Olivier $\boxed{\text{ex. 720}}$ Soit $(x,y) \in]0,1]^2$,

mq $\left(\frac{x+y}{2}\right)^{x+y} \leq x^x y^y$

On utilise la concavité de ln :

$$\ln\left(\underbrace{\frac{x}{x+y}}_{=t}\, y + x\, \underbrace{\frac{y}{x+y}}_{=1-t}\right) \geq \frac{x}{x+y}\ln y + \frac{y}{x+y}\ln x$$

$$\Rightarrow \ln\left(\frac{2xy}{x+y}\right) \geq \frac{1}{x+y}\ln\left(y^x\right) + \frac{1}{x+y}\ln\left(x^y\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2xy}\right) \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{x^y}\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2}\right) + \ln\frac{1}{xy} \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{x^y}\right)$$

$$\Rightarrow \ln\left(\frac{x+y}{2}\right) \leq \frac{1}{x+y}\ln\left(\frac{1}{y^x}\right) + \frac{1}{x+y}\ln\left(\frac{1}{x^y}\right) + \ln(xy)$$

$$\Rightarrow \ln\left(\left(\frac{x+y}{2}\right)^{x+y}\right) \leq \ln\left(\frac{1}{y^x}\right) + \ln\left(\frac{1}{x^y}\right) + \ln\left(x^{x+y}\, y^{x+y}\right)$$

$$\Rightarrow \ln\left(\left(\frac{x+y}{2}\right)^{x+y}\right) \leq \ln\left(\frac{y^{x+y}}{y^x}\right) + \ln\left(\frac{x^{x+y}}{x^y}\right)$$

$$\Rightarrow \left(\frac{x+y}{2}\right)^{x+y} \leq x^x y^y \quad \text{(par l'exp.)}$$

Arthur $\boxed{\text{ex. 734}}$ $f : \mathbb{R}_+^* \to \mathbb{R}_+^*$, $\frac{f'(x)}{f(x)} \xrightarrow[+\infty]{} \infty$

$g(x) = \ln(f(x))$

$g'(x) = \frac{f'(x)}{f(x)}$

$g(n+1) - g(n) = \ln\left(\frac{f(n+1)}{f(n)}\right)$