

B.Tech 3rd Semester Exam., 2020
(New Course)

MATHEMATICS—III

(Differential Calculus)

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.

1. Choose the correct answer of the following
(any seven) : $2 \times 7 = 14$

(a) The value of $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x}$ is

(i) 0

(ii) 1

(iii) e

(iv) $1/e$

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(Turn Over)

(b) Let $f(x) = |x|$ and $g(x) = |x^3|$, then

(i) $f(x)$ and $g(x)$ both are continuous at $x = 0$

(ii) $f(x)$ and $g(x)$ both are differentiable at $x = 0$

(iii) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$

(iv) $f(x)$ and $g(x)$ both are not differentiable at $x = 0$

(c) The value of $\nabla^2 [(1-x)(1-2x)]$ is equal to

(i) 2

(ii) 3

(iii) 4

(iv) 6

(d) If $v = xy^2\hat{i} - 2x^2yz\hat{j} - 3yz^2\hat{k}$, then the value of $\text{curl } v$ at $(1, -1, 1)$ is equal to

(i) $-\hat{j} - 2\hat{k}$

(ii) $\hat{i} + 3\hat{k}$

(iii) $-\hat{i} + 2\hat{k}$

(iv) $\hat{i} + 2\hat{j} - \hat{k}$

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(Continued)

(e) The degree of the differential equation

$$y \frac{dx}{dy} + \left(\frac{dx}{dy} \right)^2 + \sin y \left(\frac{dx}{dy} \right)^3 - \cos x = 0$$

is

- (i) 0
(ii) 1
(iii) 2

~ (iv) Cannot be determined

(f) The solution of the boundary value problem

$$(x - y^2)x dx + (x^2y - y)dy = 0, y(0) = 0$$

is

- (i) $x^2 - y^2 = 0$
(ii) $2x - y = 0$
(iii) $x - 2y = 0$

(iv) None of the above

(g) Let $P_n(x)$ be the Legendre polynomial of degree $n \geq 0$. If

$$\int_{-1}^1 P_n^2(x) dx = \frac{2}{(2n-1)}$$

then the value of (k, l) is

- (i) (1, 1) (ii) (1, 2)
(iii) (2, 1) (iv) (2, 2)

(h) The general solution of Bessel differential equation

$$x^2 y''(x) + xy'(x) + (x^2 - 64)y(x) = 0$$

is

(i) $y = AJ_8(x) + BJ_{-8}(x)$, where A and B are arbitrary constants

(ii) $y = AJ_8(x) + BY_{-8}(x)$, where A and B are arbitrary constants

(iii) $y = AJ_8(x) + J_{-8}(x)$, where A is arbitrary constant

(iv) $y = J_{3/4}(x) + Y_{3/4}(x)$

(i) The equation $p \tan y + q \tan x = \sec^2 z$ is of order

- ✓ (i) 1
(ii) 2
(iii) 0

(iv) None of the above

(j) The solution of $p \tan x + q \tan y = \tan z$ is

- ✓ (i) $\sin x / \sin y = \phi(\sin y / \sin z)$
(ii) $\sin x \cdot \sin y = \phi(\sin y / \sin z)$
(iii) $\sin x / \sin y = \phi(\sin y, \sin z)$
(iv) $\sin x / \sin y = \phi(\sin y \cdot \sin z)$

2. (a) If $y = (\sin^{-1} x)^2$, then show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

Hence find $(y_n)_0$.

(b) Find the value of

$$\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$$

3. (a) Discuss the continuity of the following function $f(x, y)$ at point $(0, 0)$:

$$f(x, y) = \begin{cases} \frac{\sin \sqrt{|xy|} - \sqrt{|xy|}}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$$

(b) For the function

$$f(x, y) = \begin{cases} \frac{xy(2x^2 + 3y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$$

check whether $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ are equal or not.

4. (a) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

(b) Obtain the second-order Taylor's series approximation to the function

$$f(x, y) = xy^2 + y \cos(x - y)$$

about the point $(1, 1)$.

5. (a) If $f = (x^2 + y^2 + z^2)^{-n}$, then find $\text{div grad } f$ and determine n , if $\text{div grad } f = 0$. <https://www.akubihar.com>

(b) Verify Green's theorem for

$$\int_C [(xy + y^2) dx + x^2 dy]$$

where C is bounded by $y = x$, $y = x^2$.

6. (a) Find the value of n for which the vector $r^n \mathbf{r}$ is solenoidal, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(b) Solve the differential equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$$

7. Solve the following differential equations :

$$7+7=14$$

(a) $p = \sin(y - xp)$. Also find its singular solution.

$$(b) \quad x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x \log x$$

8. (a) Prove that

$$2nJ_n(x) = x(J_{n-1}(x) + J_{n+1}(x)) \quad 6$$

(b) Prove that

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} P_n(1) = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right) \quad 8$$

9. Solve the following differential equations :

$$7+7=14$$

$$(a) \quad x^2 p + y^2 q = (x + y)z$$

$$(b) \quad (x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$$

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