$\boxed{938}$ (Anouk)

$f : x \mapsto \dfrac{e^{-nx}\cos(x)}{\sqrt{x}}$ continue sur $\mathbb{R}_+^*$.

en 0 : $f(x) \sim \dfrac{1}{\sqrt{x}}$ et $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$ CV (Riemann)

en $+\infty$: $|f(x)| \leq \dfrac{e^{-nx}}{\sqrt{x}} \underset{+\infty}{=} o\left(\dfrac{1}{x^2}\right)$ (Riemann)

$u = nx$
$du = n\,dx$: $\displaystyle I_n = \int_0^{+\infty} \frac{e^{-u}\cos(u/n)}{\sqrt{u/n}} \frac{du}{n}$

$$= \frac{1}{\sqrt{n}} \int_0^{+\infty} \underbrace{\frac{e^{-u}\cos(u/n)}{\sqrt{u}}}_{g_n(u)} du$$

$\forall n$, $|g_n(u)| \leq \dfrac{e^{-u}}{\sqrt{u}} = \Gamma(1/2)$ } intégrable. $\left(\Gamma(x) = \displaystyle\int_0^{+\infty} e^{-t} t^{x-1} dt\right)$

$g_n(u) \underset{n\infty}{\longrightarrow} \dfrac{e^{-u}}{\sqrt{u}}$.

$I_n \underset{n\infty}{\sim} \dfrac{\Gamma(1/2)}{\sqrt{n}}$ Par théorème de convergence dominée.

$$\int_0^{+\infty} e^{-x^2} dx = \int_0^{+\infty} e^{-u} \frac{du}{2\sqrt{u}} = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}.$$

$u = x^2$
$\sqrt{u} = x$
$\dfrac{du}{2\sqrt{u}} = dx$

donc $\Gamma(1/2) = \sqrt{\pi}$

d'où $I_n \underset{n\infty}{\sim} \sqrt{\dfrac{\pi}{n}}$

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