CAAD ETH Zurich – ATTP TU Vienna

Digital Gnomonics

Architectonics of Mathematical Thinking from the Stance of Natural Philosophy

1. Gnomon – The Beginning of Mathematics and Natural Philosophy

- Obstacle and Inaccessibility - Genesis of Theoria and Theorema
- Adjoining a Gnomon to the Directly Inaccessible
- The Symplectic Method
- Architectonics: Circumnavigation and Communicability
- Measurement and Innovation – Cartography and Calendrics
- Inversion and the Method of Shadows w.r.t. a Gnomon
- Information w.r.t. a Gnomon
- Potentiation and Closure
- Homeothesis, Homeostasis, and Homeorhesis
- Analogy and Metaphor
- Logical Conjugation and Metabole
- Stratification into Layers and Hierarchy
- Crystallization and Weaving
- Involution and Negation
- Gnomonic Replication and Gnomonic Growth

2. Gnomonic Foundations of Arithmetic

- On the Notion of Number – Arithmos and Conceptual Aphairesis
- Numbers as Multitudes and Magnitudes w.r.t. a Unit
- Unification: Numbers as Powers w.r.t. a Gnomon
- Arithmetic Systems via Logical Conjugation and Closure
- The Integer – Rational – Irrational – Real – Imaginary Cosmos
- Incommensurability and the Notion of Approximation
- Arithmetic Atoms – Prime Numbers as Gnomons
- Encoding/Decoding Bridges and the Law of Unique Factorization
- Congruence – Relative Primes – Modularity
- Polynomials and Roots – Law of Solvability w.r.t. a Gnomon
- Power and Transcendental Closure – Digital Arithmetic Gnomon
- Exponentiation and Logarithmization

3. Gnomonic Foundations of Geometry

- On the Notion of Geometric Topos as Space of Similitude
- Idempotence and the Law of Projectivity
- Conic Sections and Cross-Ratio Invariance
- Harmonic and Anharmonic Cross-Ratio
- Parallelism and the Law of Affinity
- Fulcrum and Gnomonic Linearization
- Linear Topoi and Linear Transformations
- Gnomonic Bases and the Law of Orthogonality – Geometric Eigenforms
- Metric Distance – Euclidean Gnomonic Topoi
- Angle and Area – Archimedean Gnomonic Topoi
- Variation - Least Action Principle and Geodesic Law w.r.t. a Gnomon
- Conjugation and the Atlas Principle
- The threefold classification of Geometric Gnomonic Topoi
- Intrinsic Curvature – Digital Geometric Gnomon
- Conjugation and Conformal Gnomonic Topoi
- Conjugation and Symplectic Gnomonic Topoi
- Hyperbolic Compression and Logarithmic Distance

4. Gnomonic Foundations of Algebra

- Operation and Action
- On the Notion of Algebraic Structure – Operational Closure
- Inversion and Group Structure – Symmetry and Action
- Neutral Actions and Algebraic Gnomonics
- Fixed Elements of Actions and Transitivity
- Indistinguishability and Ambiguity - Equivalence Class w.r.t. a Gnomon
- Gnomonic Orbits, Fibers, and Cells – Relativization and Quotients
- Algebraic Powers and Non-Commutativity
- Homomorphism – Partition Kernel and Image – Isomorphism
- Conjugation and Division – Ideals and Irreducibility
- Varieties and Representability of Structure
- Conjugation and Commutator Action
- Algebraic Powers and Potentiation – Observable Spectrum of Structure
- Scalarity and Modularity – Rings, Fields (Körper), Modules and Algebras
- Dilation and Contraction of Scalarity – Adjunctions
- Adjoint Functors and Natural Transformations – Categorical Structure
- Free Categorical Structure – Digital Algebraic Gnomon
- Universality of Free Structure – Generators and Relations

5. Gnomonic Foundations of Topology

- Form and Shape
- Elasticity and Plasticity
- Non-Metric Dependence and Relation via Connectivity
- Connectivity and Boundary – Digital Topological Gnomon
- Local/Global Distinction and Topological Modularity
- Coverings – Folding and Unfolding
- Cellular Decompositions and Connectivity of Meshes
- Order and Continuity – Lattices and Inverse Images
- Local and Global Homeomorphisms
- Genus and Topological Invariance
- Simple and Multiple Connectivity – Obstacles and Obstructions
- Localized Identity, Amalgamation, and Sheaves of Germs
- Covering Topoi and Connectivity of Germs
- Twisting and Winding - Deformation of Based Loops and Homotopy
- Bounding in Covering Topoi – Homology
- Knotting and Linking Loops – Topological Bonds
- Branching, Connectivity Trees and Dessins

6. Gnomonic Foundations of Analysis

- Infinitesimals and Differential Forms
- Infinitesimal Expansion and Contraction
- Limits and Convergence
- Real and Complex Differentiation
- Differential Invariants and Differential Equations
- Cocycle and Coboundary – Digital Analytic Gnomon
- Spectral Resolution – Spectral Sequence
- Covariant Differentiation and Holonomy
- Integration of Differential Forms
- Conjugation of Analytic Forms and Cohomology Classes
- Obstructions and Differential Invariants
- Modularity of Cohomology – Sheaf Cohomology
- Holomorphic Forms and Conformal Transformation
- Complex Potential Theory - Conjugate Harmonic Forms

7. **Gnomonic Foundations of Logic**

- Formal Logical Structures and Set Theory
- Syntax and Semantics
- Cardinality and Ordinality
- Boolean Logic – From Sets to Disconnected Topological Spaces
- First-Order Logic and Skolem's Paradox
- Goedelian Incompleteness and Undecidability
- Models and Types
- Axiom of Choice and Banach-Tarski Paradox
- Independence of the Continuum Hypothesis
- Cohen's Method of Forcing – Digital Logical Gnomon
- Topoi and Local Set Theories – Boolean-Valued Models
- Classification in a Logical Topos – Subobject Classifier
- Logical Conjugation - Localization of Truth and Sheaves
- From Classical Logic to Quantum Logic – Encoding and Decoding