Donc $f(x) = \sum_{n=1}^{\lfloor \frac{\pi}{2x} \rfloor} + \sum_{\lfloor \frac{\pi}{2x} \rfloor + 1}^{+\infty} \geq \underbrace{\sum_{n=1}^{\lfloor \frac{\pi}{2x} \rfloor} \frac{4}{\pi^2} \frac{\cancel{x^2} x^2}{\cancel{x^2}}}_{= \sum_{n=1}^{\lfloor \frac{\pi}{2x} \rfloor} \frac{4}{\pi^2} x^2}$

$$f(x) \geq \left\lfloor \frac{\pi}{2x} \right\rfloor \frac{4x^2}{\pi^2}$$

Donc $\forall x > 0$, $\frac{f(x)}{x} \geq \underbrace{\left\lfloor \frac{\pi}{2x} \right\rfloor}_{\underset{x \to 0}{\sim} \frac{\pi}{2x}} \frac{4x}{\pi^2}$

$\frac{f(x)}{x} \geq g(x)$ avec $g(x) \sim \frac{2}{\pi}$

$\Longrightarrow \cancel{\frac{f(x)}{x} \xrightarrow[x \to 0^+]{} 0}$

$\Longrightarrow f$ non dérivable en 0 car $\left\{ \begin{array}{l} f \text{ paire} \\ f \text{ dérivable} \end{array} \right\} \Rightarrow \underline{f'(0) = 0}$

<u>Ex 973</u> Luca

$f \in \mathcal{C}^0(\mathbb{R}, \mathbb{R})$, $a_n = \int_1^{1+\frac{1}{n}} f(t^n)\,dt$

a) <u>Déf</u> $u = t^n$, $du = n t^{n-1} dt = n u^{\frac{n-1}{n}} dt$

C^1 bijection

Alors $a_n = \int_1^{(1+\frac{1}{n})^n} \frac{1}{n} f(u) u^{\frac{n}{n-1}} du$

$\forall n \in \mathbb{N}^*$, $\left(1+\frac{1}{n}\right)^n = e^{n \ln(1+\frac{1}{n})} \leq e^{n \frac{1}{n}} \leq e$

On a donc $\left[1, \left(1+\frac{1}{n}\right)^n\right] \subset [1, e]$ $\forall n \in \mathbb{N}^*$

$\forall n \in [1, e]$

$|a_n| \leq \frac{1}{n} \int_1^{(1+\frac{1}{n})^n} \underbrace{|f(u)|}_{\leq \|f\|_\infty^{[1,e]}} \underbrace{u^{\frac{n}{n-1}}}_{\leq e^{\frac{n}{n-1}}} dt \leq \frac{1}{n} (e-1) e^{\frac{n}{n-1}} \|f\|_\infty^{[1,e]} \xrightarrow[n \to \infty]{} 0$

<u>Équivalent</u> en $n \to +\infty$

$[1,e] \to \mathbb{R}$

Posons : $v_n(x) = \begin{cases} f(x)\, x^{\frac{n}{n-1}} & \text{si } x \in \left[1, \left(1+\frac{1}{n}\right)^n\right] \\ 0 & \text{sinon} \end{cases}$

- $\forall t \in [1,e]$

$$0 \leq v_n(t) \leq f(t)\, e^{\frac{n}{n-1}} \underbrace{\leq \|f\|_\infty^{[1,e]}\, e^{\frac{n}{n-1}}}_{\text{Intégrable sur } [1,e]}$$

- $v_n \xrightarrow{CVS} x f(x)$

Par TCD, $a_n = \frac{1}{n} \int_1^e v_n(t)\,dt \longrightarrow \underbrace{\int_1^e x f(x)\,dx}_{=A} = A$

$$\boxed{a_n \sim \frac{A}{n}}$$