Ex 1451 Ulysse

$f \in \mathcal{C}^0(\mathbb{R},\mathbb{R})$

$$y'' + 2y' + 2y = f(t) \quad (E)$$

Soit y_1 et y_2 sol° périodiques de (E)

$$(y_1 - y_2)'' + 2(y_1 - y_2)' + 2(y_1 - y_2) = 0$$

$$r^2 + 2r + 2 = 0$$

$$\Delta = 4 - 4\times 2 = -4 \rightarrow \frac{-2 \pm i\sqrt{4}}{2} = -1 \pm i$$

$$y_H(t) = Ae^{-t+it} + Be^{-t-it} = e^{-t}(Ae^{it} + Be^{-it})$$

Or y_2, y_1 bornées $\Rightarrow y_1 - y_2$ bornée

$$\Rightarrow t \mapsto e^{-t}(Ae^{it} + Be^{-it}) \text{ bornée}$$

$$\Rightarrow \boxed{A = B = 0}$$

Ex 1378 Augustin

$$f : \begin{cases} E \to \mathbb{R} \\ t \mapsto f(t) \end{cases} \quad C^1$$

a) Soit $x, y \in E$

$$(\nabla f(x) | y - x) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(x)(y_i - x_i)$$

$$\psi : t \mapsto (1-t)f(x) + tf(y) - \frac{t(1-t)}{2}\|y - x\|^2$$

$$\varphi : \begin{cases} [0,1] \to E \\ t \mapsto (1-t)x + ty \end{cases}$$