128 D — a) et b) produits de Cauchy

(LUCAS)

c) • $\mathbb{E}(X_x) = \sum_{n=0}^{+\infty} \frac{n b_n x^n}{f(x)} = \frac{x}{f(x)} \sum_{n=1}^{+\infty} n b_n x^{n-1} = \frac{x}{f(x)} f'(x)$

et $f'(x) = P'(x) e^{P(x)} = P'(x) f(x)$

donc $\mathbb{E}(X_x) = x P'(x)$

• $\mathbb{E}(X_x^2) = \sum_{n=0}^{+\infty} \frac{n^2 b_n x^n}{f(x)} = \frac{x^2}{f(x)} \sum_{n=2}^{+\infty} n(n-1) b_n x^{n-2} + \frac{x}{f(x)} \sum_{n=1}^{+\infty} n b_n x^{n-1}$

$= \frac{x^2}{f(x)} f''(x) + \frac{x}{f(x)} f'(x)$

Or $f''(x) = P''(x) f(x) + P'(x) f'(x)$ ~~$= (f''(x) + P'^2(x$~~

$= (P''(x) + P'(x)^2) f(x)$

~~$\mathbb{E}(X_x^2) = x^2(P''(x) + P$~~

$\mathbb{E}(X_x^2) = x^2 (P''(x) + P'(x)^2) + x P'(x)$

$\mathbb{V}(X_x) = x^2 P''(x) + x P'(x)$

d) mq $\frac{g_d(x)}{f(x)} \xrightarrow[x \to +\infty]{} 0$

$\frac{g_d(x)}{f(x)} = \sum_{x \in I_x} \frac{b_n x^n}{f(x)} = \mathbb{P}(X_x \in I_x)$

$= \mathbb{P}(|X_x - x P'(x)| \geq x^d)$

$\overset{IBT}{\leq} \frac{\mathbb{V}(X_x)}{x^{2d}} \sim x^{\deg P - 2d} \xrightarrow[x \to +\infty]{} 0$

d'où $g_d(x) \underset{+\infty}{=} o(f(x))$