit assignment. Starting from a local graph kernel integral, it derives forward and inverse graph-local Ram operators and relates them to message-passing GNNs, spectral filters, and graph signal processing, proposing a moment/inverse framework for graph derivative jets and an inverse-transpose adjoint for local credit assignment while carefully separating known results from new contributions.

TR33. Ram-Master and Wavelet-Ram Improvements for OTFS Delay-Doppler Communications – Verification of TR14 and Extension of Ram Transform Methods for the Problems in the Goldsmith–Hadani–Molisch–Calderbank OTFS Paper (27 pp.)

Domain: Wireless / 6G *Source:* Goldsmith-Hadani-Molisch-Calderbank OTFS paper

A verification and extension of TR14's mapping of OTFS delay-Doppler communications to the Ram Transform framework. It confirms the linear time-varying channel maps correctly to a shift-variant Ram integral operator but corrects several inverse-operator claims, showing the derivative-stacked operator is not automatically unit upper triangular under Doppler and replacing overreaching invertibility claims with a corrected jet-matrix inverse, frozen upper-triangular preconditioner, or regularized inverse, then adds Wavelet-Ram multiresolution channel methods.

TR34-A. Ram-Master Transform Extensions for LEO Satellite and High-Mobility Doubly Dispersive Communications (8 pp.)

Domain: Wireless / 6G

A revised and broadened method for LEO satellite and high-mobility doubly dispersive communications, updating an earlier provisional patent formulation with the later Ram Transform toolkit. It incorporates the corrected time-varying local jet matrix of TR33 and extends the solution using Ram-Master, Wavelet-Ram, RamCS, moment-constraint, fractional-order, complex-order, and Cross-Ramlet techniques, yielding a more precise and technically defensible approach with an explicit non-overlap analysis against prior work.

TR34-B. Ram-Master and Wavelet-Ram Methods for Automotive Radar, LiDAR, and Sonar Multipath Dereverberation (8 pp.)

Domain: Radar / sensing

A revised Ram Transform method for automotive radar, LiDAR, and sonar multipath dereverberation, updating an earlier provisional patent idea with the mature Ram Transform toolkit. It incorporates the corrected time-varying local jet matrix of TR33 and extends the approach with Ram-Master, Wavelet-Ram, RamCS, moment-constraint, fractional-order, complex-order, and Cross-Ramlet techniques, delivering a more precise and broader dereverberation method plus an overlap-screening analysis against related work.

TR34-C. Ram-Master Transform Extensions for Electronic Dispersion and Nonlinear Compensation in Fiber Optic Receivers (8 pp.)

Domain: Fiber optics

A revised Ram Transform method for electronic dispersion and nonlinear compensation in ultrafast fiber-optic receivers, upgrading an earlier provisional patent formulation with the later Ram Transform extensions. It applies the corrected time-varying local jet matrix of TR33 and Ram-Master, Wavelet-Ram, RamCS, moment-constraint, fractional-order, complex-order, and Cross-Ramlet techniques to produce a more precise and defensible dispersion-compensation method, along with a non-overlap analysis relative to prior disclosures.

TR34-D. Wavelet-Ram and Ram-Master Methods for Medical Ultrasound Phase-Aberration Correction and Adaptive Beamforming (8 pp.)

Domain: Medical imaging

A revised Wavelet-Ram and Ram-Master method for medical ultrasound phase-aberration correction and adaptive beamforming, updating an earlier provisional patent idea with the full Ram Transform toolkit. It incorporates the corrected time-varying local jet matrix of TR33 and extends the solution with Ram-Master, Wavelet-Ram, RamCS, moment-constraint, fractional-order, complex-order, and Cross-Ramlet techniques, giving a more precise and broader aberration-correction method with an explicit overlap analysis against prior work.

TR34-E. Ram-Master Transform Methods for Underwater Acoustic Communication, Sonar Dereverberation, and Time-Varying Ocean Channels (8 pp.)

Domain: Underwater acoustics

This report modernizes an earlier localized differential-operator idea into a broader, more precise method for underwater acoustic telemetry and sonar dereverberation over time-varying ocean channels. It incorporates a corrected time-varying local jet matrix and extends the approach with Ram-Master, Wavelet-Ram, compressed-sensing, moment-constraint, fractional-order, complex-order, and Cross-Ramlet techniques. Buyers get an updated, technically defensible formulation with overlap analysis against prior communications work.

TR34-F. Wavelet-Ram and Ram-Master Methods for Seismic Inversion, Full-Waveform Preconditioning, and Earth Profiling (8 pp.)

Domain: Seismic inversion

This report revises an earlier localized differential-operator concept into a more precise, broader method for seismic inversion, full-waveform preconditioning, and earth profiling. It folds in a corrected time-varying local jet matrix and extends the solution with Ram-Master, Wavelet-Ram, compressed-sensing, moment-constraint, and fractional/complex-order techniques. It includes non-overlap screening against related communications work, giving purchasers a defensible, upgraded formulation for subsurface imaging.

TR35. Ram-Master Transform Methods for Deterministic Nonlinear State Estimation – Addressing Section 4.4 of TR7: Replacing and Enhancing Particle Filters and Monte Carlo Filtering (23 pp.)

Domain: State estimation

This report develops deterministic transform methods to replace or accelerate stochastic particle filters and Monte Carlo propagation in nonlinear state estimation. It reframes Bayesian filtering as an operator recursion, representing the filtering density or its score by local derivative jets, wavelet coefficients, and spectral bases, and converting transition and likelihood kernels into local moments. It draws on a Fokker-Planck density propagator, multiresolution analysis, and neural residual correction, offering an alternative to sampling-based filtering.

TR36. Ram-Master Transform Methods for Reaction–Diffusion Systems in Biology and Chemistry – Addressing Section 4.4 of TR7: GRLT Micro-Solvers, Nonlocal Terms, Gene Regulatory Networks, and Stiff Pattern-Forming Systems (21 pp.)

Domain: Computational biology

This report presents a comprehensive transform-family solver for stiff nonlinear reaction-diffusion systems arising in biology and chemistry, including Turing patterns, morphogenesis, excitable media, combustion, catalysis, and gene-regulatory networks. It treats each time step as a composition of a diffusion semigroup, a local nonlinear reaction flow, and optional nonlocal or memory operators, providing local micro-solvers for stiff reactions, spectral propagation for diffusion, and multiresolution handling of fronts, spots, and spirals. It also addresses inverse estimation of reaction and diffusion parameters from incomplete data.

TR37-A. Ram Transform and Analytic Mollifier-Ram Methods for High-Order Derivative Inverse PDE Learning – Applying the Ram Transform Family, Ram-Master Transform, Moment Constraints, and Local Inverse Operators to PDE Inverse Learning (19 pp.)

Domain: Physics-informed ML *Source:* Mollifier Layers paper

This report shows that the Mollifier Layer used in physics-informed machine learning for inverse PDE problems is a particular forward local operator with a smooth compactly supported kernel, and its derivative kernels belong to the same operator family. Making this identification unlocks a full toolkit of local partial differential operators, analytic local inverse operators, moment constraints, spectral symbols, and multiresolution variants. It covers first-order Langevin, second-order heat, and fourth-order reaction-diffusion inverse problems.

TR37-B. Composite Ram-Mollifier Operators for Multi-Term Continuous PDE Layers – Extending the P15 Discrete Composite Operator Q and Complex Q(z,zeta) to Continuous Mollifier Layers, Inverse PDE Learning, Neural Operators, and Complex-Valued Fields (28 pp.)

Domain: Physics-informed ML *Source:* Provisional patent P15

This report extends a patent's discrete real and complex composite operators to a continuous composite operator for multi-term PDE layers in inverse learning and neural operators. It shows how a general continuous linear operator with several variable-coefficient terms acting on a mollified field can be fused into a single composite kernel, giving the continuous analog of the discrete construction. The result serves as a practical fused physics output layer, applicable to complex-valued and holomorphic fields.

TR37-C. Appendix to TR38: Novelty and Prior-Art Assessment – Composite Ram-Mollifier Operators and Explicit Computational Speedup from Fused Operators (16 pp.)

Domain: Physics-informed ML *Source:* Provisional patent P15

This appendix provides a careful novelty and prior-art assessment separating what is genuinely new from known background in the composite operator work. It acknowledges that fusing several linear operators into one is already recognized in neural-network reparameterization, compiler operator fusion, and stencil-composition methods, so generic speedup should not be claimed as new. It identifies the distinctive contributions as the specific continuous fused-kernel formulation, its synthesis with mollifier derivative kernels, variable-coefficient inverse PDE residuals, and local identifiability analysis.

TR38-A. Potential Roles of Ram Transform Variants in LeWorldModel – Structured Encoders, Latent Dynamics, Inverse Diagnostics, Residual Surprise, and Self-Improving World Models (33 pp.)

Domain: World models *Source:* LeWorldModel (Maes, Le Lidec, Scieur, LeCun, Balestriero)

This report analyzes where transform-family variants can strengthen a joint-embedding predictive world-model pipeline that learns a compact latent model from raw pixels. It concludes the methods work best as structured components rather than a wholesale replacement, with the strongest roles in the encoder and tokenizer and in diagnostic or corrective modules. It describes operator-aware token front ends, coordinate-mapped kernels encoding motion, pose, and deformation, and multiscale states for longer-horizon planning.

TR38-B. Graph Local Ram Transforms for LeWorldModel – Relevance of Graph-Local Forward, Inverse, and Adjoint Ram Operators to Joint-Embedding Pixel World Models (28 pp.)

Domain: World models *Source:* LeWorldModel (Maes, Le Lidec, Scieur, LeCun, Balestriero)

This report evaluates whether graph-local forward, inverse, and adjoint operators are relevant to a pixel-based joint-embedding predictive world model. Its cautiously positive conclusion is that, while not a direct replacement for the encoder, predictor, regularizer, or planner, graph-local transforms serve as a graph-structured augmentation and diagnostic framework. Applications include patch-token graph encoding, object and scene-graph latent states, action-conditioned interaction dynamics, physical probing, surprise localization, and local inverse-dynamics diagnostics.

TR39. Cylindrical-PSF Rao/Ram Transform Shape Recovery – Explicit K'_1, K'_2 Matrices, Corrected PSF Moment, and Algebraic Solvability of Z_0, Z_X, Z_Y (9 pp.)

Domain: Computer vision *Source:* Kang, Chapter 3 (shape from defocus)

This report derives the low-order inverse transform matrices for the cylindrical defocus point-spread function used in shape-from-blur camera work, using an affine blur-radius model and exact uniform-disk moments. It corrects an error in the source chapter's second radial moment and yields a unit upper-triangular matrix with closed-form entries. It shows how the resulting equations reduce to a solvable system for local surface shape parameters, with extra images from a second projected pattern resolving ill-conditioned cases.

TR40. Ram Transform Family for Forward Modeling, Inverse Recovery, Kernel Calibration, and Coupled Systems – Theory, Camera Shape Measurement, Industrial Calibration, and Ram-Master Extensions (19 pp.)

Domain: Computer vision

This report gives a unified treatment of the local matrix relation linking a scene's derivative jet to observed image jets through a kernel-moment matrix, supporting four tasks: forward computation, inverse recovery, supervised estimation of a fixed kernel, and semi-blind kernel-parameter estimation. It shows how two differently blurred images recover shape-dependent point-spread parameters, and how a beam-splitter camera can capture both simultaneously. It covers manufacturer calibration with known shapes and patterns plus end-user local shape estimation, with Ram-Master extensions.

TR41. Octonionic Ram Transforms for Post-Quantum Cryptography – Verification of the P12 ORT-PQC Claim, Strengthened Constructions, Algorithms, and Research Roadmap (25 pp.)

Domain: Post-quantum cryptography *Source:* Provisional patent P12

This report critically verifies a provisional patent's claim that combining p-adic and octonionic transforms can support post-quantum cryptography, where non-associative twisting frustrates global quantum inversion while an authorized party decrypts locally. It concludes the machinery is a useful local, bracket-controlled, high-dimensional transform calculus but that non-associativity alone is not a security proof, since the original argument lacks a hard problem, reduction, parameters, key generation, and chosen-ciphertext protection. It offers strengthened constructions and a research roadmap.

TR42. New Ram Transform Improvements for Wireless Communications – Beyond Cohere US 11,470,485 B2 and the P33 Ram-Master OTFS Provisional (31 pp.)

Domain: Wireless / 6G *Source:* US Patent 11,470,485 B2 (Cohere)

This report analyzes a Cohere delay-Doppler patent alongside prior internal reports to identify wireless communication improvements not yet exploited in the literature or fully claimed in an existing OTFS provisional. It concludes that broad claims of applying the transform to OTFS are no longer the strongest path, since the core delay-Doppler mapping is already disclosed. It targets new value in specific post-provisional engineering that combines local moments with standards-compatible reference signals, graph-local network structure, and 6G air-interface techniques.

TR43. Deriving Forward and Inverse Ram Operators Directly from PDEs – Local Coordinate, Taylor-Jet, Boundary-Aware, and Nonlinear Algorithms (23 pp.)

Domain: Ram Transform theory

Delivers a direct, transparent route from a partial differential equation to local forward and inverse Ram operators, without first converting the PDE into a Green-kernel or Volterra/Fredholm integral equation. It presents local-coordinate, Taylor-jet, boundary-aware, and nonlinear algorithms, exploiting that a PDE is already a local operator on a finite jet, and preserves the polynomial-exactness property so truncated jets are exact for polynomial sources within stated assumptions.

TR44. A Local Calculus for Global Science: Why Ram Transforms Deserve a Broad Reconsideration (11 pp.)

Domain: Ram Transform theory

A perspective arguing that the Ram Transform family deserves broad reconsideration as a local calculus for global problems in imaging, wave propagation, fluids, quantum theory, communications, and finance. It explains the core method of exact coordinate localization, Taylor conversion to a local differential operator, and pointwise inverse recovery, and lays out a research program spanning kernel calibration, windowed and spectral transforms, graph-local operators, and neural networks.

TR45. Composite Forward and Inverse Ram Operators Q and Q': Discrete Filters, Continuous Mollifier Kernels, Ram-Master Transform Analysis, and Green-Kernel Connections (25 pp.)

Domain: Ram Transform theory *Source:* Localized and Computationally Efficient Approach to Shift-Variant Image Deblurring

Derives composite forward and inverse Ram operators Q and Q-prime for shift-variant source-kernel-response problems, from discrete filters through continuous mollifier kernels. It extends the discrete composite filter to multi-index d-dimensional and complex N-dimensional notation, then builds regularized continuous operators that replace derivative-of-Dirac kernels with differentiated smoothing kernels, fusing coefficient fields, derivative operators, and smoothing into compact kernels, and connects the construction to Green-kernel and Ram-Master transform analysis.

TR46. Bayesian Networks and Graph Local Ram Transforms – Representation, Inference, Inverse Diagnostics, and Opportunities Beyond Current BN Methods (35 pp.)

Domain: Probabilistic graphical models

Investigates how Bayesian Networks can be represented inside a Graph-Local Ram Transform framework by rewriting them as factor graphs, junction trees, or Markov blanket graphs, showing that standard inference operations such as variable elimination and belief propagation are graph-local operators. It is candid that this does not remove the known treewidth-governed hardness of exact inference; the value is an operator calculus offering local coordinates, moment expansions, difference jets, and regularized inverse diagnostics around BN operations.

TR47. Bayesian Neural Networks and Ram-Master Neural Networks – Bayesian RMNNs, Ram Moment Posterior Propagation, Inverse Uncertainty, and Practical Advantages (32 pp.)

Domain: Bayesian deep learning

Shows that Bayesian Neural Networks and Ram-Master Neural Networks can be unified by treating kernel parameters, local moment coefficients, coordinate maps, derivative-jet matrices, and regularization terms as random variables with priors and posteriors, and that Bayesian convolutional and operator-learning networks appear as special cases. It is honest that full posterior inference in high-dimensional networks remains hard, positioning the contribution as a structured operator calculus for uncertainty rather than a shortcut around Bayesian inference.

TR48. Ram Transforms: Complete Localization, Local Analytic Inversion, and a Transform Family for Kernel-Governed Problems Across Science, Engineering, Economics, and Finance (22 pp.)

Domain: Ram Transform theory

A comprehensive account of the Ram Transform family as a local analytic calculus for source-kernel-response problems across imaging, wave propagation, fluids, quantum theory, filtering, communications, economics, and finance. It emphasizes exact coordinate localization followed by Taylor, fractional-order, complex-order, exponential-polynomial, or wavelet expansions, yielding forward and inverse local series whose coefficients are kernel moments; in common truncations the local matrix is unit upper triangular, so inversion needs only a finite nilpotent recursion.

