Donc $P(S_n \geq a) = P(S_n \leq -a)$

D'où $P(|S_n| \geq a) \leq 2\exp\left(-\frac{a^2}{2\sigma_n^2}\right)$

c) $(X_k)_{k \in [\![1, 10^6]\!]}$

X_k k-ième lancer : $X_k \sim U\{-1, 1\}$

$(X=1) = P$

$(X=-1) = F$

$$X = \sum_{k=1}^{10^6} X_k$$

$P(\text{au moins } 10\,000\ P) = P(X \geq \underbrace{20000}_{=a=2\times10^4})$

$P(X \geq a) \leq \exp\left(-\frac{a^2}{2\sigma_n^2}\right)$

$$\sigma_n^2 = \sum_{k=1}^{10^6} 1 = 10^6$$

$$= \exp\left(\frac{-4\times10^8}{2\times10^6}\right) = \exp(-200)$$

1237 Théodore

Rose n'est pas fané au bout de n j

$Y = X - n$, $X > n$

$E(Y)$?

$$P(Y=k) = P(X=n+k \mid X>n) = \frac{P(X=n+k)}{P(X>n)}$$

$$= \frac{\frac{1}{2^{n+k}}}{\sum_{j=n+1}^{+\infty} \frac{1}{2^j}} = \frac{1}{2^k}$$

$Y \sim \mathcal{G}\left(\frac{1}{2}\right)$ $\quad \underline{E(Y) = 2}$

1326 Anaïs

$X \perp\!\!\!\perp Y$

$$P(X = k \mid Y + X = n) \overset{X \perp\!\!\!\perp Y}{=} \frac{P((X=k) \cap (Y+X=n))}{P(Y+X=n)}$$

$$= \frac{P(X=k) \times P(Y=n-k)}{P(Y+X=n)}$$

or $X + Y \sim \mathcal{P}(\mu + \lambda)$

$$= \frac{e^{-\lambda} \frac{\lambda^k}{k!} \times e^{-\mu} \frac{\mu^{n-k}}{(n-k)!}}{e^{-\lambda-\mu} \frac{(\lambda+\mu)^n}{n!}} = \binom{n}{k} \frac{\lambda^k \mu^{n-k}}{(\mu+\lambda)^n}$$

1238 Nathanaël

Mq $\displaystyle\sum_{k=n+1}^{3n-1} \binom{4n}{k} \geq \frac{n-1}{n} 2^{4n} \iff \sum_{k=n+1}^{3n+1} \binom{4n}{k} \left(\frac{1}{2}\right)^{4n} \geq \frac{n-1}{n}$

$$\iff \sum_{k=n+1}^{3n-1} \binom{4n}{k} \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{4n-k} \geq \frac{n-1}{n}$$

BT : $P(|X - E(X)| \geq \varepsilon) \leq \dfrac{V(X)}{\varepsilon^2}$

On considère $X \sim \mathcal{B}\left(4n, \frac{1}{2}\right)$, alors :

$E(X) = 4n \times \frac{1}{2} = 2n \qquad V(X) = 4n \times \frac{1}{2}\left(1 - \frac{1}{2}\right) = n$

On applique BT à X, $\varepsilon = n$: