The goal is to prove that we have a quasifibration
$\mathcal{B}(\mathcal{S}'\mathcal{S}) \to \mathcal{B}(\mathcal{S}'(\mathcal{S}'\mathcal{A})) \to \mathcal{B}(\langle \mathcal{S}, \mathcal{A} \rangle)$
and using this, show that if B((S.Z)) is contractible, then
$B(S'S) = B(S'S) + S_{T} = 2$
Recall: Given a monoidal category & acting on & we define the orbit
Category (8,2):
-0-3
Map(A) = Ma
Acts
C = C = C = C = C = C = C = C = C = C =
$4: 4: A + F \rightarrow G$ morphism in \mathcal{F}
L3 (A, y) ~ (A', y') if there exists a: A → A' in S s. f.
$A+F \xrightarrow{\alpha+\mu\mu} A^2+F$
$\varphi = \frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$
G
We then further defined the localisation S'It to be (3, 3x)
We then further defined the <u>localisation</u> S'X to be (3, 3×X). Where the action of 3 on 8xX is given by
We then further defined the <u>localisation</u> $S'\mathcal{X}$ to be $(S, S \times \mathcal{X})$ where the action of S on $S \times \mathcal{X}$ is given by $S \times S \times \mathcal{X} \longrightarrow S \times \mathcal{X}$
We then further defined the <u>localisation</u> $S'X$ to be $\langle S, S \times X \rangle$ where the action of S on $S \times X$ is given by $S \times S \times X \longrightarrow S \times X$ $(a, b, c) \longrightarrow (a \square b, a + c)$ Saction of S on X
We then further defined the <u>localisation</u> S'it to be (S, Sxit) Where the action of S on Sxit is given by SxSxit
We then further defined the <u>localisation</u> 3 it to be (3, 3, xit) where the action of 3 on 3xit is given by 3x3xit
We then further defined the <u>localisation</u> S'X to be (S, S, X) Where the action of S on SxX is given by SxSxX = SxX (a,b,c) > (allb, a+c) Monoided Structure As last time we will need some assumption: <u>Assumptions</u> :
G We then further defined the <u>localisation</u> S'it to be (S, Sxit) where the action of S on Sxit is given by SxSxit — Six (a,b,c) ~ (allo, a+c) nonoided As last time we will need some assumption: <u>Assumptions</u> : • S=is
G We then for these defined the <u>localisation</u> S'it to be (S, S, XE) Where the action of S on Sxit is given by SxSxit — Sxit (a, b, c) - (allo, a+c) (a, b, c) - (allo, a+c) nonvoided etudore As last time we will need some assumption: <u>Assumptions</u> : · S=is
G We then further defined the <u>localisation</u> S'it to be (S, Sxit) where the action of S on Sxit is given by SxSxit — Six (a,b,c) → (allo, a+c) Norvoided tructore As last time we will need some assumption): <u>Assermptions</u> • S=iS • VBtS: S(A, R) → S(A+B, R+B)
G We then for these defined the <u>localisation</u> S'it to be (S, S×it) where the action of S on Sxit is given by S×S×it (a,b, c) → Sxit (a,b, c) → Sint (a,b, c) → Sint As last time we will need some assumption: <u>As last time we will need some assumption</u> : <u>As is is (A,P) → SiA+B, P+B)</u> a → aridy
G We then further defined the <u>localisation</u> S ⁻ it to be <3, 5×it Where the action of S on Sxit is given by SxSxit → Sxit (a,b,c) → (allo, a+c) Mennoided As last time we will need some assumption: <u>Aosumptions</u> · S=iS · VB(S: S(A,R) → S(A+B, R+B) a → axide · sinjective

· All morphisms in & are monomorphisms
• $\forall F \in \mathcal{F}$ $S(A,A') \longrightarrow \mathcal{F}(A+F,A'+F)$
$a + id_F$
is injective
Construction: Recall that we last time showed that
$P: g^{-} {\mathcal{X}} \longrightarrow \overset{\langle g, g \rangle}{\longrightarrow} B$ $(B, F) \longmapsto B$
is cofibered. For BEODS we get that p'(B) is the collection of
pairs (B,F) in g-'Je, hence for all FEZ ~ So we identify p'(B)
with \mathcal{X} . Let $\varphi: B_o \longrightarrow B_t$
be a morphism in (S.S) represented by A+Bo > B1. Then
the cobase change functor (1. P'(B0) -> P'(B,) becomes the
translation
$T_{A}:\mathcal{Y}\longrightarrow\mathcal{Y}$
$F \longrightarrow A * F$
We are the above of the is instability of the and A tobil T
We say the action of 5 on a 13 whether it for each it coust, 1A
induces a homotopy quivalence on classifying spaces.
