$\boxed{\text{Ex } 198}$

<u>Injectivité</u> $f(x,y) = f(x',y') \Rightarrow (x,y) = (x',y')$

$f(x,y) = x^2 - \pi y^2$ $\quad f : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$

$x^2 - \pi y^2 = x'^2 - \pi y'^2$

$\underbrace{x^2 - x'^2}_{\in \mathbb{Z}} = \pi \underbrace{(y^2 - y'^2)}_{\mathbb{Z}} \in \pi \mathbb{Z}$

donc $x^2 - x'^2 = 0$ donc $\boxed{x = x'}$ $(x, x' > 0)$

et $\boxed{y = y'}$

Ainsi $f(x,y) = x^2 - \pi y^2$ comment