Nathanaël $\boxed{\text{ex. 719}}$

a/ f C^0 surjective $[0,1] \to]0,1[$?

Th. des bornes atteintes, on a $M < 1$ tq $\forall x \in [0,1]$,

$f(x) \leq M < 1$

Donc f n'atteint pas $]M, 1[\neq \emptyset$

Donc NON

b/ f C^0 surjective $]0,1[\to [0,1]$

$f : x \to \frac{1}{2}(\sin(2\pi x) + 1)$

$f(\frac{1}{4}) = 1$

$f(3/4) = 0$

OK

Arthur $\boxed{\text{ex. 726}}$ f C^1 de $\mathbb{R}_+ \to \mathbb{R}$

a/ $\ln x \to +\infty$

et $(\ln x)' \to 0$ $\boxed{\text{NON}}$

b/ $f(x) = \frac{\sin(x^4)}{x} \to 0$

$f'(x) = \underbrace{4x\cos(x^4)x^2}_{\text{dv}} - \underbrace{\frac{\sin x^4}{x^2}}_{\to 0}$ $\boxed{\text{NON}}$

c/ $f(x) \to a$

donc $f(x+1) - f(x) \to 0$

Th. acc. finis entre x et $x+1$, $\exists c_x \in]x, x+1[$ tq

$f(c_x) = f(x+1) - f(x)$

quand $x \to +\infty$,

$c_x \to +\infty$

et $f'(c_x) \to b$

Or $f'(c_x) = f(x+1) - f(x) \to 0$

donc $b = 0$

d/ $g(x) = f(x) + f'(x)$

$$e^{x} g(x) = e^{x} f(x) + e^{x} f'(x)$$

$$e^{x} g(x) = \left(e^{x} f(x)\right)'$$

$$e^{x} f(x) - f(0) = \int_0^x e^{t} g(t)\, dt$$

$$f(x) = f(0) e^{-x} + e^{-x} \int_0^x e^{t} g(t)\, dt$$

$$g(t) = l + \underbrace{\varepsilon(t)}_{\to 0}$$

$$f(x) = f(0) e^{-x} + e^{-x} l (e^{x} - 1) + e^{-x} \int_0^x e^{t} \varepsilon(t)\, dt$$

Soit $\varepsilon > 0$.

$$|f(x) - l| \leq \underbrace{|f(0) e^{-x}| + |e^{-x} l|}_{\to 0 \text{ donc } \leq \varepsilon \text{ APCR } x_1} + e^{-x} \int_0^x e^{t} \underbrace{|\varepsilon(t)|}_{\leq \varepsilon \text{ APCR } x_2} dt$$

$$|f(x) - l| \leq \varepsilon + \underbrace{e^{-x} \int_0^{x_2} e^{t} \varepsilon}_{\to 0 \text{ donc } \leq \varepsilon \text{ APCR } x_3} + \underbrace{e^{-x} \int_{x_2}^x e^{t} \varepsilon}$$

$$e^{-x}(e^{x} - e^{x_2})\varepsilon = \varepsilon - \varepsilon e^{x_2 - x} \leq \varepsilon$$

$|f(x) - l| \leq 3\varepsilon$ pour $x \geq \max(x_2, x_3)$

$f \to l$

et $f + f' \to l$ donc $f' \to 0$