Ex 1385 Ulysse

1) $\begin{cases} u(x,y) = x+y \\ v(x,y) = -x+y \end{cases}$

2) $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) = \frac{\partial^2 v}{\partial x \partial y}$

$\frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 v}{\partial x \partial y}$ (par Schwarz)

Donc $\Delta u = 0$, idem $\Delta v = 0$

3) $f(r) = \frac{1}{2\pi}\int_0^{2\pi} u(r\cos\theta, r\sin\theta)\, d\theta$

En supposant pouvoir dériver sous l'$\int$:

$f'(r) = \frac{1}{2\pi}\int_0^{2\pi}\left(\cos\theta \frac{\partial u}{\partial x} + \sin\theta \frac{\partial u}{\partial y}\right) d\theta$ u est tjs C^2, on peut rédériver.

$f''(r) = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta \frac{\partial^2 u}{\partial x^2} + 2\cos\theta\sin\theta \frac{\partial^2 u}{\partial x \partial y} + \sin^2\theta \frac{\partial^2 u}{\partial y^2}\, d\theta$

pas terminé.