

Exercice 1.116 (Quentin)

(a)

(b) $M_q \forall m,$

$$|a_m| \leq \frac{1}{2} \sum_{k=1}^m \frac{|a_{m-k}|}{k!}$$

$$\leq \frac{1}{2} \sum_{k=1}^m \frac{1}{k!} \leq \frac{1}{2} (e^1 - 1) \leq 1$$

(c) $f(z)e^z$

$$C_m = \sum_{k=0}^m a_{m-k} \frac{1}{k!} = a_0 + \underbrace{\sum_{k=1}^m \frac{a_{m-k}}{k!}}_{2a_m}$$

$$C_m = -a_m \text{ si } m \geq 1$$

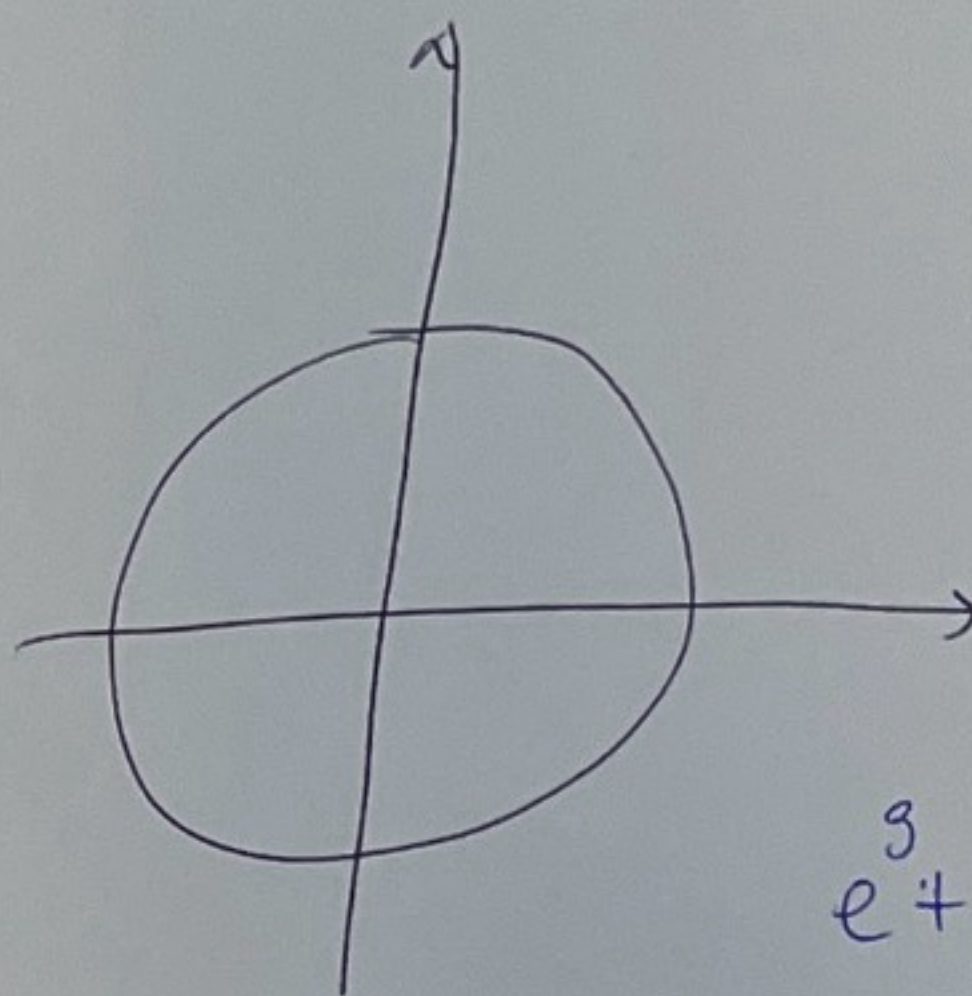
$$C_0 = a_0$$

$$f(z)e^z = \sum_{n=0}^{\infty} C_n z^n$$

$$= a_0 - \underbrace{\sum_{n=1}^{\infty} a_n z^n}_{f(z) - a_0}$$

$$f(z)e^z = 2a_0 - f(z)$$

$$f(z) = \frac{2}{e^z - 1}$$



$$e^z + 1 = 0$$

$$z = i n \pi$$

n impair

$$R = (n\pi) = \pi$$

(On montre $RCV \leq \pi$)