induces a homotopy equivalence on classifying spaces. Ex: The action of 3 on 3-12t given by
induces a homotopy equivalence on classifying spaces. Ex: The action of 3 on 3-12t given by +: 3x 8-12 -> 3-2
induces a homotopy equivalence on classifying spaces. Ex: The action of $3 \text{ on } 3^{-1} \text{ It given by}$ $+: 3 \times 3^{-1} \times \longrightarrow 3^{-1} \times$ $(A_1(B_1)) \longmapsto (B_1A_1E)$
induces a homotopy equivalence on classifying spaces. Ex: The action of S on $S^{-1}X$ given by $+: S_{\times}S^{-1}X \longrightarrow S^{-2}X$ $(A, (B, F)) \longmapsto (B, A+F)$ is invertible: To see this compare the two functors
induces a homotopy equivalence on classifying spaces. Ex: The action of S on $S' > t$ given by $+: S \times S' \times \longrightarrow S' \times t$ $(A \cdot (B_1 \neq)) \longmapsto (B, A + \neq)$ is invertible: To see this compare the two functors $T_A: S' \xrightarrow{*} \longrightarrow S' \xrightarrow{*} S_A : S' \xrightarrow{*} S \xrightarrow{*} \times (B, \neq) \longmapsto (B, A + \neq)$
induces a homotopy equivalence on classifying spaces. Ex: The action of $S on S^{-1} \Sigma t$ given by $+: S_{\times} S^{-1} \Sigma \longrightarrow S^{-1} \Sigma t$ $(A \cdot (B_{1} \mp)) \longmapsto (B, A + \mp)$ is invertible: To see this compare the two functors $T_{A}: S^{-1} \Xi \longrightarrow S^{-1} \Xi \qquad S_{A}: S^{-1} \Sigma \longrightarrow (B, F) \longmapsto (A + B, F)$. We see that they commute,

• • • •	and for each AEB we have a natural transformation
	id=> Spota
• • • •	given by the morphism
• • • •	$[A_i:d_i:d]; (B,F) \longrightarrow (A+B, A+F)$
• • • •	in 8-22. Then follows that it induces a honrotopy equivalence
Cor	on classifiquer sprieg (2.4). The functor
N.,1+, 	$T: \mathfrak{X} \longrightarrow \mathfrak{Z}^{-1}\mathfrak{X}$ $F \longmapsto (0, F)$
• • • •	induces a homotopy equivalence on classifying spaces iff g acts
• • • •	invertibly on H.
Pe:	"E" If 3 ads invertibly on 26 term by assumption we know
• • • •	that the colocie change functor
• • • •	$\Psi_{*}: \mathcal{P}'(\mathcal{B}_{0}) \rightarrow \mathcal{P}'(\mathcal{B}_{1})$
• • • •	identified with the translation functur TA: X->X
• • • •	induces a homotopy equivalence on classifying spaces. So it
• • • •	satisfies the assumptions of Quillen's theorem B for coffiberal
• • • •	functors, lience we get that
• • • •	$\mathcal{B}(p^{-1}(\mathcal{B}_0)) \to \mathcal{B}(\mathcal{G}^{-1}\mathcal{F}) \xrightarrow{\mathcal{B}_p} \mathcal{B}(\langle \mathcal{G}, \mathcal{G} \rangle)$
• • • •	is a quasifibration, with B(p-1(Bol) "B(X) Since (8.5)
• • • •	admits the mitical object O from the monoidal structure on S
	we get that B((S,S)) * ~> B(S'X) ~ B(p'(0)) & B(X),
• • • •	Rence Tinduces a Romotopy equivalence.
• • • •	"=>" If T: X - 5' X induces a homotopy equivalence, then the
• • • •	action of 3 on 26 is invertible since by the above example

we know this is the case for S'X;
$\mathfrak{X} \xrightarrow{\tau} \mathfrak{Z} \mathfrak{Z}$
$T_{A} \downarrow \qquad \forall \mid T_{A} \qquad \qquad$
$\mathcal{F}_{\mathcal{F}} = \mathcal{F}_{\mathcal{F}} = $
We now turn our focus to how & acts on give so we can understand
how the localisction of
$\begin{array}{c} \mathbf{q} : \mathbf{S}^{-1} : \mathbf{\mathcal{X}} \longrightarrow \mathbf{\mathcal{S}} : \mathbf{\mathcal{X}} \\ (\mathbf{B}, \mathbf{F}) \longmapsto \mathbf{F} \end{array}$
looks like We let I act on S'X by acting on the first component,
i.e. $+: S_{x} S'_{x} \longrightarrow S'_{x}$ (A, (B,F)) $\longmapsto (A+B,F)$
Using that this action is Riberwise with respect to q, we can extend
q to a functor
$\hat{q}: f'(S' \times) \longrightarrow (f \times)$
$(A, (B,F)) \rightarrow \mathcal{P}(B,F) = F.$
(A, (B, F)) → q(B, F)=F. To describe what q̂ does on morphisms we first need to understand
(A, (B,F)) → q(B,F)=F. To describe what q̂ does on morphisms we first need to understand how morphisms in S ⁻¹ (S ⁻¹)×) looks like. Note that
$(A, (B, F)) \rightarrow q(B, F) = F.$ To describe what \hat{q} does on morphisms we first need to understand how morphisms in $S^{-1}(S^{-1}X)$ looks like. Note that $S^{-1}(S^{-1}X) = (S, \langle S, S_X X \rangle)$
$(A, (B, F)) \longrightarrow \mathcal{Q}(B, F) = F.$ To describe what $\hat{\mathcal{Q}}$ does an morphisms we first need to understand how morphisms in $\mathcal{G}'(\mathcal{G}' \times)$ looks like. Note that $\mathcal{G}'(\mathcal{G}' \times) = (\mathcal{G}, \langle \mathcal{S}, \mathcal{S} \times \times \rangle)$ so let $\mathcal{Q}:(A, (B, F)) \longrightarrow (A', (B', F'))$
$(A, (B, F)) \mapsto q(B, F) = F.$ To describe what \hat{q} does a morphisms we first need to understand how morphisms in $S'(S'X)$ looks like. Note that $S''(S'X) = (S, \langle S, S_XX \rangle)$ so let $4:(A,(B,F)) \rightarrow (A'_{i}(B',F'))$ be a morphism in $S'(S'X)$. Then ψ is an equivalence class
(A, (B, F)) \mapsto $\mathcal{P}(B, F) = F$. To describe what $\hat{\mathcal{Q}}$ does an morphisms we first need to understand how morphisms in $\mathcal{S}'(\mathcal{S}' \times)$ looks like. Note that $\mathcal{S}'(\mathcal{S}' \times) = (\mathcal{S}, \langle \mathcal{S}, \mathcal{S} \times \times \rangle)$ so let $\mathcal{Q}: (\mathcal{A}, (\mathcal{B}, F)) \rightarrow (\mathcal{A}, (\mathcal{B}, F'))$ be a morphism in $\mathcal{S}'(\mathcal{S}' \times)$. Then \mathcal{Q} is an equivalence class of a 5-tuple $[\mathcal{V}, \mathcal{W}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3]$ with