TR49. Reassessing the Relationship Between the Information Lattice Transform, Graph Local Ram Transforms, and Ram-Master Neural Networks – A Technical Report on Concept Lattices, Graph-Local Operator Calculus, LeWorld Models, and Hybrid ILT–GLRT Architectures (28 pp.)

Domain: Ram Transform theory

Reassesses the relationship between the Information Lattice Transform and the Ram Transform family in light of graph-local operators and Ram-Master Neural Networks. Its central finding is that ILT and Graph-Local Ram Transforms are closely related at the operator level: ILT projection onto a partition is exactly a zeroth-order graph-local Ram operator on a bipartite graph, and uniform lifting is its natural adjoint, motivating hybrid ILT-GLRT architectures and world-model connections.

TR50. Mathematical Principles of the Forward and Inverse Graph-Local Ram Operator – Explanation of Equation (26) in Section 7.9 of the Ram Transform Enzyme Kinetics and Systems Biology Provisional Patent (18 pp.)

Domain: Systems biology *Source:* Ram Transform Enzyme Kinetics and Systems Biology Provisional Patent, Section 7.9 Eq. (26)

An addendum that explains the mathematics behind a specific graph-integral equation in an enzyme-kinetics and systems-biology provisional patent, interpreting the graph integral as a sum, integral, or hybrid measure over graph states. It derives forward graph-local Ram moment expansions and inverse operators for estimating graph derivative jets, covering finite differences, graph-edit and subgraph derivatives, and rule-local derivatives, with a concrete Boolean binding and phosphorylation example across reaction, regulatory, and pathway graphs.

TR51. Ram-Real and Ram-Sine Transforms Derived from Complex Ram Exponential Kernels – Eigenfunction Structure, Real/Imaginary Decomposition, Energy Compaction, and Applications in Signal/Image Processing, CFD, and Quantitative Finance (18 pp.)

Domain: Signal processing

Derives real-component transforms from the complex Ram exponential kernel: the Ram-Real Transform and the Ram-Sine (imaginary) Transform, and analyzes their eigenfunction structure, real/imaginary decomposition, and energy compaction for signal and image processing, computational fluid dynamics, and quantitative finance. It corrects a subtle motivating error, showing that for real kernels the real and imaginary parts span an invariant two-dimensional subspace acted on by a real 2x2 block, becoming separate scalar eigenfunctions only under specific eigenvalue conditions.

TR52. Magnetic Density Imaging and Field Image Tomography: A Critical Verification, Analysis, and Synthesis of US Patent 8,456,164 B2 and US Patent Application 2019/0041481 A1 (14 pp.)

Domain: Medical imaging *Source:* US Patent 8,456,164 B2 and US Patent Application 2019/0041481 A1

Critically verifies, step by step, the central derivations of a 3D magnetic density imaging patent and a related massively parallel MRI patent application, including the Fourier-domain closed-form solution, identifying where it is correct and where it needs qualification. It argues that when the source is parameterized by finitely many smooth bounded parameters and measured throughout a 3D volume, the inverse problem becomes well-posed in engineering terms despite classical multipole non-uniqueness, and notes the off-surface-coil experimental demonstration.

TR53. A Handheld Field-Image-Tomography Instrument for 3D Imaging of Human Tissue: Theory, Algorithms, Scan-Time Analysis, and a Practical Engineering Design at the $2 M Cost Target – Combined MDI / Prepolarized-MRI Imaging Based on the Field Image Principle (18 pp.)

Domain: Medical imaging

Analyzes the design of a handheld field-image-tomography instrument for 3D imaging of human tissue, with an integrated U-shaped polarizing electromagnet and hundreds of 3D-distributed magnetic and RF sensors, treating scan time at fixed voxel SNR as the primary metric under a two-million-dollar cost target. It finds pure magnetic density imaging infeasible for soft tissue due to nanosecond susceptibility decay, and identifies prepolarized low-field MRI with volumetric sensing as the feasible imaging mode.

TR54. A Cost-Optimised Handheld FIT-MDI-MRI Instrument with Stereo-Vision Registration, Polarization-Gradient Encoding, and Chunked Multi-Session Acquisition – Technical Report on the Calibration-Heavy / Hardware-Light Design Philosophy and the Comparison Against Augmentation of Existing Low-Field MRI Products (19 pp.)

Domain: Medical imaging

Develops a cost-optimized handheld field-image-tomography instrument built on a calibration-heavy, hardware-light philosophy targeting 2 mm isotropic resolution over a large tissue volume. Four choices drive down cost: stereo-vision fiducial tracking replacing an expensive robotic arm, using the electromagnet's spatially varying polarization field as the readout gradient, packing hundreds of RF coils, optically pumped magnetometers, and TMR sensors, and chunked multi-session acquisition. It also compares this approach against augmenting existing low-field MRI products.

TR55. Ram Transform Variants for Uncertainty Quantification in Artificial Intelligence – Verification of a Prior Rao/Ram-UQ Report and a Technical Synthesis for GPLVMs, Bayesian AI, Neural Operators, and Safety-Critical Systems (21 pp.)

Domain: Uncertainty quantification *Source:* Analytic Uncertainty Quantification in Probabilistic Machine Learning Using Generalized Rao Transforms

Verifies and then strengthens a prior report on using Ram/Rao Transforms for uncertainty quantification in AI, finding the earlier work promising but overstated in its claims of exactness, complexity, and generality for Gaussian Process Latent Variable Models. It presents a corrected, stronger Ram-UQ framework combining localization transforms, complex-spectral and fractional/complex-order variants, shifted-center methods, residual-adaptive refinement, and Ram-local neural operators for Bayesian AI, neural operators, and safety-critical systems.

TR56. Ram Transform Methods for Uncertainty Quantification in Artificial Intelligence Systems – A Comprehensive Theory, Algorithmic Framework, and Comparison with State-of-the-Art Uncertainty Quantification Methods (28 pp.)

Domain: Uncertainty quantification

Develops a deterministic Ram Transform framework for uncertainty quantification in AI systems, propagating uncertainty through local moment expansions, operator localization, residual correction, and physics- or data-informed constraints. It draws on a family of Ram operators including localization, complex-spectral, fractional-order, complex-order, and neural integral-equation layers to address the cost, sampling error, and interpretability limits of Bayesian, Monte Carlo, ensemble, and conformal methods. Useful for teams building trustworthy AI in medicine, autonomous systems, finance, and scientific machine learning.

TR57. The Complex-Order Ram-Master Transform (C-RMT): A Six-Parameter Framework with Log-Periodic Atoms, Discrete Scale Invariance, and Applications to Critical Phenomena (26 pp.)

Domain: Ram Transform theory

Extends the Ram-Master Transform to complex derivative order, producing a six-parameter family whose new imaginary-order parameter introduces log-periodic oscillations and discrete scale invariance orthogonal to the memory effects of fractional order. This log-periodic structure matches the signatures of stock-market crash precursors, earthquake fracture precursors, renormalization-group fixed points, and quantum phase transitions. Valuable to researchers modeling critical phenomena and discrete-scale-invariant systems.

TR58. The Fractional-Order Ram-Master Transform (F-RMT): A Five-Parameter Unified Framework with Mittag-Leffler Atoms, Fractional Ram–PDO Diagonalization, and Long-Memory Applications (25 pp.)

Domain: Ram Transform theory

Extends the Ram-Master Transform to fractional-order derivatives, giving a five-parameter framework built on Mittag-Leffler atoms and a fractional Taylor series that diagonalizes the fractional Ram operator into an algebraic multiplier relation. It supports Riemann-Liouville, Caputo, Grunwald-Letnikov, and Riesz-Feller derivatives and proves a universal reduction theorem identifying two dozen classical and modern transforms as special cases. Aimed at modeling long-memory and anomalous-diffusion phenomena.

TR59. The Octonionic-Order Ram-Master Transform (O-RMT): A Non-Associative Twelve-Parameter Framework, Cascade Cayley–Dickson Reduction, Seven-Dimensional Axis Selectivity, and the Hurwitz Terminal of the RMT Tower (22 pp.)

Domain: Ram Transform theory

Extends the Ram-Master Transform to octonionic derivative order, the terminal level of the Hurwitz tower of normed division algebras, yielding a non-associative twelve-parameter family with seven-dimensional directional selectivity. It addresses the non-associativity challenge through explicit bracketing and a cascade Cayley-Dickson reduction back to lower algebras. Intended for advanced signal-analysis researchers needing selectable oscillation directions in seven-dimensional imaginary space.

TR60. The Quaternionic-Order Ram-Master Transform (Q-RMT): A Non-Commutative Eight-Parameter Framework with Axis-Selective Log-Periodic Atoms, Cayley–Dickson Reduction, and Applications to 3D Rotational, Polarimetric, and Spinor Signal Analysis (21 pp.)

Domain: Ram Transform theory

Extends the Ram-Master Transform to quaternionic derivative order, producing a non-commutative eight-parameter family whose atoms exhibit axis-selective log-periodic oscillation along a choosable direction on the unit two-sphere. This adds a genuinely new degree of freedom inaccessible to real-fractional or complex-order calculus, together with Cayley-Dickson reduction results. Directed at 3D rotational, polarimetric, and spinor signal analysis.

TR61. The Wavelet-Weighted Ramlet (Cross-Ramlet / "Double-Wavelet") Transform: Products of Classical Wavelet Kernels with the Ramlet Exponential Atom (20 pp.)

Domain: Ram Transform theory

Generalizes the Ramlet Transform by multiplying the Ramlet exponential atom with arbitrary classical mother wavelets such as Morlet, Mexican hat, Meyer, Daubechies, Haar, Gabor, and many others, forming a double-wavelet or Cross-Ramlet kernel. It proves that on the imaginary fibre and with suitable wavelet choice the product kernel is a bona fide admissible wavelet encoding frequency, scale, and position. Useful for designing wavelet transforms that inherit both localization and exponential frequency selectivity.

TR62. The Ram-Master Transform (RMT): A Unified Framework Combining the Localized Ram Partial Differential Operator, Variable-Width Windowing, and the Cross-Ramlet Transform (25 pp.)

Domain: Ram Transform theory

Introduces the Ram-Master Transform, a unified framework combining the fully localized Ram partial differential operator and its analytic inverse, variable-width windowing that interpolates between pointwise and global limits, and the Cross-Ramlet product kernel. It proves a universal reduction theorem showing the framework subsumes the plain Ramlet, continuous wavelet, short-time Fourier, Morlet, Gabor, and chirplet transforms as parameter restrictions. A foundational reference for the four-parameter Ram-Master spectrum.

TR63. The Ramlet Transform: A Structural Identification of the RamCS Transform as a Generalized Wavelet Transform with Exponential Mother Function (21 pp.)

Domain: Ram Transform theory

Identifies the Ram Complex Spectral Transform as a generalized wavelet transform with a prescribed exponential mother function, bridging the Ramlet and continuous wavelet frameworks through their shared kernel structure. Treating the centering vector as a translation parameter yields a well-posed two-parameter transform pair with an explicit inverse. Clarifies the structural relationship between Ram spectral transforms and classical wavelet analysis.

TR64. The Polynomial-Weighted (Hermite) Ramlet Transform: Theoretical and Practical Implications of Differentiating the Ramlet Kernel with Respect to the Spectral Variable (18 pp.)

Domain: Ram Transform theory

Analyzes the transform family obtained by differentiating the Ramlet kernel with respect to the spectral variable, which introduces polynomial factors and, for Hermite-type polynomials, yields a Hermite Ramlet Transform. The central derivative-multiplier duality shows that weighting by a polynomial equals applying that polynomial in the differentiation operator to the plain Ramlet spectrum. Provides inversion formulas for every polynomial-weighted member of the hierarchy.

TR65. Proof of the Riemann Hypothesis via the Ram Transform Framework: A Comprehensive Synthesized Technical Report with Statement, Proof Steps, Verification, Explicit Assumptions, and Remaining Gaps (21 pp.)

Domain: Pure mathematics

Presents a self-contained synthesis of a Ram Transform-based approach to the Riemann Hypothesis, developing a Mellin-theta kernel, a Hilbert-Polya self-adjoint operator built from the Fractional-Order Integral Ram Transform, verification of the Carleman moment condition, and a spectral-theorem conclusion. It states all eight assumptions explicitly and honestly enumerates the four remaining technical gaps, incorporating recent numerical zero computations. Intended for external expert review rather than as a finished proof.

TR66. The Cross Wavelet–Ram-Real Transform and Cross Wavelet–Ram-Sine Transform – Global Real/Sine Multiresolution Extensions of WRRT and WRST for Transforming Functions, Operators, and Data (18 pp.)

Domain: Ram Transform theory

Extends the Wavelet-Ram-Real and Wavelet-Ram-Sine transforms to global product-kernel forms that multiply a classical inner wavelet by a damped-cosine or damped-sine Ram spectral atom. The resulting real-valued and sine-valued decompositions retain wavelet localization, scale, and vanishing-moment properties while inheriting the exponential decay and frequency selectivity of the Ramlet family, with a key result showing the two branches form a coupled pair. Includes inverse formulas, admissibility conditions, energy identities, and discrete algorithms.

TR67. The Ram-Master Transform (RMT): A Unified Framework Combining the Localized Ram Partial Differential Operator, Variable-Width Windowing, Cross-Ramlet, Cross-WRRT, and Cross-WRST Transforms (16 pp.)

Domain: Ram Transform theory

Integrates the Cross Wavelet-Ram-Real and Cross Wavelet-Ram-Sine transforms into the Ram-Master Transform by replacing the exponential atom with a conjugate-symmetric atom whose parameter choices recover the complex, real, and sine branches. This branch-generalized framework unifies numerous Ram real and sine transforms alongside the earlier complex version, with the real and sine branches shown to form a coupled two-by-two system. A consolidating reference for the full Ram-Master transform family.

TR68. Ram-Real Spectral and Ram-Sine Spectral Transforms: Real and Sine Global Spectral Extensions of the Ram Transform Framework (19 pp.)

Domain: Ram Transform theory

Develops the Ram-Real Spectral and Ram-Sine Spectral transforms as damped-cosine and damped-sine global extensions of the Ram framework, obtained from the real and imaginary quadrature components of the complex spectral kernel. These generalize the discrete cosine and sine transforms while retaining localization, moment-generating interpretation, shift-variant kernel handling, and analytic forward-inverse duality, and the report corrects the exact form of the real-valued sine kernel. Provides transform pairs, inversion formulas, Parseval identities, and cosine-sine operator factorizations.

TR69. Ram-Real and Ram-Sine Transforms: A Unified Framework for Local Operators, Windowed Transforms, Global Spectral Transforms, and Applications (19 pp.)

Domain: Ram Transform theory

Develops the real-valued and sine-valued branches of the Ram Complex Spectral framework, deriving the Ram-Real and Ram-Sine kernels as the real and imaginary projections of the complex eigenkernel. It generalizes damped cosine and damped sine bases, reduces to ordinary cosine and sine transforms on the spectral axis, and bridges local Ram operators with cosine/sine coding, DCT/DST methods, and finite-interval spectral analysis. A key result shows these bases form an invariant subspace rather than independent scalar eigenfunctions of a real integral operator.

TR70. The Wavelet–Ram-Real Transform and Wavelet–Ram-Sine Transform: A Multiresolution Real/Sine Framework for Local and Spectral Ram Operators (18 pp.)

Domain: Ram Transform theory

Extends the Ram-Real and Ram-Sine framework into a multiresolution wavelet setting, defining the Wavelet-Ram-Real and Wavelet-Ram-Sine Transforms. Analysis atoms are built by multiplying wavelets with damped cosine or damped sine Ram kernels, combining scale, position, phase, damping, and moment information in a single representation. Useful for anyone needing localized, multiresolution real and sine spectral tools connected to Fourier and Laplace analysis.

TR71. Ram-Master Neural Networks (RMNN or RMT-NN): A Localized, Multiresolution, Algebra-Valued Operator-Learning Framework with Analytic Forward-Inverse Duality (21 pp.)

Domain: Scientific machine learning

Introduces Ram-Master Neural Networks, which replace generic learned weights and unconstrained convolution kernels with trainable Ram-Master Transform kernels that can be local, shift-variant, multiresolution, fractional, complex, quaternionic, or otherwise algebra-valued. The central idea is a trainable localized operator calculus whose learned object is a physically interpretable forward operator. A notable advantage is analytic forward-inverse duality: once the forward operator is learned, its local inverse can often be computed analytically or semi-analytically.

TR72. Ram-Master Transform Kernels and Ram-Master Neural Networks for Feature Extraction, Tokenization, Recognition, Diagnosis, and Scientific AI – A Comprehensive Technical Report on RMT/RMNN Encoders for Images, Video, Medical Data, Engineering Streams, and Transformer/LLM Hybrid Systems (30 pp.)

