Ex 1430 Martin

1) $\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$

$$\begin{cases} u = x + ct \\ v = x - ct \end{cases} \longrightarrow \quad x = \frac{u+v}{2} \qquad t = \frac{u-v}{2c}$$

$g(u,v) = f(x,y)$

RdlC $\quad \frac{\partial g}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial t}\frac{\partial t}{\partial u} = \frac{1}{2}\frac{\partial f}{\partial x} + \frac{1}{2c}\frac{\partial f}{\partial t}$

$$\frac{\partial g}{\partial v} = \frac{1}{2}\frac{\partial f}{\partial x} - \frac{1}{2c}\frac{\partial f}{\partial t}$$

$$\frac{\partial^2 g}{\partial u \partial v} = \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + \cancel{\frac{1}{4c}\frac{\partial^2 f}{\partial x \partial t}} - \cancel{\frac{1}{4c}\frac{\partial^2 f}{\partial x \partial t}} - \frac{1}{4c^2}\frac{\partial^2 f}{\partial t \partial t}\right)$$

$$= \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}\right) = 0$$

$\frac{\partial^2 g}{\partial v \partial u} = \frac{\partial^2 g}{\partial u \partial v} = 0 \rightarrow \frac{\partial g}{\partial u} = C_1(u)$

, $g(u,v) = D_1(u) + k_1(v)$

où D_1 primitive de C_1

$\rightarrow \frac{\partial g}{\partial v} = C_2(v) \rightarrow g(u,v) = D_2(v) + k_2(u)$

où D_2 primitive de C_1

Ainsi, $g(u,v) = D(v) + k(u)$

donc $f(x,t) = D(x - ct) + k(x + ct)$

2) $\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = \frac{1}{\sqrt{x^2 - c^2 t^2}}$

$\rightarrow \frac{\partial^2 g}{\partial u \partial v} = \frac{1}{4\sqrt{uv}} \longrightarrow g(u,v) = 4\sqrt{uv}$

$f(x,t) = D(x + ct) + B(x - ct) + 4\sqrt{x^2 - c^2 t^2}$