$(A, (B, F)) \mapsto q(B, F) = F.$ To describe what \hat{q} does on morphisms we first need to understand how morphisms in $\mathcal{B}^{*}(\mathcal{B}^{-1}\mathfrak{X})$ looks like. Note that $\mathcal{B}^{*}(\mathcal{B}^{-1}\mathfrak{X}) = (\mathcal{B}, \langle \mathcal{B}, \mathcal{B}\mathfrak{X}\mathfrak{X}\rangle)$ so let $q:(A,(B,F)) \rightarrow (A',(B',F'))$ be a morphism in $\mathcal{B}^{*}(\mathcal{B}^{*}\mathfrak{X})$. Then q is an equivalence class of a 5-tople $[V,W_{3},Q_{1},Q_{2},Q_{3}]$ with $V,W\in obS$
$(A, (B, F)) \mapsto q(B, F) = F.$ To describe what \hat{q} does an morphisms we first need to understand how morphisms in $S^{-1}(S^{-1}X)$ looks like. Note that $S^{-1}(S^{-1}X) = (S, \langle S, S_X X \rangle \rangle$ so let $q \cdot (A, (B, F)) \rightarrow (A'_{1}(B'_{1}F'))$ be a morphism in $S^{-1}(S^{-1}X)$. Then q is an equivalence class of a 5-tople $[V_{1}W_{3}q_{1}, q_{2}, q_{3}]$ with $V_{1}We obS$ $q_{1}: V+A \rightarrow A^{2}$
$(A, (B, F)) \mapsto q(B, F) = F.$ To describe what \hat{q} does a morphisms we first need to understand how morphisms in $S^{*}(S^{*}x)$ looks like. Note that $S^{*}(S^{*}x) = (S, \langle S, S, x \rangle \rangle$ so let $q:(A,(B,F)) \rightarrow (A',(B',F'))$ be a morphism in $S^{*}(S^{*}\chi)$. Then q is an equivalence class of a 5-tople $[V,W_{3}q_{1},q_{2},q_{3}]$ with V,We obS $q_{1}: V+A \rightarrow A^{2}$ $q_{2}: W+U+B \rightarrow B^{2}$ Tn J
$(A, (B, F)) \mapsto q(B, F) = F.$ To describe what \hat{q} does on morphisms we first need to understand frow morphisms in $\mathcal{G}^{-1}(\mathcal{G}^{-1}\mathcal{X})$ looks like. Note that $\mathcal{G}^{-1}(\mathcal{G}^{-1}\mathcal{X}) = \langle \mathcal{G}, \langle \mathcal{G}, \mathcal{G} \mathcal{X} \mathcal{X} \rangle \rangle$ so let $q:(A,(B,F)) \rightarrow (A^{2}_{1}(B^{2},F^{2}))$ be a morphism in $\mathcal{G}^{-1}(\mathcal{G}^{-1}\mathcal{X})$. Then q is an equivalence class of a 5-tople $[V_{1}W_{3}V_{1}, V_{2}, V_{3}]$ with $V_{1}W_{2} obs$ $q_{1}: V+A \rightarrow A^{2}$ $q_{2}: W+V+B \rightarrow B^{2}$ $Th \mathcal{X}$

$\widehat{\mathfrak{q}}\left(\left[\mathcal{V},\mathcal{W},\varphi_{1},\varphi_{2},\varphi_{3}\right]\right)=\left[\mathcal{W},\varphi_{3}\right]$
Thus: The localization
$\hat{q}: \mathcal{J}^{-1}(\mathcal{J}^{-1}\mathfrak{X}) \longrightarrow (\mathcal{S}, \mathfrak{X})$
of the functor q, induces a questifibration
$\mathcal{B}(\mathfrak{Z}'\mathfrak{Z}) \rightarrow \mathcal{B}(\mathfrak{Z}'(\mathfrak{Z}'\mathfrak{X})) \xrightarrow{\mathcal{B}(\mathfrak{G})} \mathcal{B}(\langle \mathfrak{Z}, \mathfrak{X} \rangle)$
Pf: We wish to apply Quillen's theorem B for colibered functors, hence
We need to show
1) q is colibered
2 Every coliber lift of q induces a homotopy equivalence
③ q-1(F) ≥ 3-'3 for all FEZ
<u>Pf 1</u> : Given a morphism
$[w, \varphi_3]: \hat{q}(A, (B, F)) \rightarrow F^{2}$
we have universal lift
$\widehat{\mathbf{r}}_{\mathbf{r}} = \widehat{\mathbf{r}}_{\mathbf{r}} + \mathbf{$
$[w, \psi_3] = [o, w, id_A, id_{W+B}, \psi_3] \cdot (H_1(B, F)) = (H_1(w_FB, F))$
which is just the Universal lift of
g (B,F)=F→F° represented by q2: W+F→F°
(B,F) - (W2+B,F') represented by
$(A+B,A+F) \xrightarrow{(id, y_2)} (A+B,F^2)$
Hence q is carfibered.
