Olivier $\boxed{\text{ex. 720}}$ Soit $(x,y) \in]0,1]^2$,

mq $\left(\frac{x+y}{2}\right)^{x+y} \leq x^x y^y$

On utilise la concavité de ln :

$$\ln\Big(\underbrace{\frac{x}{x+y}}_{=t}\, y + x\, \underbrace{\frac{y}{x+y}}_{=1-t}\Big) \geq \frac{x}{x+y}\ln y + \frac{y}{x+y}\ln x$$

$$\Rightarrow \ln\left(\frac{2xy}{x+y}\right) \geq \frac{1}{x+y}\ln\left(y^x\right) + \frac{1}{x+y}\ln\left(x^y\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2xy}\right) \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{x^y}\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2}\right) + \ln\frac{1}{xy} \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{xy}\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2}\right) \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{x^y}\right) + \ln(xy)$$

$$\Rightarrow \ln\left(\left(\frac{x+y}{2}\right)^{x+y}\right) \leq \ln\left(\frac{1}{y^x}\right) + \ln\left(\frac{1}{x^y}\right) + \ln\left(x^{x+y}y^{x+y}\right)$$

$$\Rightarrow \ln\left(\left(\frac{x+y}{2}\right)^{x+y}\right) \leq \ln\left(\frac{y^{x+y}}{y^x}\right) + \ln\left(\frac{x^{x+y}}{x^y}\right)$$

$$\Rightarrow \left(\frac{x+y}{2}\right)^{x+y} \leq x^x y^y \quad \text{(par l'exp.)}$$

Arthur $\boxed{\text{ex. 734}}$ $f : \mathbb{R}_+^* \to \mathbb{R}_+^*$, $\frac{f'(x)}{f(x)} \xrightarrow[+\infty]{} -\infty$

$$g(x) = \ln(f(x))$$

$$g'(x) = \frac{f'(x)}{f(x)}$$

$$g(n+1) - g(n) = \ln\left(\frac{f(n+1)}{f(n)}\right)$$

$\exists c_n \in]n, n+1[$

$$g'(c_n) = \frac{g(n+1)-g(n)}{n+1-n} = \ln\left(\frac{f(n+1)}{f(n)}\right)$$

Or, $\lim_{n\to+\infty} g'(c_n) = -\infty$

donc $\ln\left(\frac{f(n+1)}{f(n)}\right) \longrightarrow -\infty$

donc $\frac{f(n+1)}{f(n)} \longrightarrow 0$

$\sum f(n)$ converge par d'Alembert

Martin ex 753

$T \mid [0, 2\pi] \to \mathbb{R}$

$\lambda \longrightarrow T(\lambda)$

$g(\lambda) = T(\pi+\lambda) - T(\lambda)$

$g(\pi) = T(2\pi) - T(\pi)$

$g(0) = T(\pi) - \underbrace{T(0)}_{=T(2\pi)} = -g(\pi)$

$\rightarrow g(0) = 0$, $(0, \pi)$ convient $\hat{m}$ T^{ure}

$\longrightarrow$ Wlog $g(0) > 0$ donc $g(\pi) < 0$

g continue donc par TVI, $\exists \lambda_0 \in [0, \pi]$, $g(\lambda_0) = 0$

$T(\pi+\lambda_0) = T(\lambda_0)$

Nathanaël $\boxed{\text{ex. } 718}$ $a < b \in \mathbb{R}$, $f, g \in \mathcal{C}^0([a,b], \mathbb{R})$, $g > 0$

Mq $\exists c \in [a,b]$ tq $\int_a^b f(t)g(t)\,dt = f(c)\int_a^b g(t)\,dt$

Par le th. des bornes atteintes, $\exists m \in [a, b]$ tq $f(m) = \min_{[a,b]} f$

$\exists M \in [a,b]$ tq $f(M) = \max_{[a,b]} f$