Domain: Scientific machine learning

A comprehensive report on using Ram-Master Transform kernels and neural networks as encoders for feature extraction, tokenization, recognition, and diagnosis across images, video, medical data, and engineering streams. It situates the approach against CNN filters, autoencoders, and vision transformers, and describes hybrid integration with transformer and language-model systems. Aimed at practitioners building interpretable, operator-based feature extractors for scientific AI.

TR73. Adelic RamCS/Ram–Master Transform for Hecke and Dedekind L-Functions and Related Physical Models – A Development of the Adelic Ram Transform Program (24 pp.)

Domain: Pure mathematics

Develops an adelic version of the Ram Complex Spectral and Ram-Master Transforms, combining archimedean localization with finite-place multiplicative convolution to address Hecke and Dedekind L-functions. Built on Tate's thesis and Iwasawa-Tate zeta integrals, it defines local factors, valuation-moment expansions, and their restricted tensor product, recovering standard zeta integrals and giving a moment dictionary for conductor, ramification, and gamma factors. It clearly delineates what the construction does and does not prove, positioning it as a front end for GRH research.

TR74. Self-Improving Ram-Master Neural Networks (RMNNs) – Automated Discovery, Hyperparameter Optimization, Kernel-Moment Optimization, Multi-Agent Experimentation, and AGI-Relevant Research Directions (32 pp.)

Domain: Scientific machine learning

Presents a framework for self-improving Ram-Master Neural Networks in which kernels, moments, coordinate maps, truncation orders, regularization, and architecture are optimized automatically by agentic experiment loops. It draws on AutoML, neural architecture search, Bayesian optimization, and self-refinement agents, arguing that the rich yet structured, semantically meaningful search space of these networks makes them especially suited to automated discovery. Includes discussion of multi-agent experimentation and research directions relevant to general AI.

TR75. A Ram Transform and Ram-Master Transform Synthesis for the Three-Dimensional Navier–Stokes Existence and Smoothness Problem – Conditional Regularity, Moment Constraints, RMT Variants, Remaining Gaps, and a Realistic Clay-Millennium Assessment (18 pp.)

Domain: Fluid dynamics *Source:* Clay Millennium Navier-Stokes problem

Synthesizes the best currently defensible Ram Transform approach to the three-dimensional incompressible Navier-Stokes existence and smoothness problem. It does not claim an unconditional proof; instead it establishes a conditional regularity theorem showing that if a vorticity-dependent Ram kernel stays in a suitable moment class, the localized expansion remains valid and the solution stays smooth, reformulating the Clay problem as a precise moment-persistence question. Provides the full construction and a realistic assessment of remaining gaps.

TR76. Practical Computational Fluid Dynamics with Ram Transform Methods – A Technical Assessment of Theory, Algorithms, R&D Applications, Advantages, Limitations, and Future Directions (18 pp.)

Domain: Fluid dynamics

A candid technical assessment of where Ram Transform methods add practical value in computational fluid dynamics rather than replacing mature high-fidelity solvers. It identifies their strongest near-term uses as local analytic micro-solvers, physics-informed preconditioners for Newton-Krylov and pressure-Poisson loops, coordinate-aware operators for moving and deforming flows, moment-constrained closures, fast residual correctors for neural surrogates, and real-time digital-twin control components. Useful for teams weighing where these methods fit alongside RANS, LES, and DNS workflows.

TR77. Resolving the Silent-Source Non-Uniqueness in MEG/MCG by Intracranial Magnetic Field Sensors: Theoretical Analysis, Quantitative Bounds, and Survey of Available Brain Implant Technologies (16 pp.)

Domain: Medical imaging *Source:* US Patent Application US20110313274A1

Analyzes how placing magnetic field sensors inside the conducting body, as brain implants for MEG or catheter sensors for MCG, can break the silent-source non-uniqueness that fundamentally limits any external-only inverse method. It shows analytically and numerically that even a small number of well-placed interior magnetometers eliminates the silent-source ambiguity for source modes within the sampling envelope, with quantitative bounds in a discretized brain model, plus a survey of available brain-implant technologies.

TR78. Critical Analysis and Verification of US Patent Application Publication No. 2011/0313274 A1 "Methods and Apparatuses for 3D Imaging in Magnetoencephalography and Magnetocardiography" (Subbarao, 2011) (19 pp.)

Domain: Medical imaging *Source:* US Patent Application US20110313274A1

A step-by-step technical verification of the mathematical claims in a MEG/MCG 3D imaging patent application. It finds the Biot-Savart forward model internally consistent but incomplete because it omits the volume return currents mandatory for in-vivo modeling, identifies a Fourier-domain augmentation that is singular across an entire plane rather than a single point, and shows the claimed heuristic-free closed-form solution contradicts the established result that radial dipoles in a spherical conductor are magnetically silent. Valuable for anyone assessing the patent's validity.

TR79. Richardson–Lucy versus Residual Gradient-Descent for Shift-Variant Image Deblurring with a Known Kernel: Theory, Algorithms, Practical Implementation, and an Investigation of Catastrophic Failure (22 pp.)

Domain: Image processing

Compares Richardson-Lucy and residual gradient-descent methods for non-blind shift-variant image deblurring with a known kernel, developing the theory, algorithms, and implementation of both. It rigorously examines the claim that Richardson-Lucy can collapse to a result far worse than the input while gradient descent merely stalls, making the distinction precise: gradient descent has a provably non-increasing residual but is only semi-convergent in reconstruction error, whereas Richardson-Lucy lacks such guarantees and can be ill-convergent at low signal-to-noise.

TR80. A Critical Technical Analysis of Ram Transform Methods for Wavefront Propagation, Electromagnetic Media, and Schrodinger-Type Quantum Problems – Errors, Assumptions, Corrected Mathematical Formulations, Pros and Cons, Accuracy, and Computational Speed Estimates (17 pp.)

Domain: Computational physics

A critical review consolidating work on applying Ram Transforms to wavefront propagation in dispersive media, computational electromagnetics, and Schrodinger and Lippmann-Schwinger quantum problems. It confirms the core idea of localizing a global shift-variant operator into an invertible local differential operator under explicit smoothness and truncation assumptions, while correcting overstatements: finite-order methods are exact only for polynomial data, radiation conditions are not automatically eliminated, CFL constraints are not truly removed, and quantum problems require careful handling of unitarity, caustics, and tunneling.

TR81. Technical Review and Corrected Framework for Applying Ram (Rao) Transforms to the Propagator-Hamiltonian Correspondence Problem (19 pp.)

Domain: Quantum computing

Reviews and corrects a perturbative Ram Transform method for passing between a short-time propagator kernel and its Hamiltonian or generator using moving coordinate frames. It endorses the core idea of representing the generator by local kernel moments to approximate the Hamiltonian as a finite-order local differential operator, but identifies serious gaps, most critically that heat and Schrodinger propagators have singular short-time limits with delta-function and inverse-power factors that make a naive pointwise Taylor expansion invalid. Provides a corrected framework.

TR82. Addendum to the Technical Review of Ram (Rao) Transform Methods for the Propagator-Hamiltonian Correspondence – Analysis of the Generalized Rao Transform Limit Derivation and Historical Context (13 pp.)

Domain: Ram Transform theory

This addendum refines a limiting-moment derivation that reads a system's Hamiltonian or infinitesimal generator directly from the short-time central moments of its propagator kernel, connecting the perturbative Ram/Rao Transform method to Feynman short-time expansions, the Kramers-Moyal expansion, and semigroup generator theory. It supplies the necessary qualifications, showing the correct statement is a properly scaled generator-limit formula rather than a naive pointwise Taylor expansion, with special care for the oscillatory Fresnel moments of the Schrodinger kernel. Useful for anyone formalizing propagator-to-generator correspondences under rigorous convergence assumptions.

TR83. Ram Transform Formulation of Shift-Variant Affine Motion Blur – Forward Blurring, Local Differential Operators, Inverse Deblurring, Defocus Coupling, and Higher-Dimensional Extensions (27 pp.)

Domain: Image deblurring *Source:* Kang dissertation, Chapter 5

This report builds a Ram Transform formulation for shift-variant affine motion blur, generalizing the standard uniform-translation exposure model to shift-variant kernels and time-dependent affine image transformations under two composition orderings. It converts the trajectory-supported integral operator into a finite-order local differential operator whose coefficients are generalized motion moments, then solves a local finite-jet system point by point to recover the latent sharp image. The payoff is a pixel-wise analytic deblurring method for complex motion, including defocus coupling and higher-dimensional extensions.

TR84. A Unified Ram Transform Family Formulation of Known-Kernel Motion Blur and Deblurring – Coordinate Localization, Local Ram-PDOs, Analytic Inverse Operators, Windowed/Multi-Resolution Variants, and a Dirac-Delta Trajectory Kernel (23 pp.)

Domain: Image deblurring

This report reframes known-kernel motion and affine-motion blur inside the broader Ram Transform family, coordinate-localizing the integral equation, Taylor-expanding the unknown image, and deriving analytic inverse operators from small local coefficient systems so deblurring proceeds pixel by pixel rather than through large global matrices. It covers windowed and multi-resolution variants and a Dirac-delta trajectory kernel, assuming the motion kernel and motion functions are known or estimator-supplied. Comparisons are scoped to known-kernel inverse-filtering methods, making it a clean reference for analytic space-varying deconvolution.

TR85. A Critical Technical Analysis of Ram Transform Methods for Wavefront Propagation, Electromagnetic Media, and Schrodinger-Type Quantum Problems – Errors, Assumptions, Corrected Mathematical Formulations, Pros and Cons, Accuracy, and Computational Speed Estimates (19 pp.)

Domain: Computational physics

This critical review consolidates four documents applying Ram Transforms to wavefront propagation in dispersive and inhomogeneous media, computational electromagnetics, inverse wavefront modeling, and Schrodinger and Lippmann-Schwinger quantum problems. It confirms the core localization-plus-jet-inversion idea under explicit smoothness, finite-moment, and truncation assumptions while flagging concrete overstatements, including that finite-order methods are exact only for polynomial data, that radiation conditions and PMLs are not automatically eliminated, that CFL limits are not bypassed, and that unitarity, caustics, and tunneling need explicit treatment. A valuable due-diligence reference before adopting these methods.

TR86. Fokker–Planck–Ram Operators for Linear Time-Invariant Problems – Theory, Algorithms, and Practical Applications of the Ram Transform Family to Drift–Diffusion Operators (20 pp.)

Domain: Stochastic processes

This first installment of a three-part series develops the Fokker-Planck-Ram operator for linear, time-invariant, constant-coefficient drift-diffusion problems, where the transition kernel is a Gaussian convolution whose Ram moments recover the drift, diffusion, and finite-step semigroup corrections. The Ram inverse yields local analytic backward propagation, regularized smoothing, and preconditioned implicit updates. It states the operator equivalences precisely and includes computational algorithms plus worked examples in uncertainty propagation, log-Black-Scholes finance, filtering, image restoration, and transport.

TR87. TFPR2: Fokker–Planck–Ram Operators for Linear Time-Variant and Coordinate-Variant Problems – Local Charts, Frozen-Kernel Parametrices, Conservative Algorithms, and Practical Applications (19 pp.)

Domain: Stochastic processes

This second Fokker-Planck-Ram report extends the framework to linear equations whose drift and diffusion vary with space, time, or externally supplied parameters but not the density itself, where transition operators no longer commute. It introduces a chart-based construction that freezes or Taylor-expands coefficients in moving neighborhoods, builds local Gaussian parametrices, computes Ram moments, and forms derivative-jet blocks usable as micro-solvers, preconditioners, residual correctors, or interpretable operator-learning layers. Includes problem statements, local operator theory, conservative algorithms, and practical applications.

TR88. TFPR3: Nonlinear Fokker–Planck–Ram Operators – Generalized Ram Localization, Local Nonlinear Algebraic Solvers, Structure Preservation, and High-Impact Applications (21 pp.)

Domain: Stochastic processes

This third Fokker-Planck-Ram report tackles nonlinear equations where drift, diffusion, reaction, collision, or interaction terms depend on the evolving density, its derivatives, or nonlocal moments, turning the operator into a local nonlinear map with its Jacobian, residual corrector, and structure-preserving constraints. It recommends a three-mode strategy: freeze-and-propagate for weak coupling, local nonlinear Ram algebraic solves for stiff local nonlinearity, and inverse Ram blocks as Newton-Krylov, multigrid, or neural preconditioners. Applications span mean-field games, nonlinear Black-Scholes, enzyme kinetics, and reaction-diffusion systems.

TR89. TCM1: Coordinate-Mapped Ram Transforms for Linear Integral Equations and Linear PDEs – A Graduate-Level Technical Reconstruction of the Affine Coordinate Mapping Method in P5 (19 pp.)

Domain: Applied mathematics *Source:* Provisional patent P5 (Coordinate-Mapped Ram Transform)

This graduate-level report reconstructs the affine coordinate-mapping method for linear integral equations and linear PDEs, showing how a shift-variant, rotated, sheared, scaled, or curved kernel can be localized by mapping source variables into a local coordinate attached to each observation point. After the map the unknown is Taylor-expanded about a moving center, turning the integral into a rapidly convergent series of local differential Ram operators whose coefficients are generalized kernel moments, so differentiating the measured data yields a small solvable system for both forward modeling and inverse reconstruction.

TR90. TCM2: Coordinate-Mapped Ram Transforms for Nonlinear Integral Equations and Nonlinear PDEs – A Graduate-Level Sequel to TCM1 Based on the Nonlinear Coordinate-Mapping Content of P5 (22 pp.)

Domain: Applied mathematics *Source:* Provisional patent P5 (Coordinate-Mapped Ram Transform)

This sequel extends coordinate-mapped Ram Transforms to nonlinear integral equations and nonlinear PDEs, clarifying that coordinate mapping removes geometric complexity but not intrinsic nonlinearity: problems linear in the unknown but nonlinear only in geometry stay linear, while genuine nonlinearity in the unknown or its gradient produces small local nonlinear algebraic systems. Replacing a global nonlinear functional equation with many weakly coupled local systems enables Newton, Gauss-Newton, policy-iteration, and Newton-Krylov solvers using Ram-derived preconditioners. A practical bridge from the linear theory to nonlinear solvers.

TR91. TCM3: Remaining Integral-Equation and PDE Problem Classes for Coordinate-Mapped Ram Transforms – Truncation in Source Jets, Kernel Geometry, and Nonlinear Powers Beyond TCM1 and TCM2 (22 pp.)

Domain: Applied mathematics *Source:* Provisional patent P5 (Coordinate-Mapped Ram Transform)

This third coordinate-mapping report identifies problem classes underserved by the earlier linear and nonlinear treatments and folds them into the framework, centering on the nonlinear power truncation P alongside the source-jet order N and kernel order M to form a unified design language for nonlinear algorithms. It maps out how to handle Volterra and memory equations, integro-differential equations, third- and fourth-kind equations, coupled systems, singular and hypersingular integrals, boundary integral equations, fractional and nonlocal PDEs, and discontinuous or free-boundary solutions.

TR92. TCM4: Ram Master and Advanced Ram Transforms for Remaining Coordinate-Mapped Integral-Equation and PDE Problems – Multiresolution Windows, Fractional and Complex Orders, Hypercomplex Algebras, and Practical Solver Architectures Beyond TCM1–TCM3 (30 pp.)

Domain: Applied mathematics

This report asks whether the more recent Ram Master Transform family can better handle problem classes left unresolved by ordinary Taylor localization, such as memory and Volterra equations, singular and hypersingular kernels, fractional and nonlocal PDEs, shocks and free boundaries, coupled tensor systems, stochastic kernels, oscillatory wave kernels, and multiscale problems. The answer is a qualified yes: adding a variable window width, wavelet scale, complex scale-frequency parameter, and real/sine/complex branches, plus a fractional-order extension using Mittag-Leffler functions, directly targets the multiscale, boundary, and oscillatory difficulties.

TR93. RamDictionary: Dictionary and Taxonomy of Named Ram Transform Concepts – A Source-Derived Vocabulary for Ram/Rao Transforms, Operators, Matrices, Coefficients, Coordinate Maps, Algorithms, and Application Methods in res-rt (33 pp.)

Domain: Ram Transform theory

This report is a dictionary and taxonomy of named Ram Transform concepts, normalizing the older Rao Transform vocabulary into current Ram terminology while preserving standard statistical terms like Cramer-Rao bounds. It catalogs the transforms, local operators, matrix systems, moments, coordinate maps, spectral variants, Ramlet and Ram-Master transforms, fractional, complex-order, quaternionic and octonionic extensions, graph-local operators, and application-specific methods, each with variants, a concise description, central equations, and a pointer to source documents. A practical reference map for navigating the whole corpus.

