Suite $\boxed{7.18}$ Nathanaël :

$h : x \longrightarrow f(x) \int_a^b g(t)\,dt$

$g \geq 0$ donc $\int_a^b g(t)\,dt \geq 0$ et

$$\underbrace{\int_a^b f(m) g(t)\,dt}_{h(m)} \leq \int_a^b f(t) g(t)\,dt \leq \underbrace{\int_a^b f(M) g(t)\,dt}_{h(M)}$$

h est C^0 donc par le TVI, on dispose de $c \in [a,b]$ tq
$h(c) = \int_a^b f(t) g(t)\,dt$

Olivier $\boxed{\text{Ex. } 7.15}$

$$I = \int_0^{\pi/4} \ln(1+\tan t)\,dt$$

$$= \int_0^{\pi/4} \ln\left(\frac{\sin t + \cos t}{\cos t}\right) dt$$

$$= \int_0^{\pi/4} \ln(\sin t + \cos t)\,dt - \int_0^{\pi/4} \ln(\cos t)\,dt$$

$$= \int_0^{\pi/4} \ln\left(\sqrt{2}\left(\frac{\sqrt{2}}{2}\sin t + \frac{\sqrt{2}}{2}\cos t\right)\right) dt - \int_0^{\pi/4} \ln(\cos t)\,dt$$

$$= \int_0^{\pi/4} \ln\sqrt{2}\,dt + \int_0^{\pi/4} \ln\left(\cos\left(t-\frac{\pi}{4}\right)\right) dt - \int_0^{\pi/4} \ln(\cos t)\,dt$$

$u = t - \frac{\pi}{4}$

$du = dt$

$$I = \frac{\ln 2}{2} \times \frac{\pi}{4} + \int_{-\pi/4}^{0} \ln(\cos(u))\,du - \int_0^{\pi/4} \ln(\cos u)\,du$$

$$= \frac{\ln(2) \times \pi}{8}$$