Martin $\boxed{\text{ex. 722}}$

$$f(f(x)) = 2x+1$$
$$f(f(f(x))) = f(2x+1)$$
$$f(f(f(x))) = 2f(x)+1$$

Analyse $\forall x \in \mathbb{R},\ f(2x+1) = 2f(x)+1$

$$\Rightarrow f'(2x+1) = f'(x)$$

donc $f'(2x+1) = f'(x) = f'\left(\frac{x-1}{2}\right) = f'\left(\frac{\frac{x-1}{2}-1}{2}\right)$

$$= f'\left(\frac{x-3}{4}\right)$$

$$= f'\left(\frac{x-2^n+1}{2^n}\right)$$

$$\rightarrow f'(-1)$$

Synthèse $\hookrightarrow f(x) = \lambda x + b$

$$\lambda(\lambda x + b) + b = 2x+1$$

$\Rightarrow \quad \lambda = \pm\sqrt{2} \quad$ et $\quad b = \frac{1}{\lambda+1} = \frac{1}{1\pm\sqrt{2}}$

Donc $\| f(x) = \varepsilon\sqrt{2}\,x + \frac{1}{1+\varepsilon\sqrt{2}}$ avec $\varepsilon \in \{-1, 1\}$

Nathanaël $\boxed{\text{ex. 717}}$

$x_1, \dots, x_k \in \mathbb{R}^{+*}$, Mq $\prod_{i=1}^{k} (1 + x_i^k) \geq \left(1 + \prod_{i=1}^{k} x_i\right)^k \quad (*)$

$$(*) \Leftrightarrow \sum_{i=1}^{k} \ln(1+x_i^k) \geq k \ln\left(1 + \prod_{i=1}^{k} x_i\right)$$

Or, $\frac{1}{k}\sum_{i=1}^{k} \ln(1+x_i^k) \geq \ln\left(\sum_{i=1}^{k} \frac{1}{k}(1+x_i^k)\right)$

$$\geq \ln\left(1 + \frac{\sum_{i=1}^{k} x_i^k}{k}\right)$$

(exercice non fini)

$$\geq \ln\left(1 + \left(\prod_{i=1}^{k} x_i^k\right)^{1/k}\right)$$

$$= \ln\left(1 + \prod_{i=1}^{k} x_i\right)$$