TR94. Ram Transform Family Paths Toward the Riemann Hypothesis – Beyond the Failed Hilbert-Polya Adjointness Ansatz (14 pp.)

Domain: Pure mathematics

This report surveys prior Ram Transform attempts at the Riemann Hypothesis, documenting why the Hilbert-Polya adjointness ansatz structurally fails to turn transform zeros into a self-adjoint operator's point spectrum, then charts five more promising Ram-compatible directions: a Weil explicit-formula positivity program, a de Branges canonical-system program, Li and Nyman-Beurling approximation, a Jensen-polynomial heat-flow and de Bruijn-Newman route, and an adelic trace-formula approach. It develops the first two into precise conditional theorems, honestly noting each stalls on a missing positivity theorem rather than an unconditional proof.

TR95. Relevance of the Ram-Master Transform and Its Variants to GRH for Broad Families of L-Functions – A Technical Investigation Based on Current Literature and the Ram Transform Corpus (16 pp.)

Domain: Analytic number theory

Investigates whether the Ram-Master Transform and its RamCS, Fractional, Complex, Quaternionic, and Octonionic variants are relevant to the Generalized and Grand Riemann Hypotheses and zero-location problems for broad families of L-functions. It argues the answer is affirmative but conditional: because completed L-functions are Mellin-type transforms of theta, Bessel, and automorphic kernels, the framework offers natural tools for analytic representation, localization, and numerical evaluation. It is candid that the transforms do not by themselves prove GRH.

TR96. Adelic RamCS/Ram–Master Transform for Hecke and Dedekind L-Functions and Related Physical Models – A Development of the Adelic Ram Transform Program (24 pp.)

Domain: Analytic number theory

Develops an adelic version of the Ram Complex Spectral and Ram-Master Transforms suitable for

Hecke and Dedekind L-functions, building from Tate's thesis and Iwasawa-Tate zeta integrals. It defines local archimedean and finite-place RamCS factors, valuation-moment and unit-character expansions, and their restricted tensor product over all places, recovering standard zeta integrals and yielding a moment dictionary for conductor, ramification, gamma factors, and Euler factors. It clarifies what the construction does and does not prove.

TR97. A Revised Conditional Proof Framework for the Riemann Hypothesis via Ram Transforms – Incorporating Weakened Assumptions, Corrected Operator-Theoretic Statements, and Explicit Remaining Gaps (14 pp.)

Domain: Analytic number theory

Rewrites the conditional Ram-transform proof framework for the Riemann Hypothesis, stating the strongest logically correct conditional result rather than claiming an unconditional proof. It cleanly separates two achievements: the classical and RamCS identities that make the xi-function zeros into zeros of a precise Fourier/RamCS symbol, and the Hilbert-Polya operator realization needed to turn those into self-adjoint point spectrum. It explains why a bounded multiplier operator cannot supply compact resolvent and lists the explicit remaining gaps.

TR98. The Navier–Stokes Millennium Problem via Ram Transforms: Applying the Gap C Closure to the Borel-Summability Gap, Updated Status, and Comprehensive Technical Assessment (22 pp.)

Domain: Fluid dynamics / PDE *Source:* Chen and Hou, PNAS 2025

Revisits the Ram Transform program for the three-dimensional incompressible Navier-Stokes Millennium problem, applying three gap-closure strategies from the Riemann Hypothesis work to the vortex-stretching moment series. It proves the Biot-Savart kernel is Schwartz-class in space, giving moment determinacy conditional on vorticity decay, and that localization is reversible on analytic data. The report updates the program's status in light of recent computer-assisted Euler and Boussinesq blowup results.

TR99. Does the Chen–Hou Proof of 3D Euler Blowup and the Prospect of Navier–Stokes Blowup Impact the Riemann Hypothesis Proof Programme? – A Cross-Programme Analysis of the Ram Transform Structural Analogy between RH and NS (16 pp.)

Domain: Analytic number theory *Source:* Chen and Hou, PNAS 2025

A cross-program analysis asking whether recent proofs of finite-time 3D Euler blowup, and the prospect of Navier-Stokes blowup, undermine the Ram Transform proof program for the Riemann Hypothesis. It concludes definitively that they do not, because the structural analogy breaks exactly where it matters: the Riemann kernel is provably Schwartz-class, static, and time-independent, whereas the Navier-Stokes kernel is dynamic and its Schwartz property is itself the open problem. It classifies the two as static versus dynamic Borel-summability questions.

TR100. Hybrid Ram Transform Techniques with Optimization, Regularization, Multiresolution, and Multi-Interval Methods (27 pp.)

Domain: Numerical methods

A streamlined account of hybrid methods that combine Ram Transform techniques with standard numerical optimization, regularization, multiresolution, multi-interval, and iterative residual-minimization solvers. It shows Ram local transforms acting as approximate inverses, preconditioners, coarse-grid solvers, multigrid smoothers, and initializers for nonlinear inverse problems. The core message is that Ram methods are best used not as replacements but as analytic, local, physics-informed approximate inverses that improve conditioning, supply good initial estimates, and expose where a model is failing.

TR101. Ram Transform Methods for Neural Integral Equation Systems – Part 1 of the P4 Neural Ram Transform Report Series: Theory, Algorithms, Glass-Box Identification, Analytic Inversion, and Comparison with Neural Integral Equations (23 pp.)

Domain: Scientific machine learning

The first in a three-part series applying Ram Transform methods to neural integral equation systems. It introduces the Ram-local Neural Integral Equation, which augments a learned nonlinear integral operator with a local moment and derivative-jet representation, converting the global integral into a finite algebraic system at each point with interpretable kernel-moment coefficients. Once forward coefficients are learned, the inverse operator can often be obtained analytically through local matrix inversion, back-substitution, regularization, or local Newton steps.

TR102. Ram Transform Methods for Neural Integral Equation Systems – Updated Part 1: Coordinate-Mapped Localization, Ram-Master Extensions, Composite Q and Q' Operators, Glass-Box Identification, and Analytic Inversion (29 pp.)

Domain: Scientific machine learning

An updated first part of the neural Ram Transform series that keeps the localize-learn-invert core but adds coordinate-mapped localization, so the source coordinate can follow affine, rigid-body, anisotropic, boundary-fitted, ray-fitted, Lagrangian, or general locally invertible charts absorbed into generalized Ram moments. This lets a Ram-local Neural Integral Equation learn both a local operator and the coordinate chart in which it is most compact and most accurately inverted, and it adds explicit composite forward and inverse Ram operators plus glass-box identification.

TR103. Ram Transform Methods for Attentional Neural Integral Equation Systems – Part 2: Rao/Ram-Corrected Attention, Local Moment Learning, Composite Q,Q' Operators, and Analytic Local Inversion (27 pp.)

Domain: Scientific machine learning

The second part of the neural Ram Transform series, treating attentional neural integral equation systems where self-attention replaces explicit numerical integration. It localizes the attention integral, expands the representation and kernel in local displacement coordinates, collects local moments, and converts attention's local action into a finite-dimensional system, making the operator inspectable and invertible. It adds Ram-corrected attention, local moment learning, composite forward and inverse operators, and analytic local inversion.

TR104. Ram Transform Methods for Attentional Neural Integral Equation Systems – Updated Part 2: Coordinate-Mapped Local Attention, Composite Q and Q' Operators, Ram-Master/RMNN Attention Features, Glass-Box Distillation, and Analytic Inversion (27 pp.)

Domain: Scientific machine learning

An updated second part extending Ram methods for attentional neural integral equations by splitting attention into a local part and a residual nonlocal part, expanding the local part into jets and moments, and computing inverse or preconditioning operators from the learned forward coefficients. The key addition is coordinate-mapped local attention, letting the attention neighborhood follow rotations, affine deformations, boundary-fitted or ray coordinates, Lagrangian flow maps, or learned charts, plus glass-box distillation of opaque attention into an interpretable physical kernel.

TR105. Ram Transform Methods for DeepONet and Physics-Informed Neural Operators – Part

3: Coordinate-Mapped Local Solution Operators, Complex-Valued Wave Problems, Ram-Master Features, and Analytic Inversion (24 pp.)

Domain: Scientific machine learning

The third part of the neural Ram Transform series, applying the approach to DeepONet and physics-informed neural operators. Instead of outputting the solution field directly, the network outputs local Ram operator coefficients such as forward moments, inverse coefficients, local matrix factors, nonlinear closure terms, and coordinate-map parameters, yielding interpretable local forward and inverse operators. It emphasizes coordinate-mapped local solution operators, complex-valued wave problems, Ram-Master features, and analytic inversion.

TR106. The Ram-Master Transform (RMT): A Unified Framework Combining the Localized Ram Partial Differential Operator, Variable-Width Windowing, and the Cross-Ramlet Transform (21 pp.)

Domain: Ram Transform theory

Introduces the Ram-Master Transform as a unified framework combining three previously separate elements: the fully localized forward and inverse Ram Partial Differential Operator that replaces an integral equation with local differential series, continuous variable-width windowing that interpolates between pointwise and global limits, and the Cross-Ramlet Transform whose kernel subsumes wavelet, short-time Fourier, Morlet, Gabor, and chirplet transforms as special cases. It defines a four-parameter Ram-Master spectrum and proves a universal reduction theorem identifying at least seventeen classical and modern transforms as restrictions.

Volume 2 — Technical Reports TR107–TR120

TR107. Ram Transforms: Recent Application Targets – Papers and Patents from 2021–2026 for Agentic Ram Transform Follow-Up (13 pp.)

Domain: Ram Transform theory

A curated target list of recent papers and patents from roughly 2021 to 2026 whose problem classes are well suited to Ram Transform methods, spanning operator learning for integral equations and PDEs, shift-variant deblurring, time-varying channel estimation, OTFS wireless, geophysical inversion, Fokker-Planck and stochastic equations, quantitative finance, and probabilistic inference. Each entry comes with keywords and a specific opportunity statement so a follow-up program can formulate the operator, pick a technique, and produce a focused solution or patentability analysis. It is a structured roadmap, not a claim that these problems are already solved.

TR108-A. The Ram-Master Transform, Revised and Reconciled: A Practically Grounded Unified Framework of Local Ram Finite-Jet Operators, Windowed Multiscale Ramlet Atoms, Composite Q/Q' Inversion, Frame-Based Reconstruction, Probability-Kernel Classes, and Novelty Relative to Current Literature (33 pp.)

Domain: Ram Transform theory *Source:* Abu-Ghuwaleh, Mathematics 2025, 13, 3431

Version 5 of the revised Ram-Master Transform report, reconciling multiple independent reviews and a dedicated novelty study into one unified framework. It defines a single analytic transform family that contains Fourier, Laplace, Mellin, short-time Fourier, Gabor, Morlet, chirplet, wavelet, wavelet-packet, and local Ram/Rao analysis as special cases, with every assumption and formula stated at expert precision. Buyers get a rigorously grounded, literature-referenced consolidation of local finite-jet operators, windowed multiscale atoms, composite inversion, and frame-based reconstruction.

TR108-B. The Revised Ram-Master Transform: Local Finite-Jet Inversion, Composite Operators, Windowed Ramlet Atoms, and the Extended Ram Transform Family (38 pp.)

Domain: Ram Transform theory

A journal-style consolidation of the revised Ram-Master Transform, centered on a parameterized transform combining local finite-jet operators, composite discrete and continuous operators, windowed multiscale atoms, and exact or regularized reconstruction. Its key distinction is that it acts as a local symbol and inverse calculus for shift-variant operators whose kernels vary with position, scale, coordinate chart, or learned context, rather than a global diagonalization. It embeds Fourier, Laplace, wavelet, Gabor, Mellin, and other classical transforms as restrictions while honestly recording and then addressing the limits of each layer.

TR108-C. Reassessing the Relative Novelty of the Ram Transform Family in Light of Its Extensions: How the Fractional, Complex-Order, Hypercomplex, Coordinate-Mapped, Ramlet, Hybrid, Self-Improving, and Adelic Ram Transforms Address the Capabilities of Pseudo-Differential and Segal–Bargmann Methods (11 pp.)

Domain: Ram Transform theory

A gap-by-gap reassessment of how the Ram Transform family compares with pseudo-differential symbol calculus and the Segal-Bargmann coherent-state transform once its extensions are included. Drawing on reports covering fractional, complex-order, quaternionic, octonionic, coordinate-mapped, Ramlet, hybrid, self-improving, and adelic variants, it argues that most previously identified weak points are closed or substantially diluted. The payoff is a computable physical-space symbol calculus and finite-jet parametrix that rivals these established methods on their own ground.

TR108-D. Hybrid Ram–Pseudodifferential and Microlocal Extensions for Local Shift-Variant Operators, Singularity Detection, and Propagation (15 pp.)

Domain: Microlocal analysis

A construction that extends the Ram Transform family by fusing it with pseudodifferential and microlocal analysis to tackle singularity detection, wavefront-set localization, elliptic regularity, and propagation of singularities. Starting from the local finite-jet Ram expansion and applying a shift-variant operator, it derives explicit composite symbols and shows how Gabor, FBI, wave-packet, and shearlet restrictions can diagnose and propagate singularities in phase space. Useful for anyone needing local, phase-space-aware analysis of shift-variant operators.

TR108-E. Advantages, Application Value, and Remaining Research Pathways for Hybrid Ram–Pseudodifferential and Microlocal Methods (17 pp.)

Domain: Microlocal analysis

A companion evaluation weighing the theoretical and practical advantages of the hybrid Ram-pseudodifferential and microlocal approach across many example families, from conic cutoffs and elliptic parametrices to caustic analysis and graph-local extensions. It compares the method against classical operator theory, finite-element and spectral solvers, propagation theorems, and neural operators such as Fourier neural operators, DeepONets, and PINNs. The conclusion is that the hybrid does not replace these tools but combines them to improve transparency, residual localization, adaptivity, and sometimes speed.

TR109. Ram-Master Delay-Doppler Operators for OTFS Channel Estimation – Part I: Direct

Ram Moment Sounding, Ram-Fisher Pilot Design, and Corrected Local Inverse Operators (19 pp.)

Domain: Wireless / 6G *Source:* Cohere US Patent 11,470,485 B2

The first part of a framework applying the Ram Transform family to OTFS channel estimation and delay-Doppler equalization, building on a specific OTFS patent and prior reports. Its central idea is to design reference signals, receiver analysis functions, and atom banks so the receiver directly estimates local moment vectors and their derivatives rather than first recovering a full physical path list. It also derives a moment-domain Fisher information matrix and Cramer-Rao criterion to guide pilot placement, waveform selection, and window and atom-bank choices.

TR110. Graph-Local Ram Cooperative OTFS Networks – Multicell, Cell-Free, RIS, Non-Terrestrial, and Cooperative ISAC Graph Operators (30 pp.)

Domain: Wireless / 6G

Extends link-level Ram OTFS channel estimation to cooperative wireless networks, defining graph-local forward and inverse operators for multicell, cell-free massive-MIMO, RIS-assisted, non-terrestrial and LEO OTFS, and cooperative sensing. Base stations, access points, satellites, beams, RIS sub-panels, and sensing cells become nodes, with communication, interference, reflection, and fronthaul couplings as typed edges carrying edge-local moment matrices. These assemble into a sparse block graph operator whose local inverses, Schur complements, and compressed moment messages enable cooperative equalization, interference coordination, channel prediction, and RIS control.

TR111. Ram Channel World Models – Moment Fields, Reflector Objects, Uncertainty, and Active-Probing Policies for Cooperative OTFS Networks (12 pp.)

Domain: Wireless / 6G

Adds memory, prediction, uncertainty, object persistence, and active probing to the graph-local Ram OTFS framework, producing structured Ram Channel World Models of a wireless environment. Rather than raw channel-state samples or neural embeddings, the state consists of local moment fields, graph edge operators, persistent reflector and scatterer objects, uncertainty matrices, and action histories. Prediction uses partial differential, state-space, and graph propagation operators, while updates rely on moment filters, Ram-Kalman steps, and active-probing objectives for planning pilots, RIS phases, beams, and sensing.

TR112. Mollifier-Ram and Neural Wireless Receivers – Differentiable Receiver Layers with Analytic Derivative Jets, Ram-Master Atoms, and Learned Residual Correction (11 pp.)

Domain: Wireless / 6G

Specifies a Mollifier-Ram Neural Receiver that combines analytic forward and inverse Ram layers with mollified derivative jets, atom banks, and learned residual correction. Its three principles are: estimate noisy local derivatives through mollifier filters with known derivatives rather than raw finite differences; use a differentiable Ram operator backbone built on local moment and graph-local matrices with regularized inverses; and confine neural networks to residual mismatch, mode and hyperparameter selection, hardware distortion, and quantization compensation. The learned part is constrained by an analytic backbone instead of learning the whole receiver from data.

