

$$\exists c_n \in]n, n+1[$$

$$g'(c_n) = \frac{g(n+1) - g(n)}{n+1 - n} = \ln\left(\frac{f(n+1)}{f(n)}\right)$$

$$\text{Or, } \lim_{n \rightarrow \infty} g'(c_n) = -\infty$$

$$\text{donc } \ln\left(\frac{f(n+1)}{f(n)}\right) \rightarrow -\infty$$

$$\text{donc } \frac{f(n+1)}{f(n)} \rightarrow 0$$

$\sum f(n)$ converge par d'Alembert

Martin

753



$$T: [0, 2\pi] \rightarrow \mathbb{R}$$

$$\lambda \rightarrow T(\lambda)$$

$$g(\lambda) = T(\pi + \lambda) - T(\lambda)$$

$$g(\pi) = T(2\pi) - T(\pi)$$

$$g(0) = T(\pi) - T(0) = -g(\pi)$$

$$= -T(2\pi)$$

$$\rightarrow g(0) = 0, (0, \pi) \text{ convexe } \sim \text{Tare}$$

$$\rightarrow \text{wlog } g(0) > 0 \text{ avec } g(\pi) < 0$$

$$g \text{ continue donc par IVI, } \exists \lambda_0 \in [0, \pi], g(\lambda_0) = 0$$

$$T(\pi + \lambda_0) = T(\lambda_0)$$

Nathanaël

ex. 718

$$a < b \in \mathbb{R}, f, g \in \mathcal{C}^0([a, b], \mathbb{R}), g > 0$$

$$\forall c \in [a, b] \text{ tel } \int_a^b f(t)g(t)dt = f(c) \int_a^b g(t)dt$$

Par le th. des bornes atteintes, $\exists m \in [a, b]$ tel $f(m) = \min_{[a, b]} f$

$$\exists M \in [a, b] \text{ tel } f(M) = \max_{[a, b]} f$$

Suite 718 Nathanaël :

$$h: x \rightarrow f(x) \int_a^b g(t) dt$$

$$g \geq 0 \text{ donc } \int_a^b g(t) dt \geq 0 \text{ et}$$

$$\underbrace{\int_a^b f(m) g(t) dt}_{h(m)} \leq \int_a^b f(t) g(t) dt \leq \underbrace{\int_a^b f(M) g(t) dt}_{h(M)}$$

h est li donc par le TVI, on dispose de $c \in [a, b]$ tq
 $h(c) = \int_a^b f(t) g(t) dt$

Olivier

Ex. 715

$$I = \int_0^{\pi/4} \ln(1 + \tan t) dt$$

$$= \int_0^{\pi/4} \ln\left(\frac{\sin t + \cos t}{\cos t}\right) dt$$

$$= \int_0^{\pi/4} \ln(\sin t + \cos t) dt - \int_0^{\pi/4} \ln(\cos t) dt$$

$$= \int_0^{\pi/4} \ln\left(\sqrt{2} \left(\frac{\sqrt{2}}{2} \sin t + \frac{\sqrt{2}}{2} \cos t\right)\right) dt - \int_0^{\pi/4} \ln(\cos t) dt$$

$$= \int_0^{\pi/4} \ln \sqrt{2} dt + \int_0^{\pi/4} \ln\left(\cos\left(t - \frac{\pi}{4}\right)\right) dt - \int_0^{\pi/4} \ln(\cos t) dt$$

$$u = t - \frac{\pi}{4}$$

$$du = dt$$

$$I = \frac{\ln 2}{2} \times \frac{\pi}{4} + \int_{-\pi/4}^0 \ln(\cos(u)) du - \int_0^{\pi/4} \ln(\cos t) dt$$

$$= \frac{\ln(2) \times \pi}{8}$$