Code : 100311

B.Tech 3rd Semester Exam., 2021
(New Course)

MATHEMATICS—III

(Differential Calculus)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.
(ii) There are ***NINE*** *questions in this paper.*
(iii) Attempt ***FIVE*** *questions in all.*
(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) The value of $\lim_{x\to 0} (\cos x)^{1/x^2}$ is

(i) 0 (ii) $\sqrt{e}$
(iii) $1/\sqrt{e}$ (iv) 1

(b) The function $f(x) = \sin^{-1}(\cos x)$ is

(i) discontinuous at $x = 0$
(ii) continuous at $x = 0$
(iii) differentiable at $x = 0$
(iv) None of the above

(c) If $\mathbf{v}$ is conservative force field, then the value of curl $\mathbf{v}$ is

(i) 0
(ii) 1
(iii) $\nabla\mathbf{v}$
(iv) -1

(d) If $f = 2x^2 - 3y^2 + 4z^2$, then curl (grad f) is equal to

(i) $4x - 6y + 8z$
(ii) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$
(iii) 0
(iv) 2

(e) $y = cx - x^2$ is the general solution of the differential equation

(i) $(y')^2 - xy' + y = 0$
(ii) $y'' = 0$
(iii) $y' = c$
(iv) $(y')^2 + xy' + y = 0$

(f) The solution of the boundary value problem $(x^2 - y^2)dx + 2xydy = 0,\ y(1) = 0$ is

(i) $x^2 - y^2 + x = 0$

(ii) $x^2 + y^2 - x = 0$

(iii) $x^2 + y^2 + x = 0$

(iv) None of the above

(g) Let $P_n(x)$ be the Legendre polynomial of degree $n \geq 0$. Then $\int_{-1}^{1} P_{n-2}(x)dx = 2$, if n is

(i) 0

(ii) 1

(iii) 2

(iv) None of the above

(h) The general solution of Bessel differential equation

$$xy''(x) + y'(x) + xy(x) = 0$$

is

(i) $y = AJ_n(x) + BJ_{-n}(x)$, where A and B are arbitrary constants

(ii) $y = AJ_0(x) + BY_0(x)$, where A and B are arbitrary constants

(iii) $y = AJ_n(x) + BY_{-n}(x)$, where A and B are arbitrary constants

(iv) None of the above

(i) The equation $x^2 z \frac{\partial z}{\partial x} + y^2 z \frac{\partial z}{\partial y} = x + y$ is

(i) linear

(ii) non-linear

(iii) quasi-linear

(iv) semi-linear

(j) The solution of $(y - z)p + (z - x)q = x - y$ is

(i) $xyz = \varphi(x^2 + y^2 + z^2)$

(ii) $xyz = \varphi(x + y + z)$

(iii) $x^2 + y^2 + z^2 = \varphi(x + y + z)$

(iv) $\varphi(x^2 + y^2 + z^2,\ xyz) = 0$

2. (a) If $y = \cos(m \sin^{-1} x)$, then show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

Hence find $(y_n)_0$. 7

(b) Find the value of $\lim_{n \to \infty} (4^n + 5^n)^{1/n}$. 7

3. (a) Discuss the continuity of the function $f(x, y)$ at the given point

$$f(x, y) = \begin{cases} \dfrac{x^2 + y^2}{\tan xy}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

at $(0, 0)$. 7

(b) Compute $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ for the function

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Also discuss the continuity of f_{xy} and f_{yx} at (0, 0). 7

4. (a) Use the method of Lagrange multipliers to maximize x^3y^5 subject to the constraint $x + y = 8$. 7

(b) Expand $f(x, y) = \sqrt{x+y}$ in Taylor's series up to second-order term about the point (1, 3). Estimate the maximum absolute error in the region

$$|x-1| < 0\cdot 2, \ |y-3| < 0\cdot 1$$ 7

5. (a) Find div(grad $\mathbf{r}^n$), where

$$\mathbf{r}^2 = x^2 + y^2 + z^2$$ 6

(b) Show that

$$\iint_S \mathbf{F} \cdot \hat{n} dS = \frac{3}{2}$$

where $\mathbf{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 8

6. (a) Find the directional derivative of $x^2y^2z^2$ at the point (1, 1, −1) in the direction of the tangent to the curve $x = e^t$, $y = 1 + \sin 2t$, $z = 1 - \cos t$ at $t = 0$. 7

(b) Solve the differential equation

$$\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$$ 7

7. Solve the following differential equations : 7+7=14

(a) $y = 2px + y^2p^3$

(b) $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$

8. (a) Prove that

$$(x^2 - 1)P_n'(x) = (n+1)[P_{n+1}(x) - xP_n(x)]$$ 7

(b) Prove that

$$\frac{d}{dx}\left[\frac{J_{-n}(x)}{J_n(x)}\right] = \frac{-2\sin n\pi}{x\pi J_n^2(x)}$$ 7

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8. Solve the following differential equations : 7+7=14

(a) $zp + yq = x$

(b) $(p^2 + q^2)y = qz$

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