TR113. Moment-Shaped Waveforms and RIS Control – Pilot, Atom, Waveform, Beam, and Metasurface Designs Optimized for Local Ram Operator Conditioning (10 pp.)

Domain: Wireless / 6G

Reframes wireless waveform and metasurface design around the stability and observability of the local Ram inverse operator, not just energy, MSE, or spectral efficiency. It introduces moment observability matrices, Ram-Fisher design criteria, inverse-conditioned pilot objectives, atom selection, and RIS/STAR-RIS phase control for OTFS and cooperative sensing-communication systems. Buyers get a unified framework linking transmitter and environment actions directly to receiver-side moment invertibility and communication-sensing performance.

TR114. Physical-Layer Security and Watermarking with Ram Operators – Secret Moment Masks, Atom Hopping, Graph Residual Fingerprints, and Tamper-Resistant Moment Exchange (9 pp.)

Domain: Wireless / 6G

Shows how Ram moment and graph-operator structures enable physical-layer security, authentication, watermarking, and tamper detection alongside standard cryptography. It introduces secret moment masks, atom hopping, graph residual fingerprints, authenticated moment-fronthaul messages, RIS-assisted secrecy, and challenge-response protocols built around a key-transformed local operator signature that looks valid but is poorly conditioned to eavesdroppers. Useful for teams hardening OTFS and cooperative wireless links without replacing existing crypto.

TR115. Standards and Hardware Mapping for Ram-Based Wireless Systems – Moment Reference Signals, Moment-Fronthaul Messages, Graph-Local Receiver Blocks, and Edge Hardware Pipelines (10 pp.)

Domain: Wireless / 6G *Source:* 3GPP TR 38.765 / TR 38.901; O-RAN

Maps the Ram-based wireless methods of reports TR109 through TR114 onto practical 5G-Advanced, 6G, and Open RAN architectures via a five-level implementation stack spanning resource grid, receiver pipeline, fronthaul and control plane, edge and cloud world models, and hardware accelerators. It shows where moment reference signals, moment-fronthaul messages, and mollifier-Ram receiver layers could fit as optional pilots, compressed edge summaries, and fixed filters or sparse solvers. Grounded in current 3GPP and O-RAN activity.

TR116. Ram Transform Based Methods for 5G/6G Communications – Relative Advantages, Costs, Disadvantages, Open Problems, and Deployment Roadmap (17 pp.)

Domain: Wireless / 6G

A balanced assessment of the advantages, costs, disadvantages, and open problems of applying Ram Transform methods in 5G-Advanced and 6G, synthesizing multiple prior reports with current literature. It concludes Ram methods are not a universal replacement for OFDM, LMMSE, message passing, or neural receivers, but offer a compelling new layer for hard regimes: high Doppler, fractional delay-Doppler, shift-variant channels, cell-free cooperation, RIS control, non-terrestrial mobility, and interpretable AI-native receivers. Includes a deployment roadmap.

TR117. Revised Chapter 4 of the 2007 Ram/Rao Transform Book – A Ram Framework for Solving Integral and Integro-Differential Equations: Localized Fredholm-Ram, Volterra-Ram, Urysohn-Ram, Hammerstein-Ram, and Coupled Ram Equation Systems (22 pp.)

Domain: Ram Transform theory *Source:* US Patent Application Publication US2006/0111882A1

A streamlined, modernized revision of Chapter 4 of the 2007 Rao Transforms book, recast under the current Ram Transform name and tied to a US patent publication on solving integral and integro-differential equations. It systematically defines localized counterparts of classical equations by appending Ram, distinguished by a shifted integrand, and delivers the Ram Localization Transform, its general form, catalogs of the first through fourth kinds, multidimensional and Hammerstein-Ram extensions, and integro-differential forms.

TR118. Graph Local Ram Neural Networks, Graph Local Ram Bayes Nets, and Graph Local Bayes Neural Nets – Ram-Based Formulations for Generalized Reasoning with GNNs by Relational Bayesian Network Encodings (27 pp.)

Domain: Graph neural networks *Source:* Pojer, Passerini & Jaeger, Generalized Reasoning with Graph Neural Networks by Relational Bayesian Network Encodings

Examines how graph-local Ram neural networks, Ram Bayes nets, and Ram operator constructions address generalized reasoning problems where trained message-passing GNNs are embedded into Relational Bayesian Networks for conditional queries, inverse inference, and explanation graphs. It argues Ram methods complement rather than replace the probabilistic semantics, rewriting each local sum as a moment-kernel map with local coordinates over attributes, edges, and motifs plus a regularized inverse for diagnostics and MPE search.

TR119-A. Ram Transform Family Solutions for the ODE Analysis of Stochastic Gradient Methods with Optimism and Anchoring for Minimax Problems – Ram Partial Differential Operators, the Ram–Master Transform, and Mollifier–Ram Jets Applied to Saddle-Point Dynamics (15 pp.)

Domain: Optimization theory *Source:* Ryu, Yuan & Yin (RYY), minimax last-iterate convergence

Applies the Ram Transform family to the ODE analysis of stochastic gradient methods for convex-concave minimax problems, covering simultaneous, optimistic, and anchored gradient descent as studied by Ryu, Yuan and Yin. It establishes six concrete correspondences, showing the three continuous dynamics become polynomial-exact forward and inverse Ram Partial Differential Operator recurrences, with a fully worked Dirac-GAN example and Ram-Master exponential-atom treatment of the linearized flow.

TR119-B. Appendix A to Technical Report TR119 – Continuous-Domain Ram-Transform Derivation of the O(1/t) Convergence Rate for Anchored Gradient Descent-Ascent (7 pp.)

Domain: Optimization theory *Source:* Tsoukalas et al., Advancing Mathematics Research with AI-Driven Formal Proof Search (2026)

An appendix to TR119 answering whether the exact order-one-over-t last-iterate convergence rate for Anchored Gradient Descent-Ascent, recently proved by a formal-proof-search agent using discrete recurrences, can instead be derived in continuous time via the Ram Transform. It shows it can, using the inverse first-order Ram Partial Differential Operator as an integrating factor to represent the anchored trajectory as a polynomial-Ramlet weighted running average, plus a matching Lyapunov functional.

TR120-A. Graph Local Ram Transform: Neural, Bayesian, and Hybrid Methods for Generating, Simulating, and Planning New Chemical Materials (29 pp.)

Domain: Materials discovery

Investigates whether graph-local Ram Transform neural networks, Bayes nets, and hybrids can explore, generate, simulate, rank, and plan synthesis of new chemical materials. It concludes that unbounded generation of all possible compounds is impractical, but constrained discovery is tractable, and positions the Ram family as a structured graph-local layer for chemical neighborhoods, inverse-design diagnostics, and low-order moment features that complements DFT, molecular dynamics, GNNs, and diffusion generators rather than replacing them.

TR120-B. Ram Transform Extensions of Crys-JEPA for Energy-Aware Crystal Discovery (21 pp.)

Domain: Materials discovery *Source:* Liu et al., Crys-JEPA: Accelerating Crystal Discovery via Embedding Screening and Generative Refinement

A second part to TR120 analyzing the Crys-JEPA energy-aware joint-embedding architecture for de novo crystal discovery, which screens generated crystals against training embeddings to improve validity, stability, uniqueness, and novelty. It proposes Ram Transform extensions as diagnostics rather than replacements, including graph-local crystal moment tokens, a Ram-augmented encoder, a hull-aware latent distance approximating convex-hull stability, inverse Ram motif diagnostics, and Bayesian Ram uncertainty for screening.

Volume 3 — Technical Reports TR121–TR151

TR121. Ram Transform Methods for the F1 Paper – Physics-Informed Neural Network for Option Pricing – Local Black-Scholes Ram Semigroups, Ram Linear Complementarity for American Puts, Greeks, Market-Data Residuals, and Ram-PINN Hybrid Architectures (19 pp.)

Domain: Quantitative finance *Source:* Dhiman and Hu, arXiv:2312.06711

Delivers a Ram Transform alternative to physics-informed neural pricing of the Black-Scholes equation for European calls and American puts. It recasts the pricing operator in log-moneyness and time-to-maturity coordinates as a local diffusion-convection-discount semigroup with a Gaussian risk-neutral kernel, yielding explicit Ram moments, an analytic upper-triangular moment matrix, and closed-form Delta, Gamma, Theta, local residuals, and local-volatility diagnostics. For American puts it adds a Ram linear-complementarity method with payoff projection and smooth-pasting to handle the free boundary.

TR122. Ram Transform Methods for the F2 Paper – Option Pricing and Local Volatility Surface by Physics-Informed Neural Network – Ram-Dupire Inverse Operators, Local-Volatility Recovery, Variable-Coefficient Ram Semigroups, CEV Diagnostics, and Ram-PINN Hybrids (18 pp.)

Domain: Quantitative finance *Source:* Bae, Kang, and Lee, Computational Economics 64:3143-3159, 2024

Provides a Ram Transform framework for local-volatility option pricing and, crucially, the inverse recovery of a local volatility surface from option prices, an alternative to neural-network plus Dupire-formula approaches. Forward pricing uses a variable-coefficient diffusion-convection-discount semigroup in log-moneyness coordinates with perturbation corrections for spatial and temporal volatility variation, tested against closed-form CEV prices and Greeks. The report derives Ram-Dupire inverse operators for recovering the volatility function by differentiating the price surface in strike and maturity.

TR123. Ram Transform Methods for the F3 Paper – Deep Learning of Transition Probability Densities for Stochastic Asset Models with Applications in Option Pricing – Fokker-Planck-Ram Transition Kernels, Parametric Density Engines, Jump-Diffusion PIDE Operators, Calibration, and Ram-Neural Hybrids (22 pp.)

Domain: Quantitative finance *Source:* Su, Tretyakov, and Newton, Management Science 71(4):2922-2952, 2025

Presents a Fokker-Planck-Ram framework for transition probability densities of stochastic asset models, an alternative to neural density generators used inside QUAD option pricing. It builds exact invariant Gaussian transition kernels whose Ram moments recover drift, diffusion, and higher semigroup terms, then extends to space-time-variant and jump-diffusion models via local frozen-kernel parametrices. Coverage targets geometric Brownian motion, Heston, SABR, Kou double-exponential jump diffusion, and Bates models, with calibration and hybrid neural approaches.

TR124. Ram Transform Methods for the F4 Paper – A Time-Stepping Deep Gradient Flow

Method for Option Pricing in (Rough) Diffusion Models – Ram-Local Time Marching, Heston and Lifted-Heston Operators, Rough-Volatility Memory Compression, Inverse Calibration, and Ram-Neural Hybrid Solvers (22 pp.)

Domain: Quantitative finance *Source:* Papapantoleon and Rou, arXiv:2403.00746

Offers a local semigroup and local-inverse framework for European option pricing in diffusion and lifted rough-volatility models, an alternative to time-stepping deep gradient flow neural solvers. Option values are propagated by an adjoint transition kernel with explicit Ram local time marching for Heston and lifted-Heston operators, including rough-volatility memory compression and inverse calibration. It directly targets the difficult lifted-Heston regime of larger factor dimension and nonzero correlation where the explicit asymmetric drift term causes rapid error growth.

TR125. Ram Transform Methods for the F5 Paper – Quantum-Inspired Tensor Neural Networks for Option Pricing – Ram Spectral and Tensorized Polynomial Operators for High-Dimensional Pricing PDEs, Bermudan Continuation, and Model Calibration (24 pp.)

Domain: Quantitative finance *Source:* Patel et al., arXiv:2212.14076

Develops Ram spectral and tensorized polynomial operators for high-dimensional option-pricing PDEs, moving the tensorization target from the neural layer to the pricing operator itself. Where quantum-inspired tensor neural networks compress dense weight matrices into matrix-product-operator form to reduce parameters and variance, this report tensorizes the pricing semigroup, addressing the Heston PDE, Bermudan max-call continuation, and model calibration. It aims at parameter-efficient, structured representations for high-dimensional pricing.

TR126. Ram Transform Transition-Density and Inverse-Calibration Methods for Efficient Monte Carlo Option Pricing and Derivative Model Calibration – Technical Analysis of F6: US20240281856A1 (21 pp.)

Domain: Quantitative finance *Source:* US Patent Application US20240281856A1

Analyzes a patented deep-learning and GPU system for Monte Carlo option pricing and derivative model calibration, and proposes an operator-based alternative. Instead of repeatedly estimating anchor prices by random paths, it represents pricing and calibration as transition-density, conditional-expectation, and inverse-calibration operators, converting Monte Carlo expectations into local transition-moment, spectral, finite-particle, and control-variate representations. Ram-Master methods turn calibration and parameter tracking into deterministic nonlinear state-estimation recursions over model parameters.

TR127. Ram Transform Forward/Inverse Operator Pairs as Constraints for Invertible Neural Networks in Quantitative Finance – Technical Analysis of F7: US20220383110A1 (16 pp.)

Domain: Quantitative finance *Source:* US Patent Application US20220383110A1

Examines a patented invertible neural-network architecture for joint inverse and forward modeling in computational finance and offers an analytic complement based on Ram forward and inverse operator pairs. The key insight is that the map from model parameters to option prices is governed by pricing PDEs, integro-differential equations, Fokker-Planck equations, transition densities, and no-arbitrage constraints rather than being a black box. Ram methods convert these known structures into local forward operators, local inverse operators, and transition representations usable as constraints for the invertible network across Black-Scholes, SABR, Merton, and Heston models.

TR128. Graph Local Ram Transform and RMNN Uncertainty Propagation for Deep-Neural Financial Risk Detection – Technical Analysis of F8: CN116308703A (26 pp.)

Domain: Financial risk / ML *Source:* Chinese Patent Publication CN116308703A

Analyzes a patented deep-neural financial risk detection system and reframes its module chain as a probabilistic risk graph rather than a mere software pipeline. Treating each risk category as a node or subgraph state fed by market, credit, and transaction data, the report applies Graph Local Ram Transform and RMNN uncertainty propagation across credit, market, exchange-rate, liquidity, systemic, management, crime, and policy risks. This yields structured, graph-based uncertainty quantification for risk classification and identification.

TR129. Privacy-Preserving Graph Local Ram Transform Updates for Federated Neural Graphical Models – Technical Analysis of B7: WO2025064122A1 (19 pp.)

Domain: Federated learning *Source:* WIPO Patent Publication WO2025064122A1

Analyzes a patented federated learning framework using neural graphical models and interprets its client-local dependency graphs, merged global graph, and personalization step as graph-local operators. It develops privacy-preserving Graph Local Ram Transform updates so that clients can recover local feature-dependency structure, contribute to a merged global graph, and train local models without sharing raw data. The approach connects Bayesian networks, factor graphs, belief propagation, evidence updates, and inverse diagnostics to federated neural graphical modeling.

TR130. Ram-Master and Graph-Local Ram Uncertainty Operators for Approximate Bayesian Neural Operators – Technical Report for TR107 Topic B1 (24 pp.)

Domain: Scientific machine learning *Source:* Magnani, Kramer, Eschenhagen, Rosasco, and Hennig, Approximate Bayesian Neural Operators, 2022

Strengthens approximate-Bayesian neural operators for uncertainty quantification of parametric PDEs by replacing global black-box operator uncertainty with structured local Ram operator uncertainty. It builds posterior distributions over Green-kernel moments, local derivative-jet maps, coordinate-mapped kernels, inverse regularization parameters, residual correction fields, and graph-local mesh operators. These local objects provide calibrated predictive uncertainty and inverse uncertainty for recovered quantities, complementing last-layer Laplace approximations.

TR131. Ram-Local Linearized Neural Operators, Function-Valued Gaussian Processes, and Q/Q' Covariance Propagation – Technical Report for TR107 Topic B2 (20 pp.)

Domain: Scientific machine learning *Source:* Magnani, Pfortner, Weber, and Hennig (LUNO / NOLA), 2024

Complements linearized Laplace approximations that turn trained neural operators into function-valued Gaussian processes with analytically linearized local Ram operators and covariance propagation. Where the linearized method pushes a Gaussian weight posterior through a first-order model to produce a Gaussian process over output fields, this report makes that posterior local, physical, inverse-aware, and interpretable through Ram Transform, RMNN, and Graph Local Ram Transform methods. It targets structured, resolution-aware uncertainty over predicted functions.

TR132. Graph-Local Ram Transform Reasoning for Relational Bayesian Network Encodings of Graph Neural Networks (26 pp.)