Pf of 2: $(\hat{q}'(F) \cong 3'3)$ The objects of $\hat{q}'(F)$ is of the form
(A, (B, F)) with A, BEODS. A Morphism
$\mathfrak{P}:(A,(\mathcal{B},F)) \to (A^2,(\mathcal{B}^2,F))$
in g-'(F) is represented by
$(V, 0, 9_1, 9_2, id_F)$

• • • •	which corresponds directly to
• • • •	$(V, \varphi_1, \varphi_2)$ $(A, B) \longrightarrow (A, B)$
• • • •	in 3-18.
• • • •	Pfolz: Assume weare given a morphism F-F in (3,2)
• • • •	represented by q3: W+F ->F? We then have that the
• • • •	cofiber lift
• • • •	$\hat{\varphi}_{3}:\hat{q}^{-1}(F)\rightarrow\hat{q}^{-1}(F')$
	is given by Twi 5-13 - 5-19
	$(A,B) \rightarrow (A,W+B)$
• • • •	Life and Rugha for our corrier provale induces a
• • • •	P l'an animale service en anima en l'anter service en anima en l'anter service en anima en la service en anima en
• • • •	nomotopy equivalence with homotopy muche manced by
• • • •	
<u>Thm:</u> 11.19	If the category (3.2) has contractible classifying space, then
<u>Thm:</u> 11.19	If the category $\langle 3, 2 \rangle$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'JE$
<u>Thm:</u> 11.19	If the category $(3,2)$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'X$ $(A,B) \longrightarrow (A,B+F)$
<u>Thm:</u> 11.19	If the category $(3,2)$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'X$ $(A,B) \mapsto (A,B+F)$ induces a Promotopy equivalence on classifying spaces for all First.
<u>Thm:</u> 11.19 PE:	If the category (S,Z) has contractible classifying space, then the functor $F: S'S \longrightarrow S'Z$ $(A,B) \mapsto (A,B+F)$ induces a Romotopy equivalence on classifying spaces for all First. Consider the following (non-commototive!) square
<u>Thm:</u> 11.19 PE:	If the category $\langle S, d \rangle$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'Y$ $(A, B) \longrightarrow (A, B+F)$ induces a Promotopy equivalence on classifying spaces for all FeSt. Consider the following (non-commotative!) square $(A,B) \longrightarrow (A, CB,F)$
<u>Thm:</u> 11.19 <u>PE:</u>	If the category $\langle S, Z \rangle$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'Z$ $(A, B) \mapsto (A, B+F)$ induces a Romotopy equivalence on classifying spaces for all Fett. Consider the following (non-commotorize!) square $(A,B) \longrightarrow (A, CB,F)$ $(B,A) S'S \longrightarrow S'(S'Z) (O_1(B,G))$ 1
<u>Thin:</u> 11.19 <u>PE:</u>	If the category $\langle S, Z \rangle$ has contractible classifying space, then the functor $i_{F}: S^{-1}S \longrightarrow S^{-1}X$ $(A, B) \mapsto (A, B+F)$ induces a homotopy equivalence on classifying spaces for all Fest. Consider the following (non-commototive!) square $(A, B) \longmapsto (A, CB, F)$ $(B, A) S^{-1}S \longrightarrow S^{-1}(S^{-1}X) (O_{1}(B, G))$ $T \qquad T$
<u>Thin:</u> 11.19 <u>PE:</u>	If the category $\langle 3, d \rangle$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'Y$ $(A, B) \mapsto (A, B+F)$ induces a Romotopy equivalence on classifying spaces for all Fett. Consider the following (non-commototive!) square $(A,B) \longrightarrow (A, (B,F))$ $(B,A) S'S \longrightarrow S'(S'X) (O, (B, 6))$ $T \qquad 1$ $(A,B) S'S \longrightarrow S'X$ (B,G)
