$$-\frac{1}{2}\arcsin\left(\frac{\sqrt{x}}{2}\right)$$

donc $= \pm\frac{1}{2}\frac{\sqrt{1-x/4}}{\frac{\sqrt{x}}{2}} = \pm\frac{1}{2}\sqrt{\frac{4-x}{x}} - \frac{1}{2}\arcsin\left(\frac{\sqrt{x}}{2}\right)$

$$-\frac{1}{2}\arcsin\left(\frac{\sqrt{x}}{2}\right).$$

$$y(x) = \left(\pm\frac{1}{2}\sqrt{\frac{4-x}{x}} + C\right)\frac{\sqrt{x}}{|x-4|^{3/2}}$$

$$y(x) = \pm\frac{1}{2}\cdot\frac{1}{4-x} + C\frac{\sqrt{x}}{(4-x)^{3/2}}$$

~~On doit avoir $f(0) = 1 \Rightarrow$ fixe C ?~~

<u>Ex 1171</u> : $\sum_{n\geq 0} tr(A^n) z^n$ (Algèbre)

$|tr(A^n)| \leq n \cdot \|A^n\|_\infty \leq \cancel{n \times C|||A^n|||}$

~~avec $|||A||| = \sup \frac{\|Ax\|}{\|x\|}$~~ $= m$

En trigonalisant A (possible dans $\mathbb{C}$).

$$|tr(A^n)| \leq n\Big(\underbrace{\sup_{\lambda\in Sp(A)}|\lambda|}_{=M}\Big)^n.$$

donc pour $|z| < \frac{1}{M}$, $|zM| < 1$.

donc $R = Rcv(\Sigma) \geq \frac{1}{M}$.

$\chi_A(x) = \det(xI_p - A)$.

$$(xI_p - A)^{-1} = \frac{1}{x}\left(I_p - \frac{A}{x}\right)^{-1} = \frac{1}{x}\sum_{n=0}^{+\infty}\left(\frac{A}{x}\right)^n$$

(pour $|x| > M$)

Soit z tq $|z| < \frac{1}{M}$.

$$z(I_p - zA)^{-1} = \sum_{n=0}^{+\infty} A^n z^n.$$

$$Tr(z(I_p - zA)^{-1}) = \sum_{m=0}^{+\infty} tr(A^n) z^m$$

$$\boxed{R = \frac{1}{M}}$$

$$\chi_M(x) = x^m - tr(M)x^{p-1} + \dots + (-1)^p \det(M)^p.$$

$$\chi_M^{(p)}(0) = -(p-1)!\, tr(M)$$

donc $tr(I_p - zA)^{-1} = \frac{-1}{(p-1)!} \cdot \chi^{(p)}_{(I_p - zA)^{-1}}(0)$.

$$\chi^{(p)}_{(I_p - zA)^{-1}}(0) = \det(0 - (I_p - zA)^{-1})$$

$$= \frac{1}{\det(I_p - zA)} = \frac{1}{\chi_A(\frac{1}{z}) z^n}.$$

$$\sum_{n=0}^{+\infty} Tr(A^n) z^n = z \times \frac{-1}{(p-1)!} \times \frac{1}{\chi_A(\frac{1}{z}) z^n}.$$

<u>version prof :</u>

$A = PTP^{-1}$ $T = \begin{bmatrix} \lambda_1 & & * \\ & \ddots & \\ & & \lambda_p \end{bmatrix}$ $A^n = PT^nP^{-1}$

$$tr(A^n) = \lambda_1^n + \dots + \lambda_p^n.$$

$$\sum_{n \geq 0} tr(A^n) = \sum_{n \geq 0} (\lambda_1 z)^n + \dots + (\lambda_p z)^n$$

CV si $\left.\begin{array}{l} |z\alpha_1| < 1 \\ \vdots \\ |z\alpha_p| < 1 \end{array}\right\} \Leftrightarrow |z| < \min_{i \in [\![1,p]\!]} \frac{1}{\alpha_i}$

$$= \frac{1}{\max_i |\alpha_i|}$$

$$\sum \operatorname{tr}(A^n) z^n = \sum_{i=1}^{p} \frac{1}{1-\alpha_i z} = f(z)$$

$$R \geq \frac{1}{\max|\alpha_i|} = \alpha.$$

<u>Si $R > \alpha$</u> : $\Rightarrow f(z)$ serait continue sur $D(0,R)$

et donc en $\alpha = \frac{1}{|\alpha_{i_0}|}$

or en $z \to \frac{1}{\alpha_{i_0}}$, $|f(z)| \to +\infty$.

$\Rightarrow$ contradict°.

$$R = \alpha = \frac{1}{\max|\alpha_i|}$$

$$\chi_A(z) = \prod_{i=1}^{p} (z - \alpha_i) \qquad \frac{\chi_A'(z)}{\chi_A(z)} = \sum_{i=1}^{p} \frac{1}{z - \alpha_i}$$

$$\frac{\chi_A'(1/z)}{\chi_A(\frac{1}{z})} = \sum_{i=1}^{p} \underbrace{\frac{1}{\frac{1}{z} - \alpha_i}}_{= \frac{z}{1 - z\alpha_i}}$$

donc $f(z) = \dfrac{\chi_A'(1/z)}{z\,\chi_A(1/z)}$.