Domain: Graph neural networks *Source:* Pojer, Passerini & Jaeger, PMLR 2024

Develops a Ram Transform response to compiling trained graph neural networks into Relational Bayesian Networks, mapping conditional probability tables, relational probability formulas, edge priors, one-hot constraints, factor messages, and most-probable-explanation search neighborhoods onto local graph kernels. It shows how a discriminative node-classification network becomes a full generative probabilistic model supporting conditional queries, inverse inference from labels to latent attributes, unobserved-structure inference, and model-level explanation, with graph-local message-passing and inference reformulations.

TR133. Ram-Local Bayesian RMNN Uncertainty Propagation for Physics-Informed Material Property Regression (23 pp.)

Domain: Materials informatics *Source:* Li et al., Scientific Reports 2024

Casts Bayesian uncertainty quantification for physics-informed material-property regression in Ram-local terms, using creep rupture life prediction across stainless steel, nickel superalloy, and titanium datasets as the benchmark. It positions Ram Master Neural Networks against standard networks, quantile regression, NGBoost, Gaussian processes, deep ensembles, Monte Carlo dropout, and variational and MCMC Bayesian networks, and shows how physics-informed features and loss terms help most on small datasets while addressing the sampling cost of posterior inference.

TR134. Ram-Local Latent Particle Filtering and Inverse Observation Operators for Real-Time Data Assimilation (20 pp.)

Domain: Data assimilation *Source:* Mucke, Bohte & Oosterlee, Scientific Reports 2024

Connects particle and Kalman filtering to Ram-local state-transition and inverse observation operators for real-time data assimilation with uncertainty quantification. The key observation is that the latent Bayesian prediction step of a deep latent-space particle filter is already a Ram integral operator, which lets the report reframe latent time-stepping and filtering, demonstrated on viscous Burgers dynamics, water-wave generation over a submerged bar, and multiphase pipe-flow leak localization.

TR135. Graph-Local Ram Transform Covariance Operators for Neural-Network Uncertainty Estimation via Factor Graphs (23 pp.)

Domain: Neural network uncertainty *Source:* US Patent Application US20240394506A1

Reformulates post-hoc neural-network uncertainty estimation by replacing factor-graph uncertainty propagation with graph-local Ram forward and inverse covariance operators. Working from a patent and companion paper that convert a trained network into a factor graph and propagate input uncertainty through Jacobian-based factors to target layers, it shows the network computational graph equipped with uncertainty variables is itself a Ram operator, supporting covariance estimation without modifying or retraining the original network and compatible with residual and transformer-like flows.

TR136. Ram Transforms: Recent Application Targets, Volume 2 – Papers and Patents from 2024–2026 for Agentic Ram Transform Follow-Up, with an Agentic Automation Program (47 pp.)

Domain: Ram Transform theory

Catalogs recent 2024 to 2026 papers and patents that present problem classes suited to systematic Ram Transform application, filtered by commercial licensing value and scientific visibility. Coverage spans 6G integrated sensing and communication, Zak-OTFS and AFDM waveforms, PDE surrogates, computational lithography, battery management, quantitative finance, automotive radar, wearable sensing, quantum device control, weather and climate AI, and robotics world models, plus a Riemann Hypothesis section, with keyword-tagged targets and an agentic automation program for follow-up.

TR137. Invertible Delay–Doppler and Chirp-Domain Waveforms for 6G Integrated Sensing and

Communication – Closed-Form Ram Pulse and Waveform Families for Zak-OTFS, AFDM, and Continuous-Aperture ISAC, with Ram–Fisher Pilot Design and Exact Sensing Inversion (37 pp.)

Domain: Wireless / 6G

Unifies four families of 6G integrated sensing and communication waveforms, Zak-OTFS delay-Doppler, AFDM chirp, continuous-aperture, and legacy waveform adaptation, under a single shift-variant Ram integral operator with localized kernel and jet matrix. For each target it derives an explicit forward sensing operator and an explicit inverse communication operator, delivering division-free local equalization in the underspread regime plus Ram-Fisher pilot design and exact sensing inversion, replacing the iterative numerical channel inversion used across the current literature.

TR138. Analytic Localized Inversion of the Hopkins Kernel for Computational Lithography – Training-Free Forward/Inverse Ram Factorization of the Partially-Coherent SOCS Imaging Operator for Curvilinear Inverse Lithography and Source-Mask Optimization (37 pp.)

Domain: Computational lithography

Constructs an analytic, training-free localized forward and inverse factorization of the partially-coherent Hopkins sum-of-coherent-systems imaging operator used in computational lithography, a map that is bilinear in the mask and linear in the source. It charts the aerial image in a local Ram coordinate system, assembles a per-point local Hopkins-Ram tensor from Taylor-jet moment matrices, and gives an explicit two-layer local inverse, offering a closed-form alternative to gradient-descent and learned-surrogate inverse lithography and source-mask optimization.

TR139. Exact Distribution-of-Relaxation-Times Inversion and Certifiable Battery State Estimation – Regularization-Free Analytic Inversion of the Impedance Kernel, Fractional Ram Operators for Constant-Phase Behavior, and Microcontroller-Grade SOC/SOH/Safety Estimators (37 pp.)

Domain: Battery management

Provides a regularization-free analytic inversion of the electrochemical impedance kernel for distribution-of-relaxation-times analysis, exploiting that after a logarithmic change of variable the kernel is shift-invariant and reduces to algebraic division by a closed-form Beta-function spectral symbol on an admissible contour. Constant-phase behavior is handled with a Mittag-Leffler fractional Ram atom, and the results support microcontroller-grade, certifiable state-of-charge, state-of-health, and safety estimators for batteries and fuel cells with error bars the usual Tikhonov methods lack.

TR140. Invertible Physiological Transfer Operators for Wearable Sensing – Exact Noise-Stable Ram Deconvolution for PPG-to-Blood-Pressure and Interstitial-to-Blood-Glucose Inverse Problems, with Provable Error Bounds for Certifiable Devices (34 pp.)

Domain: Wearable sensing

Builds closed-form invertible physiological transfer operators for wearable health sensors, recovering true blood-borne signals from smoothed, delayed, shift-variant tissue-optical or interstitial measurements via exact noise-stable Ram deconvolution. Using a unit-upper-triangular local moment matrix whose unit determinant guarantees an exact inverse, it derives operators including a glucose de-lag operator that generalizes a single-pole correction to arbitrary order and time-varying lag, with provable error bounds suited to regulatory certification for PPG-to-blood-pressure and interstitial-to-glucose problems.

TR141. Analytic Volterra and Feynman–Kac Kernels for Pricing, Calibration, and Market Simulation – Closed-Form Ram Forward/Inverse Operators for Rough-Volatility Calibration, XVA, Multi-Asset Bermudan Hedging, and Arbitrage-Free Volatility Surfaces (33 pp.)

Domain: Quantitative finance

Shows that the two central kernels of contemporary quantitative finance, the rough-volatility Volterra kernel and the Feynman-Kac transition kernel, both belong to the Ram operator class with explicit analytic inverses. The Volterra kernel inverts by fractional differentiation to recover driving noise and roughness in closed form with a spectral roughness estimator, while the Gaussian semigroup inverts by upper-triangular moment back-substitution, enabling auditable, arbitrage-aware rough-volatility calibration, XVA, multi-asset Bermudan hedging, and volatility-surface construction with analytic sensitivities.

TR142. Exact-Inverse Chirp and Multiscale Operators for Automotive Radar and FMCW Lidar – Closed-Form Fractional-Fourier and Ramlet Operators for Interference Mitigation, Chirp-Nonlinearity Correction, 4D Super-Resolution, and Micro-Doppler Inversion (34 pp.)

Domain: Automotive radar

Develops closed-form forward and inverse Ram Transform operators for the four dominant signal-recovery problems of automotive FMCW radar and lidar, all built on a localized shift-variant Ram integral with a unit-determinant jet matrix giving exact division-free reconstruction. It treats mutual chirp interference as an estimable operator eigen-atom subtracted while preserving co-located echo energy, corrects transmit-chirp nonlinearity as an analytically invertible time-warp, and addresses 4D super-resolution and micro-Doppler inversion, positioned against six recent academic and patent targets.

TR143. Analytic Noise-Channel Inversion and Closed-Form Pulse Synthesis for Quantum Computing – Ram Operators for Lindblad Noise-Model Construction, Runtime Error Mitigation, and Closed-Form Entangling-Gate Pulse Design (33 pp.)

Domain: Quantum computing

A closed-form operator theory that jointly tackles quantum noise-channel modeling and entangling-gate pulse design. It derives Pauli-Lindblad error rates as kernel moments of the Liouville generator and, by ordering the Pauli-transfer matrix by weight, obtains an analytic, training-free error-mitigation channel as a terminating Neumann series that needs no channel sampling or learned model. Includes diamond-distance and fidelity bounds, moment-matrix conditioning analysis, noise-robust derivative estimators, and closed-form Molmer-Sorensen gate synthesis.

TR144. Invertible Observation, Closure, and Downscaling Operators for AI Weather Stacks – Closed-Form Ram Forward/Inverse Operators for Data Assimilation, Coarse-Graining, and Generative Downscaling in Machine-Learning Weather and Climate Models (36 pp.)

Domain: Weather/climate modeling

Supplies the missing invertible, certifiable operators that machine-learning weather and climate models lack, addressing the observation, subgrid-closure, and downscaling maps. Using the Ram Transform, it builds a forward derivative-jet operator whose moment matrix is block unit upper triangular with unit determinant and an explicit inverse, enabling analytic data assimilation, a semigroup-consistent coarse-graining law, and downscaling that provably reconstructs the input. Useful for anyone needing bounded-error, physics-consistent alternatives to black-box denoisers and generative samplers.

TR145. Exactly Invertible Tokenizers and Latent Dynamics for World Models and Autonomous-Driving Data Engines – Closed-Form Ram Encoder/Decoder Pairs and Local Forward/Inverse

Latent-Dynamics Operators for Video World Models, Trajectory Prediction, and Auto-Labeling (34 pp.)

Domain: World models / autonomous driving

Provides exactly invertible tokenizers and latent-dynamics operators for video world models and autonomous-driving data engines, replacing the separately trained, lossy decoders and forward-only dynamics used across current systems. It defines a Ram tokenizer whose decode step is the computed operator inverse, requiring no second network and no reconstruction, perceptual, or adversarial losses, plus regularized inverses near ill-conditioned regions. Enables intent inference and auto-labeling that forward-only architectures cannot support.

TR146. Analytic Invertible Surrogates and One-Shot Inverse Design for Industrial Computer-Aided Engineering – Closed-Form Ram Elliptic-Kernel Forward/Inverse Operators for CFD, Structural, and Multiscale-Material Surrogates and One-Shot Shape Inverse Design (33 pp.)

Domain: Computer-aided engineering

Replaces data-hungry, forward-only, opaque learned surrogates in industrial computer-aided engineering with an analytic, training-free, closed-form-invertible Ram-operator apparatus for CFD, structural, and multiscale-material problems. By localizing the elliptic or parabolic generator into a finite derivative-jet operator, it gives the steady solution as a resolvent and the time-dependent solution as an exact matrix-exponential semigroup that satisfies the semigroup identity by construction. Includes boundary-augmented inversion for operator nullspaces plus conditioning and error analysis, and supports one-shot shape inverse design.

TR147. The Ram-Master Transform: A Local Operator Calculus for Shift-Variant Kernels – Finite-Jet Inversion, Composite Operators, Extended Transform Families, and a Hybrid Microlocal Extension (64 pp.)

Domain: Ram Transform theory

Consolidates and extends prior work into a single unified local operator calculus for shift-variant integral, differential, and learned operators. Built on windowed exponential atoms whose Taylor coefficients are local kernel moments, its finite derivative jets admit exact or regularized matrix inversion, and it embeds Fourier, Laplace, Gabor, wavelet, Morlet, chirplet, Mellin, and other transforms as restrictions of one parameter space while giving exact local deblurring filters. Extended layers add fractional, complex-order, hypercomplex, graph-local, mollifier, and PDE-parametrix atoms, plus a hybrid microlocal extension.

TR148. Ram Spectral Operators and the 2024–2026 Zero-Density Landscape: A Ram-Master-Transform Research Program Toward the Riemann Hypothesis (33 pp.)

Domain: Pure mathematics

An honest research-program and survey report that reformulates zeta-function questions in the Ram spectral and microlocal transform framework without claiming to prove the Riemann Hypothesis. It rigorously surveys the 2024-2026 analytic frontier, including zero-density estimates, the Guth-Maynard large-value bound, the Tao-Trudgian-Yang exponent machinery, and moment and random-matrix conjectures. Every conjectural or unverified step is explicitly flagged, cleanly separating proven identities from proposed program directions.

TR149. Ram Transform Methods for Finite-Time Singularity Formation in Three-Dimensional Navier–Stokes and Euler Flows (37 pp.)

Domain: Pure mathematics

A research-program and survey report that reformulates three-dimensional Navier-Stokes and Euler evolution in the Ram Transform frame, without claiming to resolve global regularity or finite-time blowup. It surveys the 2024-2026 singularity-formation landscape, carefully tagging each result by evidentiary status from peer-reviewed and computer-assisted proofs to preprint numerics. It assembles the Stokes Ram eigenoperator, a complex operator-integral treatment of the nonlinearity, a moment-constraint hierarchy, and Borel/Carleman resolution and microlocal tools.

TR150. The Multimodal Ram Transform: Forward and Inverse Operators for Bimodal and L-Modal Shift-Variant Kernels, with Extensions to N-D, Complex, and Ram-Master Settings (27 pp.)

Domain: Ram Transform theory

Extends the single-mode Ram Transform to multimodal shift-variant kernels whose point-spread functions have two or more separated lobes, as arise in multipath propagation, double-exposure imaging, layered defocus, reverberation, and secondary reflections. It derives a verified bimodal forward identity in which each mode contributes its own Ram-coefficient block at its own source center, correcting a kernel re-centering subtlety that otherwise causes double-counting. Includes extensions to L-modal, N-dimensional, complex, and Ram-Master settings.

TR151. The Panel (Domain-Decomposed) Ram Transform for Wide Shift-Variant Kernels: A Block-Tridiagonal Forward and Inverse Operator for Large-Sigma Gaussians, with Extensions to General Kernels and N Dimensions (23 pp.)

Domain: Ram Transform theory

Introduces a panel, or domain-decomposed, Ram Transform for wide shift-variant kernels such as large-scale Gaussians, where a single Taylor expansion would require unstable high-order derivatives of noisy observed signals. By partitioning the wide kernel support into adjacent panels each expanded about its own center, it keeps the jet order low, yielding a centered three-term coupling and a block-tridiagonal inverse solvable by the Thomas algorithm. Includes extensions to general kernels and N dimensions.

Volume 4 — Technical Reports TR152–TR159

TR152. A Unified Domain-Decomposition and Multimodal Framework for the Ram Transform – Panels, Modes, Source-Kernel Parameter Decoupling, Computational Complexity, and the Ram-Master Complex-Spectral Extension (36 pp.)

Domain: Ram Transform theory

Part I of a three-part series that unifies two ways of extending the shift-variant Ram Transform beyond a single expansion center: the multimodal approach (one center per kernel lobe, coupled into a block-bidiagonal chain) and the panel domain-decomposition approach (contiguous tiles coupled into a block-tridiagonal chain). It proves a unification theorem showing both are instances of one block-banded jet calculus built purely from the kernel's incomplete moments, with source-kernel parameter decoupling and a complex-spectral extension. Useful for anyone handling wide or multi-peaked shift-variant convolution operators.

TR153. Multi-Panel Ram Methods for Shift-Variant Deblurring in Two- and Three-Dimensional Imaging and Optics – Aberration Point-Spread Functions, Boundary Handling by Reflection, and Large-Panel-Count Kernels (38 pp.)

Domain: Medical imaging

Part II of the series, applying the unified panel framework to practical two- and three-dimensional shift-variant deblurring where the point-spread function changes across depth, field, or sensor. It builds a per-point block stencil (nine panels in 2-D, twenty-seven in 3-D) from incomplete panel moments, assembles a sparse block-banded forward operator, and inverts it with block Thomas, ADI, or block Gauss-Seidel solvers, including reflection-based boundary handling and large-panel-count kernels. Valuable for restoration of aberration PSFs, motion blur, turbulence, and 3-D microscopy.

TR154. Nonlinear and Fokker-Planck-Ram Extensions of the Unified Multi-Panel and Multimodal Ram Framework – Domain-Decomposed Nonlinear Integral Equations, Kramers-Moyal Moment Identification, and Mean-Field (McKean-Vlasov) Density Evolution (33 pp.)

Domain: Ram Transform theory

The final part of the series, extending the unified multi-panel and multimodal jet calculus from linear to nonlinear integral operators, where the kernel depends on the unknown function itself (General, Urysohn, and Hammerstein equations). It shows the block-banded machinery carries over once the per-panel forward map is linearized, and adds Fokker-Planck-Ram density evolution, Kramers-Moyal moment identification, and mean-field McKean-Vlasov dynamics. Of interest to those solving nonlinear shift-variant integral equations and stochastic density-evolution problems.

TR155. Kernel Identification and Calibration for Multi-Panel and Multi-Modal Ram Transforms – Supervised, Semi-Blind, and Inverse-Consistency Estimation of Per-Panel Incomplete Moments, with Cross-Center Persistent Excitation and Mode-Center Recovery (41 pp.)

Domain: Image restoration

Extends single-center kernel identification and calibration methods to the multi-panel and multimodal block-banded forward map, exploiting the fact that per-panel incomplete moments and source center-jets are disjoint so the problem splits into a linear calibration stage and a linear restoration stage. It develops supervised per-panel moment calibration as a stacked block least-squares problem, semi-blind inverse-consistency elimination, joint mode-center and moment recovery, and a cross-center persistent-excitation condition governing identifiability. Useful for practitioners needing to estimate an unknown shift-variant blur or kernel from data.

TR156. Multi-Panel and Multi-Modal Ram Operators for Linear Partial Differential Equations – Block-Banded PDE-Jet Systems from Coefficient Jets, Boundary Conditions as Reflection Closures, and Complex-Node Quasi-Trefftz Solvers for Oscillatory Equations (36 pp.)

Domain: Numerical PDEs

Part I of a two-part series that ports the structured panel and mode calculus to linear partial differential equations, representing the solution by Taylor jets about several panel or mode centers. The PDE's local action, built from coefficient jets via a Leibniz identity, couples the observed field jet to the field jets at each center through a block-banded system closed by boundary conditions expressed as reflection rows and solved by block Thomas. It adds a complex-node quasi-Trefftz variant for oscillatory equations, improving on single-interval PDE-to-Ram solvers.

TR157. Nonlinear and Inverse Multi-Panel Ram Operators for Partial Differential Equations – Block-Newton Solvers, Fokker-Planck-Ram Evolution, and Cross-Center Identification of Spatially-Varying PDE Coefficients (43 pp.)

Domain: Numerical PDEs

Part II of the PDE series, extending the direct PDE-to-Ram method to nonlinear equations and to coefficient identification within the multi-panel framework. Nonlinear PDEs yield block-banded nonlinear algebraic systems (via Cauchy products and Faa di Bruno on jets) solved by block-Newton or Gauss-Newton iterations whose Jacobian keeps the band sparsity, so each step is a block-Thomas solve. Standout results include Fokker-Planck-Ram evolution and cross-center persistent-excitation identification of spatially varying PDE coefficients, useful for nonlinear modeling and inverse problems.

TR158. Calibration and Identification of Shift-Variant Coefficients of Linear Partial Differential Equations by Ram-Operator Methods – Single-Panel, Multi-Panel, and Multi-Modal Supervised, Semi-Blind, and Inverse-Consistency Estimation under a Cross-Center Persistent-Excitation Condition (45 pp.)

Domain: Inverse problems

Recasts the inverse problem of identifying shift-variant coefficient functions of a linear PDE from observed field data as a calibration process, using the fact that the local Ram PDE-jet identity is bilinear: linear in the solution jet when coefficients are known, and linear in the coefficient jets when the solution is known. Writing coefficients in a linearly parameterized model turns calibration into a well-posed linear regression whose design matrix comes entirely from known solution derivative jets. It develops single-panel, multi-panel, and multimodal supervised, semi-blind, and inverse-consistency estimation under a cross-center persistent-excitation condition.

TR159. Calibration and Identification of Coefficients of Nonlinear Partial Differential Equations, and the Unification of Kernel and Differential-Operator Calibration – Newton-Ram Estimation, Joint Coefficient-Solution Recovery, Fokker-Planck Drift-Diffusion Identification, and Panel-Local Model Discovery (45 pp.)

Domain: Inverse problems

Part II on PDE coefficient calibration, covering nonlinear PDEs and unifying kernel and differential-operator calibration. It distinguishes three nonlinearity types by their calibration structure: equations nonlinear in the solution but linear in the unknown coefficients (such as Burgers with unknown viscosity) remain a linear least squares with a Faa di Bruno design, while other cases require Newton-Ram estimation and joint coefficient-solution recovery. It also treats Fokker-Planck drift-diffusion identification and panel-local model discovery, valuable for data-driven modeling of nonlinear PDE systems.

Volume 5 — Technical Reports TR160–TR180

TR160. Ram Calculus for Single-Robot Task Planning: Theory, Algorithms, Examples, and an Assessment (17 pp.)

Domain: Robotics

Assesses how the local operator calculus of Ram Calculus applies to single-robot task planning framed as a graph problem, drawing on graph-local, Bayesian-network, factor-graph, world-model, and optimal-control operators. It works through grid and costmap navigation and belief-space planning-as-inference examples, treats factor graphs in depth, and delivers algorithms, a benchmarking plan, and a candid pros-and-cons assessment showing where the approach genuinely helps, such as policy evaluation, entropy-regularized planning, uncertainty propagation, fast replanning, and interpretable learned planners.

TR161. Ram Calculus for Power System State Estimation (13 pp.)

Domain: Power systems *Source:* Ngo et al., arXiv:2312.17738

Recasts power system state estimation as a graph-operator inverse problem, replacing a learned graph-neural-network diffusion with the exact physics operator and an analytic, division-free, explicitly invertible graph-local Ram operator pair. It proves that on radial distribution feeders the Ram inverse is exact and runs in linear time with no training, that topology changes perturb only a local neighborhood without retraining, and that each bus carries a certifiable estimate, offering an alternative to the referenced physics-informed GNN benchmark.

TR162. Ram Calculus for Context Graphs: LLM Reasoning and Codebase Intelligence (Combined, Web-Updated) (14 pp.)

Domain: LLM reasoning / RAG *Source:* LinearRAG

Evaluates whether the graph-local theory and tools of Ram Calculus benefit context graphs that supply reasoning-ready context to language models and agents, covering context selection, provenance, and verifiable reasoning, plus codebase retrieval, change-impact analysis, refactoring, and verification. Combining and updating two prior reports with fresh web research, it finds a close fit, showing that recent linear-system reformulations of graph retrieval match the closed-form linear inverse the Ram operator computes, and generalizes it with locality, adjoints, incremental updates, interpretability, and confidence measures.

TR163. Ram Calculus for In-Situ Quantum Hamiltonian Learning (12 pp.)

Domain: Quantum computing *Source:* Liu, Wu and Niu, arXiv:2510.07818

Treats in-situ quantum Hamiltonian learning by recognizing that the per-block coefficients-to-signal map is an oscillatory-response kernel equivalent to a two-by-two Ram-Sine invariant subspace and a short-time propagator-to-Hamiltonian moment map. It replaces the referenced iterative Cramer-Rao-saturating estimator with an analytic, localized, explicitly invertible forward and inverse operator pair that recovers each local Hamiltonian patch in closed form, offering a training-free route to the same coefficient recovery.

TR164. Verifiable, Watermarked, Authorized-Use QA over the Ram Calculus Corpus (8 pp.)

Domain: Verifiable QA

Details, at the equation level, two technical cores that distinguish a Ram-Calculus-grounded question-answering assistant from a generic retrieval or fine-tuned chatbot: a derivation verifier that checks answers against operator definitions and self-consistency properties to produce a calibrated verification score with quantified false-accept and false-reject behavior, and a provenance watermarking and tamper-evident logging scheme with keyed, license-identifying signals and a hash-chained audit log. It supplies detection tests, false-positive bounds, and algorithms, and states plainly what these mechanisms do and do not guarantee.

TR165. The Analytic Ram-Operator Core plus Learned Residual Architecture (8 pp.)

Domain: Scientific machine learning

Develops generically, at the equation level, a recurring architecture that pairs an exact, local, analytically invertible Ram-operator core enforcing a problem's governing structure with a learned residual that absorbs only what the core misses. It proves a residual-smallness result explaining the architecture's data efficiency, gives a total-error decomposition with a certifiable core-truncation term plus a learned term, shows how invertibility, conservation constraints, and uncertainty propagation are preserved, and supplies seven algorithms spanning hybrid inversion, training, dynamics, uncertainty quantification, and domain transfer.

TR166. The Discrete Shift-Variant Ram Transform (10 pp.)

Domain: Ram Transform theory

Develops the previously unexplored discrete shift-variant Ram transform, completing the family alongside the discrete shift-invariant and continuous shift-variant theories. By localizing the shift-variant kernel, defining position-dependent discrete factorial moments, and expanding the source via Newton's forward-difference formula, it derives a closed-form discrete shift-variant forward operator and a local difference-jet matrix system, proving the coefficient matrix is unit-upper-triangular with unit determinant under a moment-variation truncation.

TR167. The Ram Sampling Theorem for Known Shift-Variant Channels (15 pp.)

Domain: Wireless / 6G

Assesses whether the Ram Transform can make a precise statement about recovering a signal from a known shift-variant channel, something the classical Nyquist-Shannon theorem cannot do because it relies on Fourier diagonalization that fails for time-varying filtering. Using a local operator, it delivers exact recovery under a local-polynomial assumption plus an explicit error bound for analytic signals, and judges the claim substantially correct subject to three qualifications concerning known channel moments, frozen-coefficient truncation, and the polynomial basis-expansion model as the closest classical analog.

TR168. Agentic Automation Program for Ram Transform Research: Updated for the Results of TR137–TR167 (21 pp.)

Domain: Research automation

Updates the agentic research automation program of TR136 to account for the thirty-one reports produced since, folding in new Ram Transform method classes including panel and multimodal transforms, nonlinear and Fokker-Planck extensions, kernel and PDE-coefficient calibration, and the discrete shift-variant transform. It revises the topic-to-method assignment rules with three new problem corners, expands the retrieval corpus and screening to deduplicate against solved targets, adds equation-level self-checks to verification, and enforces umbrella-first patent filing order.

TR169. Graph-Local Ram-Calculus Dynamic Power-System State Estimation (10 pp.)

Domain: Power systems

Extends the graph-local Ram-Calculus formulation of static power-system state estimation to dynamic estimation, dynamic estimation with unknown inputs, topology-aware estimation with unobservable buses, and continual multi-grid reconfiguration. This fully detailed edition derives each operator step by step, separating the exact tree-elimination inverse from the truncated graph-local Neumann inverse on meshed grids, derives the electromechanical swing as an exact two-by-two Ram-Sine block with a damped correction, and adds five worked numerical examples with computed results.

TR170. Near-Field Wideband ISAC and Blind Delay-Doppler Channel ID via Ram Calculus (8 pp.)

Domain: Wireless / 6G

Extends the Ram-Calculus ISAC framework beyond its far-field, known-channel assumptions in two verified directions. For near-field 6G ISAC it derives the Fresnel aperture phase from first principles, showing the first aperture difference yields angle and the second yields range in closed form, proves a near-field local-recoverability theorem, and extends it to wideband. For blind and semi-blind delay-Doppler channel identification it casts joint signal-and-channel recovery as a bilinear Ram problem with an alternating estimator, proves identifiability up to the blind gauge, and includes full derivations and worked numerical examples.

TR171. Nonlinear Dynamical-Response and Coordinate-Conditioned Shift-Variant Ram Inversion (7 pp.)

Domain: Quantum device characterization

Tackles two hard operator-inversion problems: recovering higher-order nonlinear response and susceptibility kernels of a driven quantum system from its measured time evolution via a multilinear local-jet Volterra expansion, and per-pixel closed-form deconvolution when the point-spread function varies smoothly with field position in computational imaging. Delivers forward and inverse operators, using an analytic calibrated coordinate map rather than a learned network for the position-dependent case. Extends prior linear and shift-invariant work into the nonlinear and spatially varying regimes.

TR172. Analytic Invertible Ram Kernels for Yield-Curve Construction and Rough-Volatility Calibration (6 pp.)

Domain: Quantitative finance

Joins analytic pricing-kernel and PDE-coefficient calibration methods to build two regulator-auditable finance engines. It constructs a no-arbitrage yield curve in closed form as an analytic operator calibrated to bond and swap prices under the Heath-Jarrow-Merton drift condition, and calibrates rough-volatility models with jumps by analytically inverting the fractional Volterra kernel to recover the Hurst exponent, jump intensity, and term structure. Both target markets that demand auditable, arbitrage-consistent, locally exact results.

TR173. Ram-Calculus FMCW Radar/Lidar Receiver Operators for Nonlinear Chirp and Interference (6 pp.)

Domain: Radar / Lidar

Extends closed-form radar and lidar beat-domain receiver inversion to two real-world impairments: nonlinear transmit chirp from laser or oscillator tuning, and mutual interference from other radars. It reconstructs the instantaneous-frequency warp analytically to render a nonlinear chirp locally linear with no interpolation-kernel error, and excises interferers by a single division in a fractional transform domain, an invertible removal rather than crude magnitude zeroing. Includes forward and inverse operators, algorithms, and an experiment plan against established receiver baselines.

TR174. Contactless and Multi-Frequency Physiological Inverse Operators via Ram Calculus (6 pp.)

Domain: Medical sensing

Extends analytic, invertible physiological sensing operators beyond contact optics into two new front-ends: contactless millimeter-wave reconstruction of the arterial pressure waveform from sub-millimeter skin displacement, treated as a shift-variant electromagnetic and motion operator, and multi-frequency thoracic bioimpedance inversion for compartmental fluid and relaxation-time distributions. It derives forward and inverse operators with closed-form recovery and error bounds, plus algorithms and experiment plans, emphasizing the certifiability that medical-device licensees value.

TR175. Analytic Green's-Function Operator Surrogates via Panel Ram Calculus (6 pp.)

Domain: Scientific computing *Source:* Yoo et al., NeurIPS 2025, arXiv:2511.01924

Provides an analytic alternative to learned Green's-function surrogates for linear PDEs, representing the Green's function as a panel-local jet whose moments are the local inverse of the PDE operator, with boundary conditions via reflection folding and irregular geometry via coordinate mapping. Coefficients are calibrated once and a small learned residual absorbs the remainder. This expanded edition proves polynomial-exactness and the discrete Green's-function identity and includes worked one-dimensional Poisson examples reaching machine precision and demonstrating second-order convergence.

TR176. Ram-Calculus Contact-Rich Robot Dynamics Inversion (6 pp.)

Domain: Robotics *Source:* Xu et al., CoRL 2025, arXiv:2508.15755

Frames contact-rich robot dynamics, the regime where manipulators and legged robots make and break contact, as a natural operator-inversion problem, opening robotics as a new application domain. Within a smooth contact mode the forward map from torque to acceleration and its inverse share local moments recovered exactly for polynomial trajectories, while contact events are handled as a shift-variant impact map. This expanded edition derives the mode-local operators and impact map with energy interpretation and includes worked numerical examples reaching near machine precision.

TR177. Invertible Ram-Calculus Closure Operators for the Atmospheric Boundary Layer (5 pp.)

Domain: Atmospheric science

Gives an analytic, invertible version of the atmospheric-boundary-layer eddy-diffusivity mass-flux closure that recent work casts as a learnable convolution kernel. The turbulent flux is represented by a shift-variant operator whose local moments are the eddy diffusivity and counter-gradient mass-flux terms, with a stability-dependent coordinate map, calibrated once from large-eddy-simulation data. The closure is invertible between flux and gradient and consistent with coarse-graining, and includes a closure-recoverability theorem, algorithms, and an experiment plan.

TR178. Ram-Calculus Inverse Calibration of Rough-Heston-with-Jumps Volatility (6 pp.)

Domain: Quantitative finance

A dedicated, fully detailed method for inverse calibration of rough-Heston volatility with jumps. The model's characteristic function solves a fractional Riccati equation that is diagonalized in the fractional transform domain, so the Hurst exponent, forward variance curve, jump intensity, and size moments are read off directly, with a learned residual absorbing market-idiosyncratic remainder and a two-panel construction jointly calibrating the VIX and SPX surfaces. Includes forward and inverse operators, a joint identifiability theorem, a block-Newton iteration for the jump nonlinearity, and detailed error analysis.

TR179. A Ram-Calculus Solution of Fredholm-Neural-Network Problems for Forward and Inverse Elliptic PDEs (16 pp.)

Domain: Numerical PDEs *Source:* Georgiou, Siettos, Yannacopoulos, arXiv:2507.06038

Reformulates and solves the forward and inverse elliptic PDE problems handled by explainable Fredholm neural networks, whose weights are computed rather than trained to realize a fixed-point iteration for a second-kind integral equation. The key insight is that the closed-form, division-free local inverse the network approximates by iteration already exists analytically as a finite nilpotent Neumann series from a unit-upper-triangular jet matrix. Draws systematically on the full report corpus to cover Poisson, Helmholtz, semi-linear, and inverse-source problems.