<u>Thin:</u> 11.19 PE:	If the category $\langle 3,3 \rangle$ has contractible classifying space, then the functor $i_{F}: g^{*}g \longrightarrow g^{-}\chi$ $(A,B) \mapsto (A,B+F)$ induces a Biomotopy equivalence on classifying spaces for all Fe2t. Consider the following (non-commotodime!) square $(A,B) \longrightarrow (A,(B,F))$ $(B,A), g^{*}g \longrightarrow g^{*}(g^{*}\chi)$ $(O_{1}(B,G))$ $\left[\begin{array}{c} Y \\ Y \\ (A,B) \end{array}\right] \xrightarrow{i_{F}} g^{*}(g^{*}\chi)$ $(O_{1}(B,G))$ $(A,B), g^{*}g \longrightarrow g^{-}\chi$ (B,G) $(A,B) \longmapsto (A,B+F)$
<u>Thin:</u> 11.19 PE:	If the cotegory $\langle S, Z \rangle$ has contractible classifying space, then the functor $F: S'S \longrightarrow S'\mathcal{X}$ $(A, B) \mapsto (A, B+F)$ induces a Romotopy equivalence on classifying spaces for all Fest. Consider the following (non-commototive!) square $(A, B) \longmapsto (A, (B, F))$ $(B, A) S'S \longrightarrow S'(S'\mathcal{X}) (O_i(B, 6))$ $f = \int T \int I$ $(A, B) S'S \longrightarrow S'2 (B, G)$ $(A, B) \stackrel{i_F}{\longrightarrow} S(A, B+F)$ $(A, B) \stackrel{i_F}{\longrightarrow} S(F, F)$
Pe:	If the category $\langle S, I \rangle$ has contractible classifying space, then the functor $i_{F}: S'S \longrightarrow S'X$ $(A, B) \mapsto (A, B+F)$ induces a Romotopy equivalence on classifying spaces for all Fext. Consider the following (non-commototive!) square $(A, B) \longrightarrow (A, (B, F))$ $(B, A) S'S \longrightarrow S'(S'X) (O, (B, 6))$ $I \longrightarrow (A, B) \longrightarrow (A, B+F)$ $(A, B) S'S \longrightarrow S'X (B, G)$ $(A, B) \longrightarrow (A, B+F)$ $(A, B) \longmapsto B(J_{F}\circ X) \cong B(T \circ i_{F})$

•	•	••••	$T \circ i_F(A,B) = (0, (A,B+F))$
•	•	• •	$j_F \cdot Y(A,B) = (B, (A,F))$
•	•	•••	We introduce a third functor
•	•	•••	K _F : 5 ⁻ '3 → 5 ⁻ '(5 ⁻ '3)
•	•	•••	$(A,B) \mapsto (B,(B+A,B+F)),$
•	•	•••	and see that we have natural transformations
•	•	•••	(0, B, id, id): j= 08=>k= To i= (B, 0, id, id, id)
•	•	• •	which implies that $B(j_F \circ \delta) \cong B(T \circ i_F)$.
•	•	• •	Claim 2: jr induces a homotopy equivalence on classifying spaces
•	•	•••	Pf: We have assumed B((S, E)) = * So by 11.18 we have
•	•	• •	a grasifibration
•	•	• •	$B(J^{-}J) \xrightarrow{B(J^{-})} B(J^{-}(J^{-}\mathcal{F})) \longrightarrow \mathcal{H}$
•	•	• •	Claim 3: 8 induces a Romotopy equivalence on classifying spaces
•	•	• •	PP: Voy=id so in particular id => 208
•	•	• •	Claim 4: T induces a homotopy equivalence on classifying spaces
•	•	•••	PP: From example 11.16 we get that I acts invertibly on
•	•	• •	S'zt, so the desired follows by 1117
•	•	• •	So it follows that Blie) is a Promotopy equivalence.
•	•	• •	
•	•	•••	
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