TR180. Ram-Calculus Approaches to the Open Problems of TR179 (12 pp.)

Domain: Numerical PDEs

Explores the five open problems left by the preceding report on Fredholm-neural-network-style elliptic PDE solving, offering concrete constructions, proposed theorems, and an honest status for each. It brings automatic curved-geometry panelization close to resolution by importing harmonic-map, conformal, and NURBS-enhanced boundary parameterizations into an automatic panelization pipeline, and defines a unified boundary-integral operator as the boundary trace of the panel Green's operator, the Dirichlet-to-Neumann map. Draws on both the report corpus and current 2023 to 2026 research.

Volume 6 — Technical Reports TR181–TR203

TR181. A Ram-Calculus Solution of the Precursor Fredholm-Neural-Network Problems (11 pp.)

Domain: Ram Transform theory *Source:* Georgiou, Siettos & Yannacopoulos, Fredholm Neural Networks, arXiv:2408.09484

A companion to two prior reports, this work applies Ram Calculus to three problem classes from a precursor Fredholm Neural Networks paper that were not previously covered: ordinary-differential-equation two-point boundary-value problems recast as Green's-function Fredholm integral equations, standalone linear and nonlinear Hammerstein-type integral equations, and the inverse problem of identifying an unknown integral kernel from input-output data. Each solution is derived from a systematic re-examination of the corpus and verified numerically, with the ODE Green's-function kernel shown to be the Ram panel inverse.

TR182. Inversion of General Variable-Coefficient Differential Operators with Polynomial Coefficients (17 pp.)

Domain: Applied mathematics

This report develops a local inversion method for general linear variable-coefficient differential relations whose coefficients are arbitrary polynomials, generalizing the one-dimensional Ram Transform. It shows the forward map from the local jet of the unknown function to that of the observed data is a tall, lower-banded matrix that is invertible by a finite, division-free back-substitution using high-order derivatives from the top downward, never inverting the operator as a whole. Explicit entry formulas and exact recovery for polynomial inputs are proven.

TR183. Local Distributed Parameterization, Compactification, and the Extension of Ram Kernel-Calibration Theory to General Polynomial-Coefficient Operators (10 pp.)

Domain: Ram Transform theory

Building on general operator inversion and kernel-calibration theory, this first part of a two-part series articulates the unifying idea that smoothness lets a function be parameterized by its local derivative jet rather than global interval data, with the operator's support compactified by expanding about the coefficient centroid. It extends four operating modes (forward, inverse, supervised calibration, and semi-blind elimination) to general polynomial-coefficient operators, showing that supervised calibration of the coefficients is a linear least-squares problem governed by an explicit persistent-excitation condition.

TR184. Ram-Master Transform of the General Operator Equation and a Roadmap of Corpus Extensions (8 pp.)

Domain: Ram Transform theory

The second part of a two-part series applies the Ram-Master Transform, a single atom combining a window, a wavelet factor, and an oscillatory or decaying exponential, to both sides of a general polynomial-coefficient operator equation. Because the transform is linear, integration by parts moves derivatives off the noisy data onto smooth test atoms and the exponential factor turns the operator into an algebraic symbol, yielding robust multiscale calibration and inversion. It also lays out a prioritized roadmap of extensions enabled across the full corpus.

TR185. Ram-Calculus Core Ideas Applied to Commonly-Taught Differential Equations (16 pp.)

Domain: Applied mathematics

This report distills the core ideas of Ram Calculus and applies them to differential equations central to undergraduate and graduate curricula, adding the technique of substituting truncated Taylor series for non-polynomial terms so that transcendental nonlinearities fit the polynomial-coefficient framework. Worked examples include the nonlinear pendulum, Bratu and Liouville-Gelfand boundary-value problems, heat, transport, and wave equations, Sturm-Liouville and time-independent Schrodinger eigenvalue problems, and the Euler-Bernoulli beam, each with explicit local-jet formulations and textbook-method comparisons.

TR186. Ram Calculus for In-Context World Modeling in Robotic Control (18 pp.)

Domain: Robotics *Source:* Wang et al., In-Context World Modeling for Robotic Control (ICWM), 2026

This report reformulates a 2026 In-Context World Modeling paper for robotic control through Ram Calculus, showing that its test-time self-probing phase, in which a robot executes random safe motions to infer its configuration, is exactly a Ram supervised-calibration problem for a local sensorimotor operator. It verifies the original claims against the source paper and gives a corrected, expanded derivation with an explicit least-squares estimator for the sensorimotor Jacobian. Probing diversity is identified as a persistent-excitation requirement.

TR187. The Ram Calculus of Graphs and Networks Applied to Reasoning, Planning, and Searching (13 pp.)

Domain: Graph algorithms

This report shows that the Ram Calculus of graphs and networks does not replace shortest-path solvers like A-star or Dijkstra but supplies a closed-form, graph-local, auditable operator layer for the linear, smooth, inferential, and incremental substrates around search. Verified examples include tree and DAG cost-to-go by finite inverse, entropy-regularized soft values as admissible and consistent heuristics, topology-local replanning, Bayesian-chain inference, and multi-query amortization, plus new results on Sherman-Morrison edge-edit updates and explainable search saliency traces.

TR188. Ram Calculus of Graphs and Networks: Scope, Status, and Potential (Whitepaper) (17 pp.)

Domain: Graph algorithms

This whitepaper introduces Ram Calculus of graphs and networks as a focused branch of the wider program, treating graph computation as a family of local forward, inverse, adjoint, update, and sensitivity operators on nodes, edges, motifs, paths, and temporal states. It maps concrete opportunities across shortest paths, planning, Bayesian and factor graphs, graph neural networks, power grids, wireless and financial networks, and more, offering exact finite inverses on trees and DAGs, certified soft heuristics, low-rank topology updates, and glass-box sidecars for learned systems. It honestly marks what is verified versus aspirational.

TR189. Ram Calculus for Robot Control When Self-Generated ICWM Probing Is Not Available (18 pp.)

Domain: Robotics *Source:* Wang et al., In-Context World Modeling for Robotic Control (ICWM)

This report addresses the complementary regime to In-Context World Modeling: robot-control settings where a self-generated probing history is unavailable, unsafe, too slow, too costly, or forbidden, such as surgical and micro-robotic manipulation, on-orbit servicing, hazardous environments, high-speed cells, one-shot fragile grasping, and contact-rich tasks. It accepts the diagnosis that a policy still needs the system configuration but supplies Ram-Calculus alternatives to active probing for recovering the local sensorimotor map without disturbing the initial state.

TR190. The Graph-Global Ram Transform (GGRT): A Master Transform on Graphs, with a Restriction Theorem Unifying Multiresolution Graph Transforms (7 pp.)

Domain: Graph signal processing

This report defines the Graph-Global Ram Transform, a single parameterized coefficient map built on the graph-local Ram operator using a spectral master atom of window, wavelet, and complex-exponential factors applied to a graph operator's eigenvalues via functional calculus. Its central result is a restriction theorem proving that the Graph Fourier Transform, heat and diffusion operators, the Spectral Graph Wavelet Transform, windowed graph Fourier, ChebNet and GCN polynomial filters, and Kron reduction are all exact instances, unifying multiresolution graph transforms under one master transform.

TR191. The Graph-Global Ram Transform (GGRT): Applications, Windowing/Wavelet/Oscillatory Kernels, and Extensions (Part II) (6 pp.)

Domain: Graph signal processing

This companion to the Graph-Global Ram Transform develops its practical uses, giving recipes for each of the three atom factors, spectral windowing, wavelet and frame design, and the decaying-oscillatory complex shift, with example settings. It catalogs applications across graph signal processing and graph neural networks, including pooling, denoising, community detection, meshes, molecules, brain and traffic networks, and PDE-on-graphs, defines an extended transform family with fractional-Laplacian, mollified, learned, and directed-magnetic variants, and shows how it enables a genuine directional graph transform.

TR192. A Unified Multi-Modal, Multi-Panel, and Double-Domain-Decomposition Ram Calculus (17 pp.)

Domain: Ram Transform theory

This report adds a third decomposition axis to the Ram Transform framework, Double-Domain Decomposition, which tiles the output domain into cells and computes the full Ram inverse only at each cell center, propagating interior values and derivative jets by Taylor expansion. Combined with the existing multi-modal decomposition for separated kernel structure and multi-panel decomposition for wide support, it yields three composable axes that cut cost when the measured output and kernel parameters vary slowly and a controlled approximation error is acceptable.

TR193. Ram Calculus Methods for Single-Phase Microgrid Fault Location (16 pp.)

Domain: Power systems *Source:* Duan, Zhang & Cheng, A Novel Method of Fault Location for Single-Phase Microgrids

Re-reads the Duan-Zhang-Cheng single-phase microgrid fault-location method through the Ram

Calculus corpus, showing that its linear feature model (peak transient oscillation magnitude versus sensor-to-fault distance) is the first-order case of a graph-local Ram inverse family. Reframes the least-squares distance solve, RMSE candidate ranking, and post-fault coefficient updates as a weighted graph-local inverse with uncertainty, posterior scoring, and online calibration, adding multi-path structure and a physics-based kernel model.

TR194. Ram Calculus for Single-Phase Microgrid Fault Location: A Transparent, Physics-Grounded, and Identifiability-Aware Re-derivation and Extension (11 pp.)

Domain: Power systems *Source:* Duan, Zhang & Cheng, A Novel Method of Fault Location for Single-Phase Microgrids

A transparent, physics-grounded re-derivation of single-phase microgrid fault location that recovers the transient pole and residue, and hence line resistance and electrical distance, as explicit algebraic functions of measured signal derivatives, replacing the empirical peak-magnitude linear fit. It honestly reports the main gains as improved transparency, identifiability analysis, graph structure, and certified uncertainty rather than lower localization error, and uses a Ram-Sine invariant subspace to decode damped transients into decay and frequency pairs.

TR195. Ram Calculus Methods for the Resolution Limit of Single-Photon LiDAR (20 pp.)

Domain: Medical imaging *Source:* Chan et al., CVPR 2024, Resolution Limit of Single-Photon LiDAR

Reads the CVPR 2024 single-photon LiDAR resolution-limit paper through the Ram Calculus corpus, showing its analysis is already close to a local-kernel calculation: the delay-field linearization is a first-order jet, the boxcar-to-Gaussian pixel replacement is moment matching, the effective pulse width is a second-moment statement, and the maximum-likelihood variance is a Fisher-information computation. Confirms rather than overturns the original result while supplying a broader, more transparent operator-design view of the bias-variance resolution trade-off.

TR196. Ram Calculus for the Resolution Limit of Single-Photon LiDAR: A Moment-Exact Rederivation, Jet-Order Reconstruction Upgrades, and a Second Observable in the Timestamp Width (11 pp.)

Domain: Medical imaging *Source:* Chan et al., CVPR 2024, Resolution Limit of Single-Photon LiDAR

A moment-exact re-derivation of the single-photon LiDAR resolution trade-off, showing the U-shaped mean-squared-error curve is Ram-shaped end to end, with the effective per-pixel pulse variance computed exactly for boxcar pixels so the Gaussian approximation becomes unnecessary. Its headline contribution is a jet-order reconstruction upgrade that moves beyond the degree-one delay-map model, plus identification of the effective kernel moment rule with the classical depth-from-defocus blur-variance relation and a second observable in the timestamp width.

TR197. Ram Calculus for Solid-State DC Transformers (21 pp.)

Domain: Power electronics

Applies Ram Calculus to solid-state DC transformers, arguing that ideal switching waveforms are exactly piecewise-polynomial panel jets, so the Ram machinery that only approximates generic smooth signals becomes exact here. Delivers a transparent, controllable calculus for transferred power, soft-switching boundaries, RMS and peak currents, core loss, EMI spectra, and control response, replacing truncated-Fourier tools such as fundamental-harmonic approximation and dynamic phasors, and extends to calibration, persistent excitation, graph-local operation, and certified uncertainty.

TR198 (Part I). Ram Calculus for Trajectory Optimization, Part I: The Jet-Flow Correspondence, Exact Variational Sensitivities, and Certified Coarse-Step Accuracy for DDP and the iLQR (13 pp.)

Domain: Robotics *Source:* Li & Todorov (2004); Tassa, Mansard & Todorov (2014)

Establishes the jet-flow correspondence for trajectory optimization, treating the one-step map as the local jet solution operator of the dynamics to obtain a high-order discrete map, its exact first- and second-order variational sensitivities, and a certified truncation budget from a single construction. Demonstrates coarse-step bias removal for Differential Dynamic Programming and the iterative Linear-Quadratic Regulator, giving certified accuracy at larger step sizes without the usual finite-difference derivative overhead.

TR198 (Part II). Ram Calculus for Trajectory Optimization, Part II: Beyond-Quadratic Value Models, Structured and Graph-Local Backward Passes, Muscle-Model Jet Surrogates, Control Limits and Contact Events, Uncertainty Propagation, and Comparative Assessment (11 pp.)

Domain: Robotics *Source:* Li & Todorov (2004); Tassa, Mansard & Todorov (2014)

Extends the jet-flow trajectory-optimization calculus with a beyond-quadratic value model from a third-order jet that cuts cost-to-go approximation error by over seven times in genuinely non-quadratic regions, and a structured backward pass identified with a block-bidiagonal panel inverse that matches dense Newton solves at far lower cost. Adds machine-precision muscle-model jet surrogates, control-limit and contact-event handling, uncertainty propagation, and a comparative assessment, pointing toward graph-local linear-time scaling for high-degree-of-freedom systems.

TR199. Ram Calculus Across the Hierarchy: From Real and Discrete Signals through Ram-Master Features to Graphs, Networks, and Intelligent Systems (Whitepaper 2) (16 pp.)

Domain: Ram Transform theory

A whitepaper surveying Ram Calculus as a local jet and moment operator calculus that turns one governing equation into a finite system of same-point derivative relations and inverts it algebraically by back-substitution on a unit-upper-triangular matrix without dividing by an operator. It traces the program across raw signals in real, complex, quaternionic, and octonionic values, feature representations, graphs and networks, and intelligent-system reasoning and planning, and verifies and corrects eight specific claims about activation functions such as tanh, erf, Gaussian, and GELU.

TR200. Ram Calculus Across Signal, Feature, Symbol, Graph, and Agent Hierarchies (Whitepaper 3) (23 pp.)

Domain: Ram Transform theory

A whitepaper assessing the reach of Ram Calculus from low-level signals to symbolic, graph, planning, and agent representations, positioning it as a coherent algebraic language for forward and inverse operators, finite jets, windowed and multiscale coefficients, and hybrid neural-operator architectures. It relates the framework to physics-informed networks, neural operators, transformers, graph neural networks, and world models, while deliberately correcting overstatements about repeated differentiation and about the special properties of the tanh activation.

TR201. Vectorial Hopkins-Ram Local Inverse for High-NA EUV STCC with Polarization and Mask-3D Effects (9 pp.)

Domain: Computational lithography

A draft derivation extending the scalar Hopkins-Ram bilinear local inverse to the vectorial, thick-mask transmission cross-coefficient functional governing aerial-image formation in high-numerical-aperture EUV lithography. It localizes the polarization-resolved imaging functional into matrix-valued unit-upper-triangular moment matrices, derives an explicit local block inverse recovering the effective mask near-field, handles mask-3D depth coupling and field-position shift-variance by multi-panel Thomas coupling, and states an exactness class with a stated truncation error bound plus a learned-residual calibration wrapper.

TR202. RamResReps.com: A Detailed Plan for an Automated Ram-Calculus Research-Report Pipeline and Storefront (14 pp.)

Domain: Business / operations

A detailed, costed business plan for an automated pipeline and storefront that finds recently published papers and patents, applies Ram Calculus to each using the corpus as retrieval-augmented reference material, and lists the resulting reports for sale with source citations. It specifies two editions per item, a first draft and an expert-reviewed revision, and estimates per-draft compute cost, thousand-draft budgets under different model and caching configurations, and wall-clock production timelines for the requested output rate.

TR203. Ram Calculus for iLQR, iLQG, and Control-Limited DDP Trajectory Optimization (19 pp.)

Domain: Robotics *Source:* Li & Todorov (2004); Tassa, Mansard & Todorov (2014)

Investigates two trajectory-optimization papers behind a public C++/Eigen iLQR implementation, identifying where Ram Calculus adds modeling, derivative, uncertainty, local-inverse, multi-panel, and double-domain-decomposition tools to the same workflow without replacing the methods. A reproducible pendulum experiment shows analytic Ram-style finite jets reproduce finite-difference Jacobians to numerical precision while cutting derivative time about sixteen-fold and end-to-end iLQR wall time about twofold, and gain-compression tests map when panel spacing helps or fails.